

# MONOCHROMATIZATION OF $e^+e^-$ COLLIDERS WITH A LARGE CROSSING ANGLE

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## Abstract

The relative center-of-mass energy spread  $\sigma_W/W$  at  $e^+e^-$  colliders is  $O(10^{-3})$ , which is much larger than the widths of narrow resonances  $J/\psi$ ,  $\psi(2S)$ ,  $\Upsilon(1S)$ ,  $\Upsilon(2S)$  and  $\Upsilon(3S)$  mesons and some others. Its reduction would significantly increase the resonance production rates and open up great opportunities in the search for new physics. In this paper, we propose a new monochromatization method for colliders with a large crossing angle (which can provide a high luminosity). The contribution of the beam energy spread to  $\sigma_W$  is canceled by introducing an appropriate energy–angle correlation at the interaction point;  $\sigma_W/W \sim (3-5) \times 10^{-6}$  appears possible. Limitations of the method are considered.

## INTRODUCTION

The point-like nature of the electron and a narrow energy spread are important advantages of  $e^+e^-$  colliders. The energy spread occurs due to synchrotron radiation (SR) in the rings as well as beamstrahlung (BS) at the IP. The energy spread due to SR depends mainly on the beam energy  $E_0$  and magnetic radius of the ring  $R$ , and only weakly on the specific design of the collider: for rings without damping wigglers  $\sigma_E/E \propto E_0/\sqrt{R}$ . The invariant mass spread for the existing and planned rings  $\sigma_W/W = (1/\sqrt{2})\sigma_E/E \sim (0.35-0.5) \times 10^{-3}$ . This spread is much greater than the widths of the narrow  $e^+e^-$  resonances  $J/\psi$ ,  $\psi(2S)$ ,  $\Upsilon(1S)$ ,  $\Upsilon(2S)$ ,  $\Upsilon(3S)$  and the Higgs boson, see Table 1. The resonance width  $\Gamma$  is the full width at half maximum, so one should compare  $\Gamma/m$  and  $\text{FWHM} = 2.36 \sigma_W/W \approx (0.8-1.2) \times 10^{-3}$ .

Table 1: Width of Some Narrow  $e^+e^-$  Resonances

	$J/\psi$	$\psi(2S)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$H(125)$
$m$ , GeV/ $c^2$	3.097	3.686	9.460	10.023	10.355	125
$\Gamma$ , keV	93	300	54	32	20.3	4200
$\Gamma/m$ , $10^{-5}$	3	8	0.57	0.32	0.2	3.4
$2.36\sigma_W/\Gamma$	$\sim 35$	$\sim 13$	$\sim 180$	$\sim 310$	$\sim 500$	$\sim 30$

Reducing the spread of the invariant mass can significantly increase the production rate of resonances. The study of rare and forbidden processes sensitive to new physics is one of the promising directions for particle physics. Therefore,  $J/\psi$  and  $\Upsilon$  factories with a narrow invariant-mass spread would be good candidates for future experimental facilities. In the absence of a background, for the same luminosity the monochromatization lowers the branching limit proportionally to  $\sigma_W$ . In the case of a large continuum background (such as the tauonium production), the signal-to-noise ratio

$S/\sqrt{B} \propto (\mathcal{L}_{\text{int}}/\sigma_W)/\sqrt{\mathcal{L}_{\text{int}}} = \sqrt{\mathcal{L}_{\text{int}}/\sigma_W}$ ; therefore, the integrated luminosity required to observe a narrow resonance with a very small coupling ( $\propto \Gamma_{e^+e^-}$ ) is  $\mathcal{L}_{\text{int}} \propto (1/\sigma_W)^2$ .

The first consideration of energy monochromatization in  $e^+e^-$  collisions dates back to mid-1970s [1]. In the proposed scheme, beams collide head-on and have a horizontal or vertical energy dispersion at the interaction point (IP), opposite in sign for the  $e^+$  and  $e^-$  beams. As a result, the particles collide with opposite energy deviations,  $E_0 + \Delta E$  and  $E_0 - \Delta E$ , and their invariant mass  $W \approx 2\sqrt{E_1 E_2} \approx 2E_0 - (\Delta E)^2/E_0$  is very close to  $2E_0$ . This monochromatization scheme was considered by many authors in 1980s–1990s [2–4] (and references therein) for use in  $c\text{-}\tau$  and  $B$ -factory projects (i.e., in the energy range of the  $\psi$  and  $\Upsilon$  resonances); however, none of the proposals were implemented as they led to a loss of luminosity. For  $B$ -factories working at wide  $\Upsilon(4S)$ , the high luminosity was more important.

## A NEW METHOD OF MONOCHROMATIZATION

The new generation of circular  $e^+e^-$  colliders (DAΦNE, SuperKEKB,  $c\text{-}\tau$  at BINP, FCC-ee, CEPC) rely on the so-called “crab-waist” collision scheme [5], where the beams collide at an angle  $\theta_c \gg \sigma_x/\sigma_z$ . Due to reduced collision effects in this scheme the luminosity can be higher by a same factor of 20–30. Below we propose a new method of collision monochromatization [6], which nicely works at a large crossing angle.

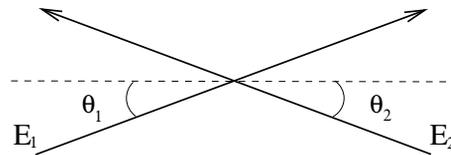


Figure 1: Collisions with crossing angles.

The invariant mass of the produced system ( $c = 1$ ) in collisions with a crossing angle, Fig. 1, the invariant mass spread

$$W^2 = (P_1 + P_2)^2 \approx 2E_1 E_2 (1 + \cos(\theta_1 + \theta_2)). \quad (1)$$

By differentiating this formula while assuming that the energies and the angles are independent and setting  $\theta_1 = \theta_2 = \theta_c/2$  and  $E_1 = E_2 = E$ , we find the relative mass spread

$$\left(\frac{\sigma_W}{W}\right)^2 = \frac{1}{2} \left(\frac{\sigma_E}{E}\right)^2 + \frac{1}{2} \frac{\sin^2 \theta_c}{(1 + \cos \theta_c)^2} \sigma_\theta^2, \quad (2)$$

where  $\theta_c$  is the beam crossing angle,  $\sigma_E$  is the beam energy spread, and  $\sigma_\theta$  is the beam angular spread at the IP,

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which is determined by the horizontal beam emittance  $\varepsilon_x$  and beta function  $\beta_x^*$  at the IP:  $\sigma_\theta = \sqrt{\varepsilon_x/\beta_x^*}$ . For head-on collisions, the second term vanishes, and the mass resolution is determined solely by the beam energy spread. In the aforementioned colliders with the crab-waist scheme, the contribution of beam energy spread is also dominant.

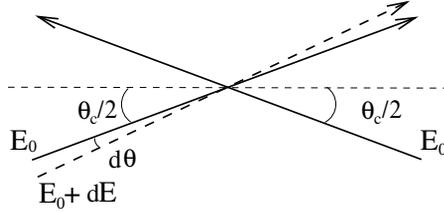


Figure 2: Collisions with the energy-angle correlation.

The presently proposed monochromatization method is based on the fact that the invariant mass  $W^2$  depends on both the beam energies and their crossing angle. The second term in Eq. (2) reflects the natural stochastic beam spread due to the horizontal beam emittance and cannot be avoided; however, the first term can be suppressed very significantly, as we shall demonstrate. In the proposed method, we provide the beams with an angular dispersion such that a beam particle arrives to the IP with a horizontal angle that depends on its energy: the higher the energy, the larger the angle, Fig. 2. We can choose such a dispersion that when a particle coming from the left, with energy  $E_0 + dE$  and angle  $\theta = \theta_c/2 + d\theta$ , collides with a particle coming from the right, with the nominal (average) energy  $E_0$  and angle  $\theta_c/2$ , they produce the same invariant mass as two colliding particles that both have the nominal energies and angles,  $E_0$  and  $\theta_c/2$ . From Eq. (1), we obtain the required condition:

$$(E_0 + dE)E_0(1 + \cos(\theta_c + d\theta)) = E_0^2(1 + \cos \theta_c). \quad (3)$$

In the linear approximation, this gives the required angular dispersion (the same for both particles)

$$d\theta_i = \frac{1 + \cos \theta_c}{\sin \theta_c} \frac{dE_i}{E_0}. \quad (4)$$

In what follows, the exact dispersion relation [Eq. (3)] will be called “nonlinear”, while Eq. (4) will be referred to as “linear” dispersion. Taking the first derivative of  $W^2$  [Eq. (1)] and substituting the linear dispersion from Eq. (4), one can confirm that the resulting variation of the invariant mass due to the beam energy spread is indeed zero. Note that the proposed monochromatization method works even for unequal beam energies.

If  $W$  were a product of two functions that depend, respectively, only on the parameters of the left or the right colliding particles, then the contribution of the beam energy spreads  $\sigma_{E_i}$  to  $\sigma_W$  could have been completely zeroed using the nonlinear dispersion [Eq. (3)]. However, the contributions of the two beams are not factorized due to the  $\cos(\theta_1 + \theta_2)$  term in Eq. (1). Therefore, a second-order contribution from the energy spread remains; below, we will find it for both the linear and nonlinear dispersions.

Since the first derivative of  $W$  is zero, we must use the quadratic term of the Taylor series proportional to  $\sigma_E^2$  and results are the following [6].

For ideal nonlinear dispersion [Eq. (3)], the mass spread due to the beam energy spread and the shift of the average mass  $\Delta W$  are

$$\left(\frac{\sigma_W}{W}\right)_E = \frac{\sigma_E^2}{2E^2} \frac{1 + \cos \theta_c}{\sin^2 \theta_c}, \quad \frac{\Delta W}{W} = 0. \quad (5)$$

For linear dispersion (Eq. (4))

$$\left(\frac{\sigma_W}{W}\right)_E = \frac{\sigma_E^2}{2E^2} \left[ \left(1 + \frac{1 + \cos \theta_c}{\sin^2 \theta_c}\right)^2 + \left(\frac{1 + \cos \theta_c}{\sin^2 \theta_c}\right)^2 \right]^{1/2}, \quad (6)$$

$$\frac{\Delta W}{W} = \frac{\sigma_E^2}{2E^2} \left(1 + \frac{1 + \cos \theta_c}{\sin^2 \theta_c}\right). \quad (7)$$

The total invariant mass spread is the sum of the residual contribution of the energy spread [Eq. (5) or Eq. (6)] and the second term of Eq. (2), which is due to the angular spread:

$$\left(\frac{\sigma_W}{W}\right)^2 = \left(\frac{\sigma_W}{W}\right)_E^2 + \frac{1}{2} \frac{\sin^2 \theta_c}{(1 + \cos \theta_c)^2} \sigma_\theta^2 \quad (8)$$

These formulas have been verified by direct simulation.

The dependence of the invariant mass spread on the collision angle  $\theta_c$  is shown in Fig. 3. It can be seen that linear

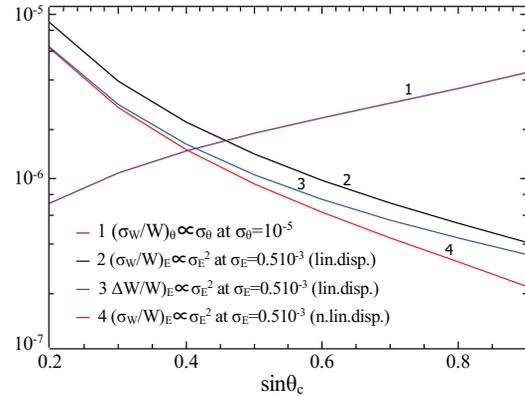


Figure 3: Monochromaticity of collisions vs collision angle.

dispersion works almost as well as the best-case nonlinear dispersion. One can see that at  $\sin \theta_c \sim 0.5$  and  $\sigma_\theta \sim 10^{-5}$  one can have  $\sigma_W/W \approx 2.5 \times 10^{-6}$ , which is about 150 to 200 times better than that at current and past  $e^+e^-$  storage rings. The angular spread, determined by the horizontal emittance, will be a limiting factor.

## LIMITATIONS

### Radiation in the Final Quadrupoles

The required angular dispersion at the IP [Eq. (4)] leads to a large horizontal beam size in the final quadrupoles magnets that are located at the distance  $F$  in the places with high dispersion  $D_x \sim F(1 + \cos \theta_c)/\sin \theta_c$ , which leads to strong synchrotron radiation and, as a consequence, to an additional energy spread and deterioration of the horizontal emittance  $\varepsilon_x$ . For the energy spread  $\sigma_E/E = 0.5 \times 10^{-3}$

## CONCLUSION

- A new method of monochromatization is proposed, which works at large crossing angles, where  $e^+e^-$  colliders can provide maximum luminosity due to the crab constriction scheme.
- This method of monochromatization does not require an increase of the spot size at the IP, therefore the luminosity may be lower only due to larger crossing angle ( $\sim 6 \times \theta_c$  (Super-KEKB)), that can be compensated partially by an increase of particles in the bunch:  $\mathcal{L} \propto N(Nf)/(\sigma_z \sigma_y \tan \theta_c/2)$  [5].
- This method of monochromatization works well in the region of very narrow resonances  $J/\psi, \psi', \Upsilon, \Upsilon', \Upsilon'', \Upsilon'''$ , can increase their production rate 30–100 times, that opens a great opportunity to search for new physics in  $10^{13-14}$  decays of these resonances.
- The present work is just an idea and preliminary consideration. The next step requires the efforts of the accelerator designers.
- HEP direction': energy frontiers, intensity frontiers (high  $\mathcal{L}$ )  $\Rightarrow$  high  $\mathcal{L}$  + monochromatization.

## ACKNOWLEDGMENTS

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and  $\sin \theta_c = 0.5$  the r.m.s. angular spread at the IP due to the dispersion is  $\sigma_{\theta,d} = ((1 + \cos \theta_c)/\sin \theta_c)(\sigma_E/E_0) \approx 1.87 \times 10^{-3}$ . Larger crossing angles are preferable. To understand the importance of these effects, we have estimated the equilibrium energy spread and horizontal emittance when they are caused only by these effects but their damping is provided by synchrotron radiation in the rings. The result is the following [6]

$$\Delta \left( \frac{\sigma_E}{E} \right)^2 \approx \frac{40 r_e \gamma^2 R}{F^2} \left( \frac{1 + \cos \theta_c}{\sin \theta_c} \right)^3 \left( \frac{\sigma_E}{E} \right)^3 \propto \frac{E^5}{\sqrt{R}}; \quad (9)$$

$$\Delta \varepsilon_x = 170 \frac{r_e \gamma^2 R}{\beta_x} \left( \frac{1 + \cos \theta_c}{\sin \theta_c} \right)^5 \left( \frac{\sigma_E}{E} \right)^3 \propto \frac{E^5}{\sqrt{R}}. \quad (10)$$

For example: for  $E = 5$  GeV,  $R = 500$  m,  $\sin \theta_c = 0.5$ ,  $\sigma_E/E = 0.5 \times 10^{-3}$ ,  $F = \beta_x = 1$  m, we get  $\Delta(\sigma_E/E)^2 = 3.6 \times 10^{-11}$ ,  $\Delta \varepsilon_x = 2.1 \times 10^{-9}$  m. Thus, the effect on the energy spread is negligible, and the increase of the horizontal emittance is two times smaller than the SuperKEKB emittance. Due to strong dependences of this effect on the energy we conclude that this method of monochromatization works well only at  $W < 10$  GeV, it is good for  $\Psi(nS)$ ,  $\Upsilon(nS)$  region, but not for  $H(125)$ .

### Beam-Beam Attraction

During beam collisions particles are attracted to the opposing beam, their collision angles increase that can influence the invariant mass. However, a detailed examination of this effect [6] unexpectedly shows that beam attraction does not affect the invariant mass of the colliding particles, since the energy simultaneously increases so that  $dW = 0!$

### Detector Magnetic Field

Synchrotron radiation of particles in the magnetic field of the detector in the presence of a large dispersion function can lead to an unacceptable increase of the the horizontal emittance. This must be taken into account.

### Chromatic Section

Obtaining of a large dispersion function  $D_x \sim 5$  m needs quit long dispersion section in order to exclude the increase of the horizontal emittance. Estimation (in some model) gives the following increase of the equilibrium horizontal emittance due radiation in the chromatic section [6]

$$\Delta \varepsilon_x \sim 6500 \frac{r_e \gamma^2 R F^4}{L^5} \left( \frac{1 + \cos \theta_c}{\sin \theta_c} \right)^5. \quad (11)$$

For  $E = 5$  GeV,  $R = 500$  m,  $\sin \theta_c = 0.5$  and  $F = 1$  m, we get  $\Delta \varepsilon_x \sim 10^{-9}$  m at  $L = 230$  m. It would be 2 times shorter for  $F = 0.7$  m and  $\sin \theta_c = 0.7$ .