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EQUILIBRIUM BUNCH DENSITY DISTRIBUTION WITH MULTIPLE ACTIVE AND PASSIVE RF CAVITIES

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Abstract

This paper describes a method to get the equilibrium bunch density distribution with an arbitrary number of active or passive RF cavities in uniform filling. This method is an extension of the one presented by M. Venturini which assumes a passive harmonic cavity and no beam loading in the main RF cavity [1].

INTRODUCTION

In the last decade, following the growing interest for the usage of harmonic cavities (HC) in the light source community, there were many attempts to develop analytic methods to obtain the equilibrium bunch density. First by assuming a uniform filling pattern [1, 2] and then with non-uniform filling pattern [3–5].

All these methods assume that the RF system is composed of two cavities: the main RF cavity (MC) is assumed to be a perfect sine wave (active cavity with no beam loading) and the HC is a passive one (no generator voltage). For most of theses methods, it should be straightforward to extend them to a general case of a RF system with an arbitrary number of cavities with both beam loading and generator voltage, which can then be applied to all cases: MC, passive and active HC.

This paper provides the steps needed to first treat the active HC case and then to add the beam loading of the main cavity to the method developed by Venturini in his paper [1]. Because the IPAC paper format is quite short and that the method described here is an extension, we will use the same notation as in [1] and the reader should refer to that paper for further information on the base method.

The method described here with the general case, considering N active or passive cavities, has been added to `mbtrack2` [6]. Finally, the equilibrium bunch profile obtained with this method for the SOLEIL upgrade case [7] is compared to tracking using `mbtrack2`.

EQUILIBRIUM BUNCH PROFILE

In this section, the method used (corresponding to appendix B of [1]) to obtain the equilibrium bunch profile $\rho_0(z)$ is succinctly presented.

The equations of motions for the synchrotron oscillations are given by:

$$\frac{dz}{dt} = \alpha c \delta, \tag{1}$$

$$\frac{d\delta}{dt} = \frac{eV_{rf}(z) - U_0}{E_0 T_0}, \tag{2}$$

where z is the longitudinal distance from the synchronous particle, α is the momentum compaction factor, E_0 is the reference energy, $\delta = \frac{E-E_0}{E_0}$ is the relative energy deviation, U_0 the losses per turn and V_{rf} is the total RF voltage. From these equations of motions, a scaled potential $u(z)$ can be defined as:

$$u(z) = -\frac{1}{\alpha c \sigma_\delta^2 E_0 T_0} \int^z \left(eV_{rf}(z') - U_0 \right) dz', \tag{3}$$

where e is the elementary charge, σ_δ is the energy spread and T_0 is the revolution period. As we are looking for the equilibrium conditions, we can write the bunch profile at equilibrium $\rho_0(z)$ as:

$$\rho_0(z; F, \Phi) = \frac{e^{-u(z;F,\Phi)}}{\int_{-\infty}^{\infty} e^{u(z';F,\Phi)} dz'}, \tag{4}$$

where F and Φ are bunch form factors which depends on the bunch profile. For which the following normalisation condition is verified:

$$\int_{-\infty}^{\infty} \rho_0(z) dz = \int_{-\infty}^{\infty} \frac{e^{-u(z)}}{\int_{-\infty}^{\infty} e^{u(z')} dz'} dz = 1. \tag{5}$$

In [1], the author assumes the total RF voltage V_{rf} to be the sum of a perfect main RF cavity (neglecting beam loading) providing a voltage $V_1(z)$ and a passive 3rd harmonic cavity so the total RF voltage V_{rf} is:

$$\begin{aligned} V_{rf}(z) &= V_1(z) + V_3(z) \\ &= V_1 \sin(k_1 z + \phi_1) + V_3(z), \end{aligned} \tag{6}$$

where $k_1 = \frac{\omega_1}{c}$ is the RF wave number, ω_1 is the RF frequency and ϕ_1 is the phase of the MC. The expression $V_3(z)$ for the beam loading voltage in the passive 3rd harmonic cavity $V_3(z)$ is detailed in Appendix A of [1] and is given by:

$$V_3(z) = -2 I_{avg} R_s F \cos\psi \cos(k_3 z + \psi - \Phi), \tag{7}$$

where I_{avg} is the total beam current, R_s is the shunt impedance of the cavity, ψ is the cavity tuning angle and $k_3 = \frac{3\omega_1}{c}$ is the third harmonic wave number. This expression assumes that the beam has a uniform filling and that the cavity impedance is narrow impedance described by the resonator model. So the scaled potential $u(z)$ is expressed as:

$$u(z) = \frac{U_0 z}{\alpha c \sigma_\delta^2 E_0 T_0} - \frac{e}{\alpha c \sigma_\delta^2 E_0 T_0} \left[V_1 \int^z \sin(k_1 z' + \phi_1) dz' - 2 I_{avg} R_s F \cos\psi \int^z \cos(k_3 z' + \psi - \Phi) dz' \right]. \tag{8}$$

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After integration, and using the condition $u(z = 0) = 0$ to find the value of the integration constant, it gives:

$$u(z; F, \Phi) = u_0 z + u_1[\cos(k_1 z + \phi_1) - \cos\phi_1] + u_3 \cos\psi F[\sin(k_3 z + \psi - \Phi) - \sin(\psi - \Phi)], \quad (9)$$

where $u_0 = \frac{U_0}{(\alpha\sigma_\delta^2 E_0 C)}$, $u_1 = \frac{eV_1}{(\alpha\sigma_\delta^2 E_0 C k_1)}$, $u_3 = \frac{2eI_{avg}R_s}{(\alpha\sigma_\delta^2 E_0 C k_3)}$ and $C = cT_0$ is the ring circumference. The condition for the energy balance of the synchronous particle, $eV_{rf}(z = 0) = U_0$, also gives us the following equation:

$$V_1 \sin\phi_1 = U_0 + 2I_{avg}R_s F \cos\psi \cos(\psi - \Phi). \quad (10)$$

From there, to be able to obtain ρ_0, one needs to find an expression for F and Φ as $u(z) = u(z, F, \Phi)$. To do this, one can compute the Fourier transform of ρ_0 at the third harmonic k_3:

$$\tilde{\rho}_0(k_3) = F e^{j\Phi} = \int_{-\infty}^{\infty} e^{jk_3 z}\rho_0(z)dz. \quad (11)$$

Taking the real part and the imaginary part of the last equations, together with the condition for the energy balance of the synchronous particle, it allows to numerically compute F, Φ and ϕ_1 for a given R_s and ψ. Then the bunch equilibrium profile ρ_0 is obtained using Eq. (4).

ADAPTATION FOR ACTIVE CAVITIES

The idea of this section is to adapt Venturini's method to be able to use it for active cavities. To achieve this, the results obtained in the previous section are modified in order to take into account this addition. The voltage of the harmonic cavity, now active, is the sum of the generator voltage and of the beam loading part:

$$V_3(z) = V_g \cos(k_3 z + \theta_g) - \frac{2I_{avg}R_s F}{1+\beta}\cos\psi\cos(k_3 z + \psi - \Phi), \quad (12)$$

where V_g is the generator voltage amplitude and θ_g the generator phase of the active HC in the cosine definition (seen from the beam axis in phasor space).

The impedance $Z(\omega)$ of the active HC is slightly changed compared to the impedance of a passive HC, it is now expressed in term of the loaded quality factor $Q_L = Q/(1+\beta)$ instead of the unloaded quality factor Q to take into account the coupling coefficient β:

$$Z(\omega) = \frac{R_s}{1 + jQ_L(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r})}, \quad (13)$$

which reduces the amplitude of the beam loading voltage by a factor $\frac{1}{(1+\beta)}$ and also impact the definition of the tuning angle ψ and:

$$\tan\psi = Q_L\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right). \quad (14)$$

This change introduces a modification of the scaled potential $u(z)$ to a new scaled potential taking into account active HC u_a:

$$u_a(z) = \frac{U_0 z}{\alpha c\sigma_\delta^2 E_0 T_0} - \frac{e}{\alpha c\sigma_\delta^2 E_0 T_0}\Bigg[V_{10}\int^z \sin(k_1 z' + \phi_1)dz' + V_g\int^z \cos(k_3 z' + \theta_g)dz' - \frac{2I_{avg}R_s F}{1+\beta}\cos\psi\int^z \cos(k_3 z' + \psi - \Phi)dz'\Bigg]. \quad (15)$$

Keeping the same definitions for u_0, u_1 and u_3, after integration and using the condition $u_a(z = 0) = 0$ to find the value of the integration constant, it gives:

$$u_a(z) = u_0 z + u_1\Big[\cos(k_1 z + \phi_1) - \cos\phi_1\Big] + \frac{u_3}{1+\beta}F\cos\psi\Big[\sin(k_3 z + \psi - \Phi) - \sin(\psi - \Phi)\Big] - \frac{u_3 V_g}{2I_{avg}R_s}\Big[\sin(k_3 z + \theta_g) - \sin\theta_g\Big]. \quad (16)$$

The new energy balance condition for the synchronous particle, $(z = 0, \delta = 0)$, is given by:

$$V_1\sin\phi_1 + V_g\cos\theta_g = \frac{U_0}{e} + \frac{2I_{avg}R_s F}{1+\beta}\cos\psi\cos(\psi - \Phi). \quad (17)$$

Then using the equations $\tilde{\rho}_0(k_3) = F_3 e^{i\Phi_3}$, it is possible to solve numerically to find θ_g, F, Φ.

GENERAL CASE OF N CAVITIES WITH BEAM LOADING

In this section, we generalize to the case of N active (or passive) cavities with beam loading. The total RF voltage can be expressed as the sum of the generator voltage and of the beam voltage for each cavity:

$$V_{rf}(z) = \sum_{i=1}^{N} V_i(z) = \sum_{i=1}^{N} V_{g_i}\cos(k_i z + \theta_{g_i}) - \frac{2I_{avg}R_{s_i}F_i}{1+\beta_i}\cos\psi_i\cos(k_i z + \psi_i - \Phi_i), \quad (18)$$

where the index i corresponds to quantities for the cavity of the ith cavity.

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This change introduces a modification of the scaled potential $u(z)$ to a new scaled potential u_b:

$$u_b(z) = \frac{U_0 z}{\alpha c \sigma_\delta^2 E_0 T_0} - \frac{e}{\alpha c \sigma_\delta^2 E_0 T_0} \sum_{i=1}^{N} \left[V_{g_i} \int^z \cos(k_i z' + \theta_{g_i}) dz' - \frac{2 I_{avg} R_{s_i} F_i}{1+\beta_i} \cos\psi_i \int^z \cos(k_i z' + \psi_i - \Phi_i) dz' \right], \quad (19)$$

which gives, keeping the same definition of u_0, after integration:

$$u_b(z) = u_0 z + \sum_{i=1}^{N} -u_i \left[\sin(k_i z + \theta_{g_i}) - \sin\theta_{g_i} \right] + u_{b_i} F_i \cos\psi_i \left[\sin(k_i z + \psi_i - \Phi_i) - \sin(\psi_i - \Phi_i) \right], \quad (20)$$

where we introduced the constants $u_i = eV_{g_i}/(\alpha\sigma_\delta^2 E_0 C k_i)$ and $u_{b_i} = 2eI_{avg}R_{s_i}/(\alpha\sigma_\delta^2 E_0 C k_i(1+\beta_i))$.

The new energy balance condition for the synchronous particle, $(z = 0, \delta = 0)$, is given by:

$$\sum_{i=1}^{N} V_{g_i} \cos\theta_{g_i} = \frac{U_0}{e} + \sum_{i=1}^{N} \frac{2 I_{avg} R_{s_i} F_i}{1+\beta_i} \cos\psi_i \cos(\psi_i - \Phi_i) \quad (21)$$

Then using the system of equations $\forall i \in N, \tilde{\rho}_0(k_i) = F_i e^{j\Phi_i}$, it is possible to solve numerically to find θ_1 and all the form factors F_i and Φ_i. This algorithm has been added to the `mbtrack2` collective effect library [6].

COMPARISON WITH TRACKING

Now we compare the bunch profiles obtained by using this method and by tracking in the SOLEIL upgrade case [7]. We consider a normal conducting (active) main cavity and a superconducting passive 3rd harmonic cavity with the parameters in Table 1. The phasor diagram of the main RF cavity is shown in Fig. 1.

The tracking is done using 10^6 macro-particles with `mbtrack2` using the algorithm described in [8]. As shown in Fig. 2, there is a very good agreement between both methods for the two detunning values shown here.

However it is important to stress that finding a solution with this analytical method does not say anything about its stability. The stability of the solution should be checked with a self-consistent method like tracking or Vlasov analysis. But if no solution exists, then the beam cannot be stable with this parameter set.

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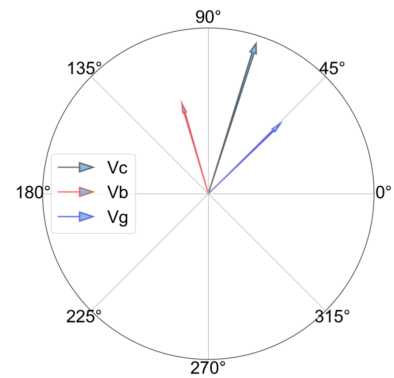

Figure 1: Phasor diagram of the main RF cavity, representing the cavity phasor $\tilde{V}_c = \tilde{V}_b + \tilde{V}_g$, the beam phasor $\tilde{V}_b$ and the generator phasor $\tilde{V}_b$.

Table 1: RF Cavities Parameters

Parameters	MC	HC
Harmonic number m	1	3
Shunt impedance R_s $(M\Omega)$	19.6	9000
Quality factor Q_0	34 000	10^8
Loaded quality factor Q_L	6 000	10^8
Detuning $(f_r - mf_1)$ (kHz)	-100	90 and 85
Generator voltage V_g (MV)	1.05	0
Generator phase θ_g (°)	44.2	0
Beam voltage V_b (MV)	0.97	0.52 and 0.56
Beam phase θ_b (°)	106.3	89.9
Cavity voltage V_c (MV)	1.7	0.52 and 0.56
Cavity phase θ_c (°)	72.4	89.9

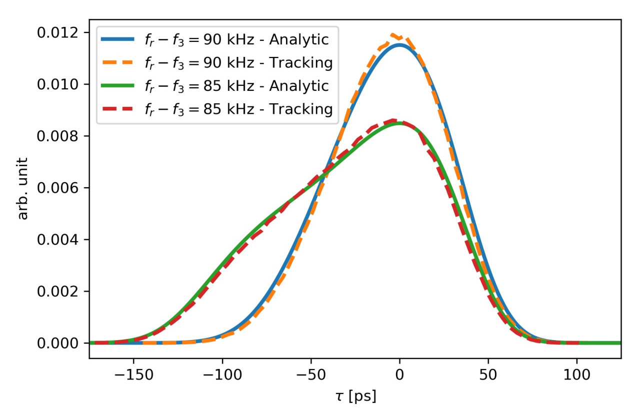

Figure 2: Equilibrium bunch profile for SOLEIL upgrade parameters at 500 mA in uniform filling for two different detunning, $f_r - f_3 = 85$ kHz and 90 kHz. SOLEIL upgrade parameters can be found in [7].

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