

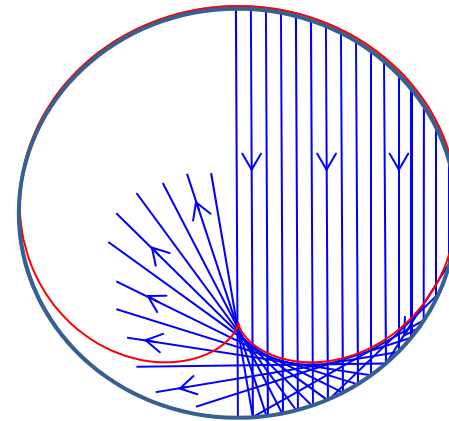
First Experimental Measurements of the Caustic Nature of Trajectories in Bunch Compressors

Tessa Charles^{1,2},

Jonas Björklund Svensson^{3,4}, Andrea Latina¹, Sara Thorin⁴

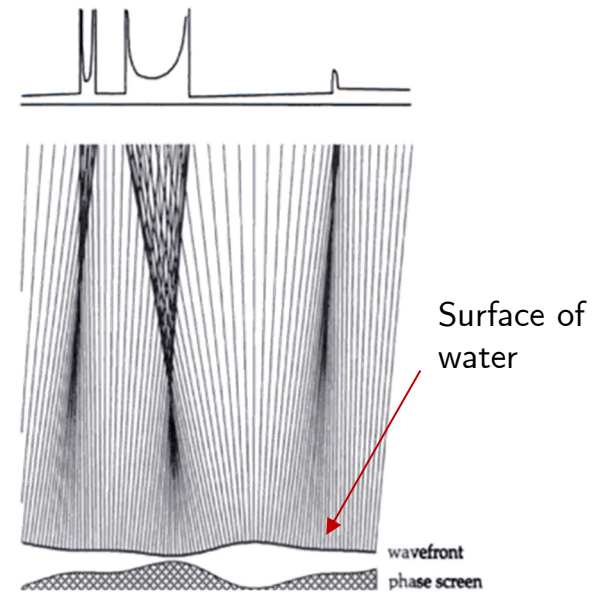
1. European Organization for Nuclear Research (CERN), Geneva, Switzerland
2. School of Physics, University of Melbourne, 3010, Victoria, Australia
3. Lund University, Lund, Sweden
4. MAX IV Laboratory, Lund, Sweden





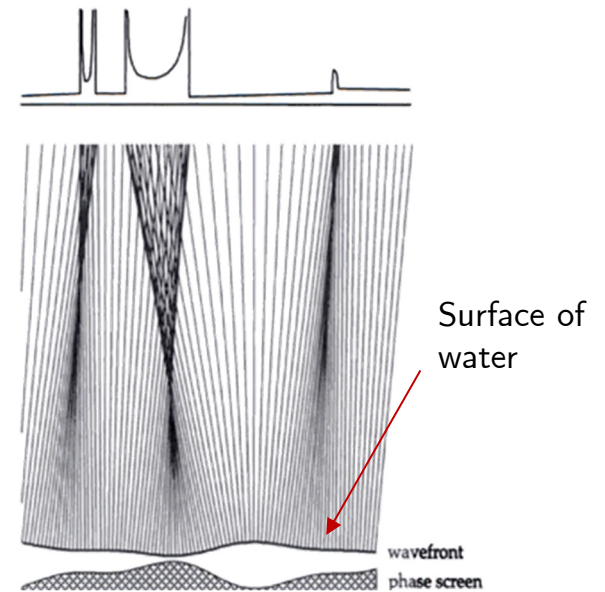
The envelope of trajectories forms the caustics lines.

Caustics lines are an envelope of trajectories,
or the projection of that envelope on another surface.



J.F. Nye "Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations", (Taylor & Francis, Philadelphia, 1999).

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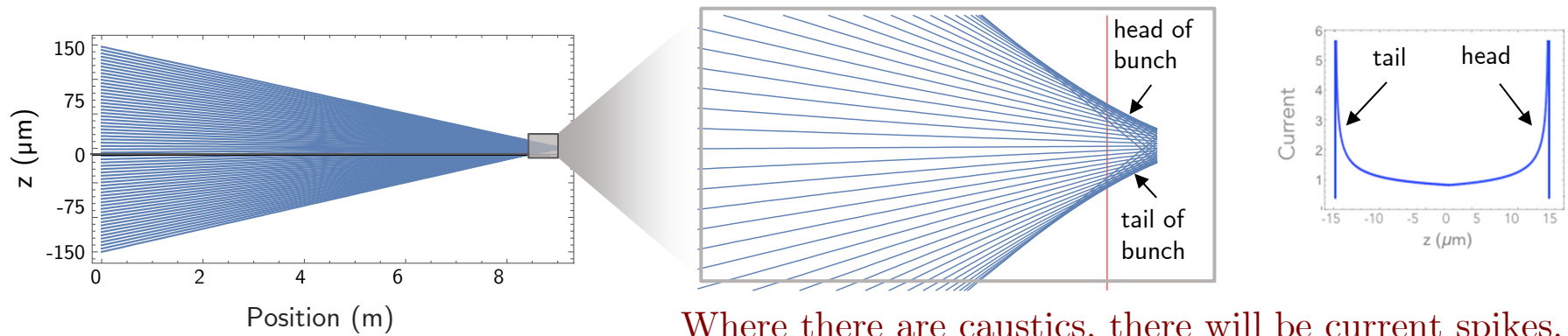
J.F. Nye "Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations", (Taylor & Francis, Philadelphia, 1999).

Outline

1. Introduction of caustics
2. Example of the usefulness of caustics
3. MAX IV experimental set up & results

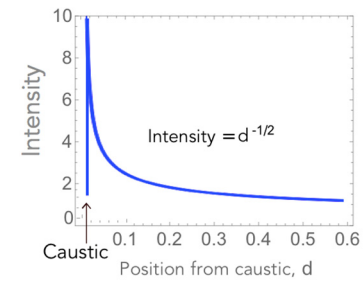
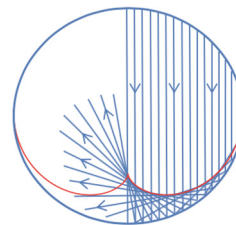
Caustics in accelerator physics

Electron trajectories in a bunch compressor:



Where there are caustics, there will be current spikes.

Optical caustics:



Caustics are singularities in the density of families of trajectories.

Caustics fall into catastrophe theory

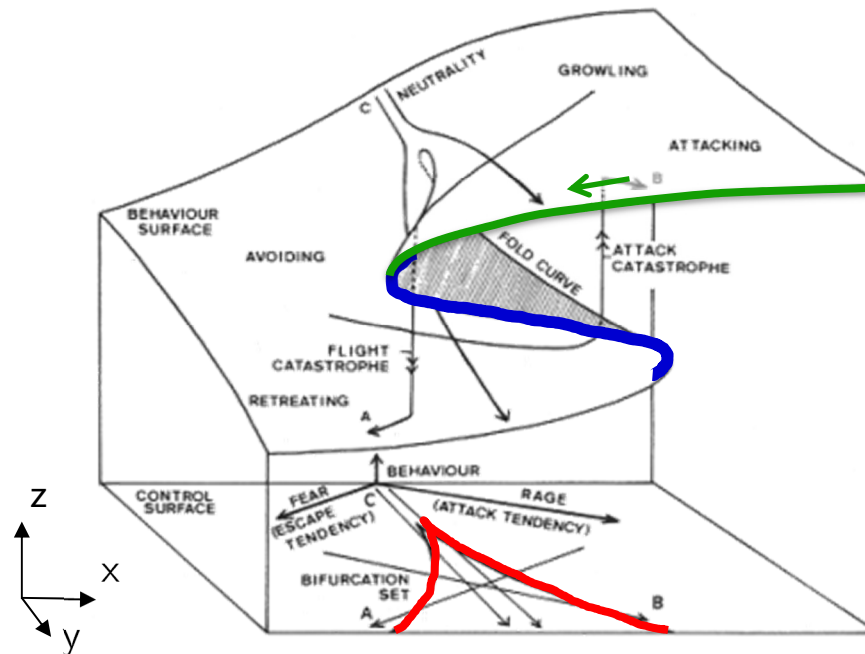
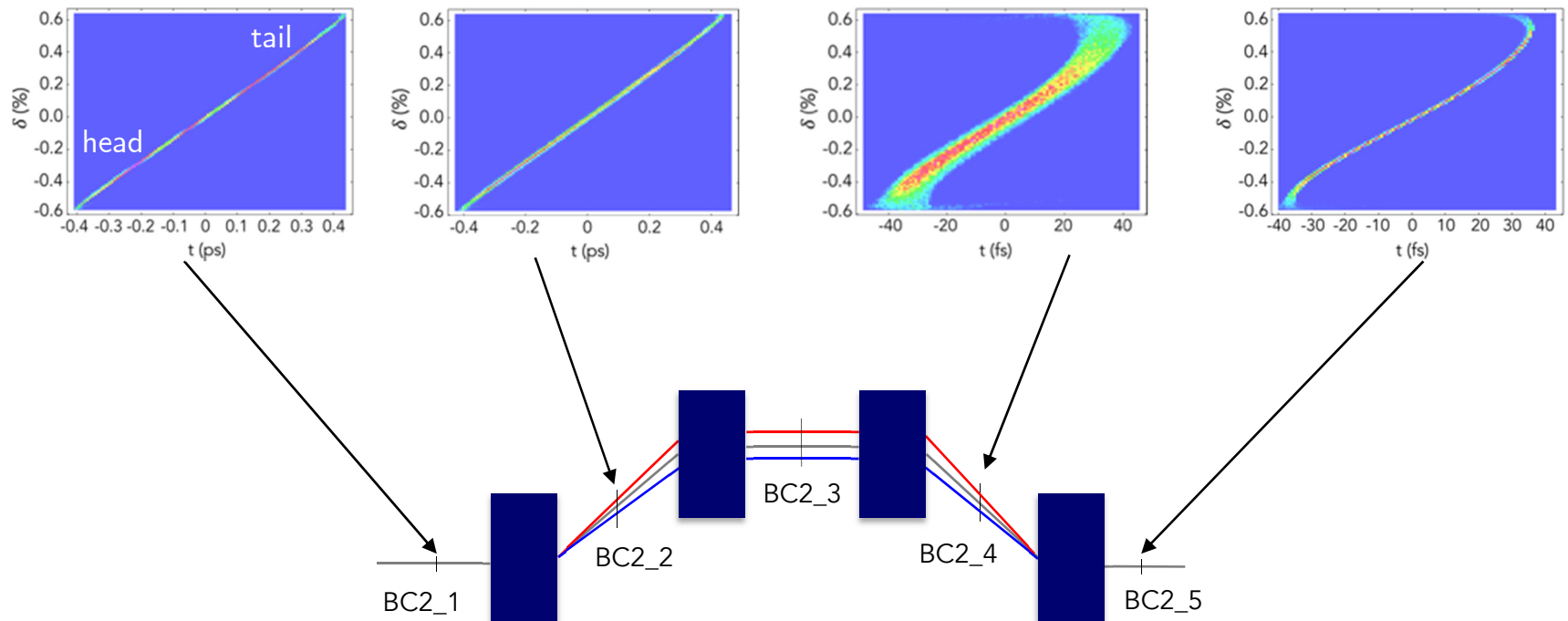
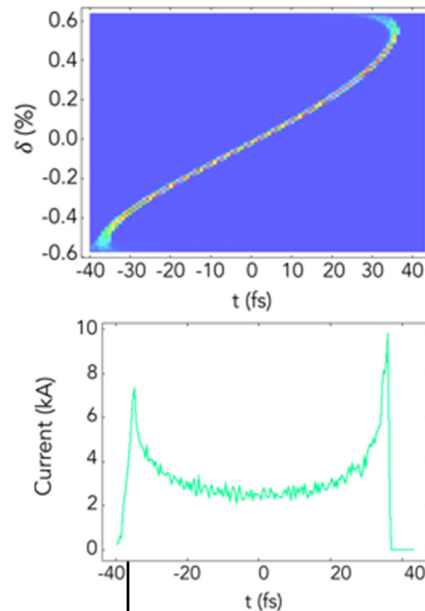


Figure: EC Zeeman (1976) Catastrophe Theory in Scientific American.

Phase space evolution through a bunch compression



Phase space evolution through a bunch compression

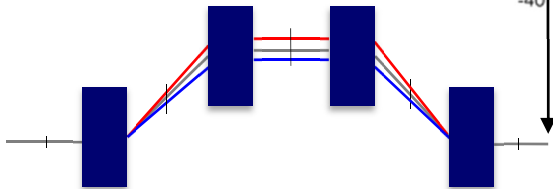


Energy spread induced by CSR:

λ = linear charge density

$$\frac{dE}{cdt} = \frac{-2e^2}{4\pi\epsilon_0(3R^2)^{1/3}} \int_{\tilde{z}-z_L}^{\tilde{z}} \frac{d\lambda}{dz} \left(\frac{1}{\tilde{z}-z} \right)^{1/3} dz$$

Current spikes are problematic, leading to greater **CSR-induced emittance growth**.

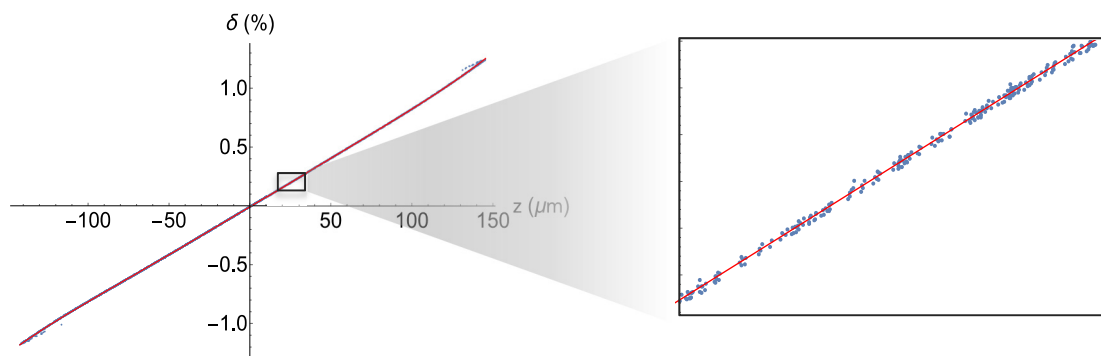


Caustic expression

$$\tilde{z}_c(z_i) = z_i + \frac{\delta(z_i)(-1 + T_{566}(-2 + \delta(z_i))\delta'(z_i) + U_{5666}(-3 + \delta(z_i)^2)\delta'(z_i))}{\delta'(z_i)}$$

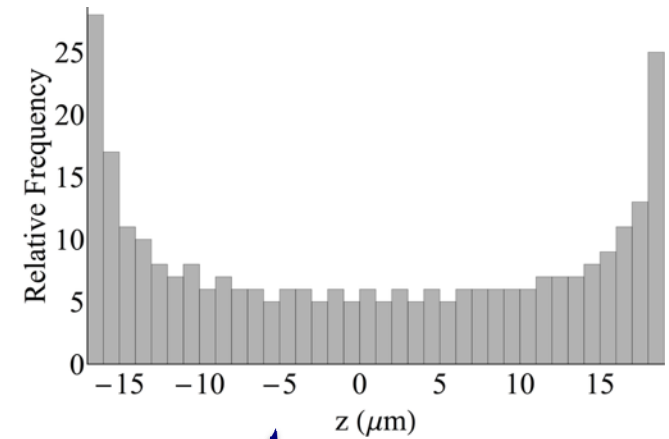
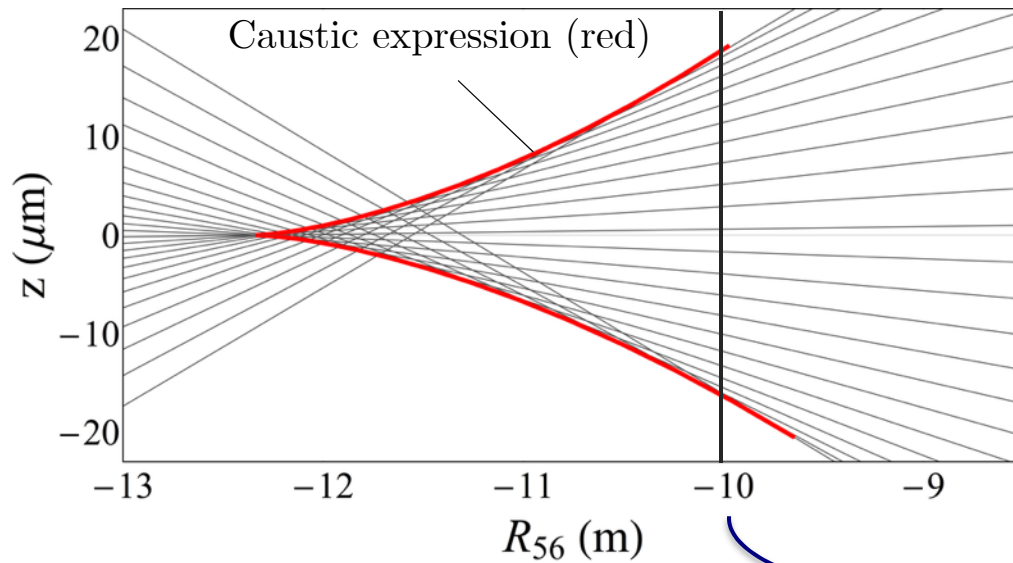
$$\tilde{R}_{56}(z_i) = \frac{-1 - 2T_{566}\delta'(z_i) - 3U_{5666}\delta'(z_i)}{\delta'(z_i)}$$

T. K. Charles *et al.* (2016) Phys. Rev. Accel. Beams, **19**, 104402



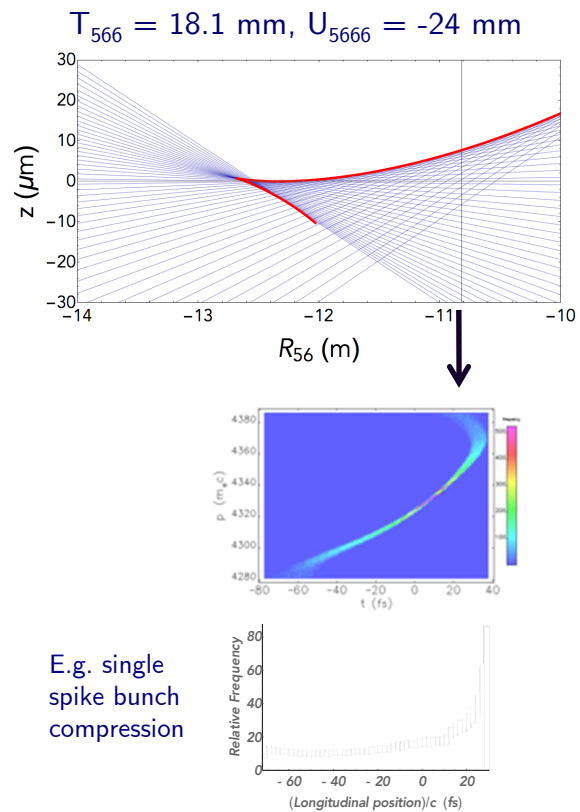
$$\delta(z_i) = h_1 z_i + h_2 z_i^2 + h_3 z_i^3$$

Caustic expression for a chicane

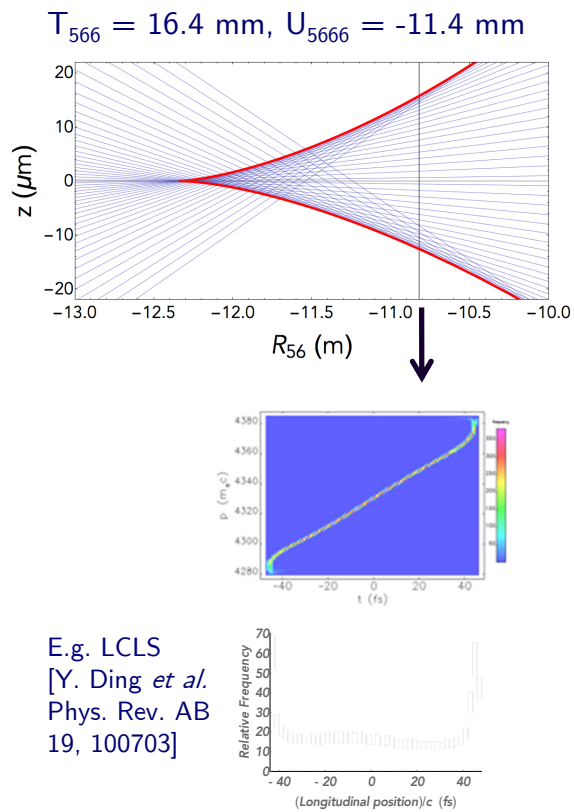


$R_{56} = -10$ mm

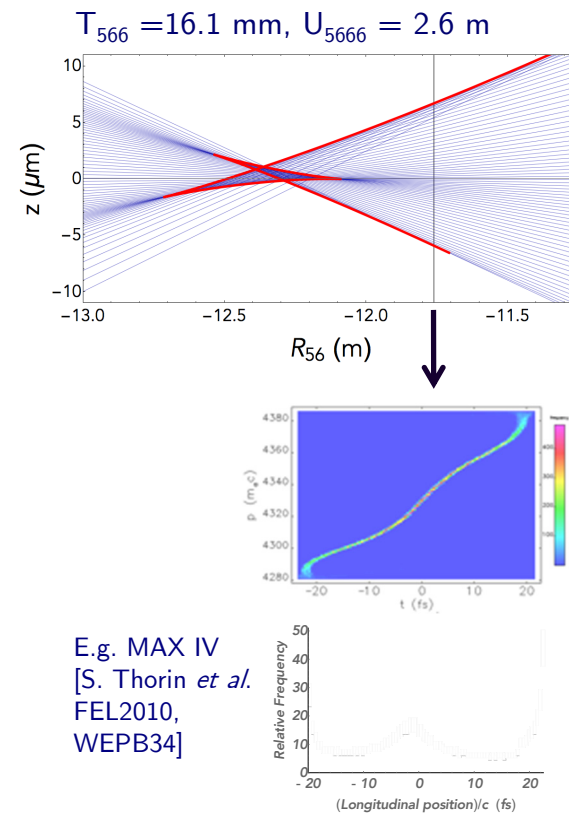
Fold



Cusp



Butterfly

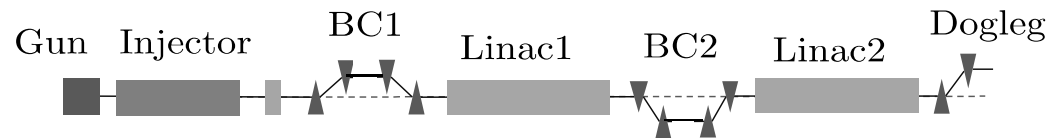
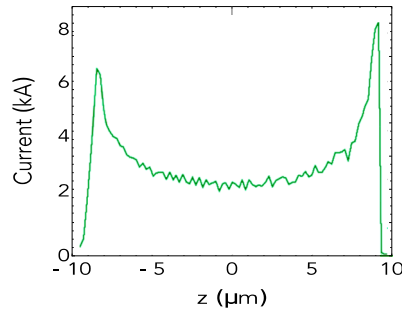
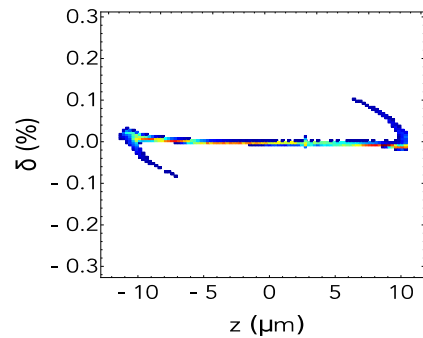


Example of caustics

Current horn suppression

Current horn suppression

An S-band FEL Linac example



Elegant simulations of longitudinal phase space and current profile at the end of Linac2.

Caustic condition

$$R_{56}s \frac{d(\delta(z))}{dz} \Big|_{z=z_i} + T_{566}s \frac{d}{dz} (\delta^2(z)) \Big|_{z=z_i} + U_{5666}s \frac{d}{dz} (\delta^3(z)) \Big|_{z=z_i} + s_{BC} = 0.$$

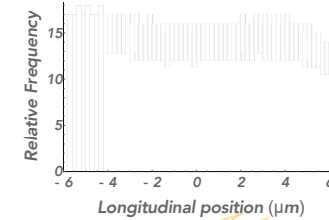
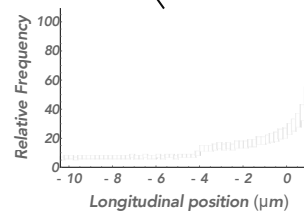
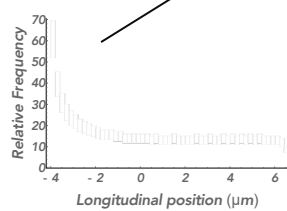
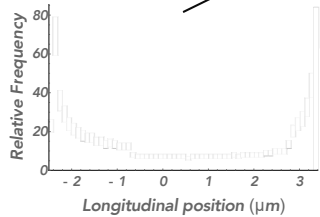
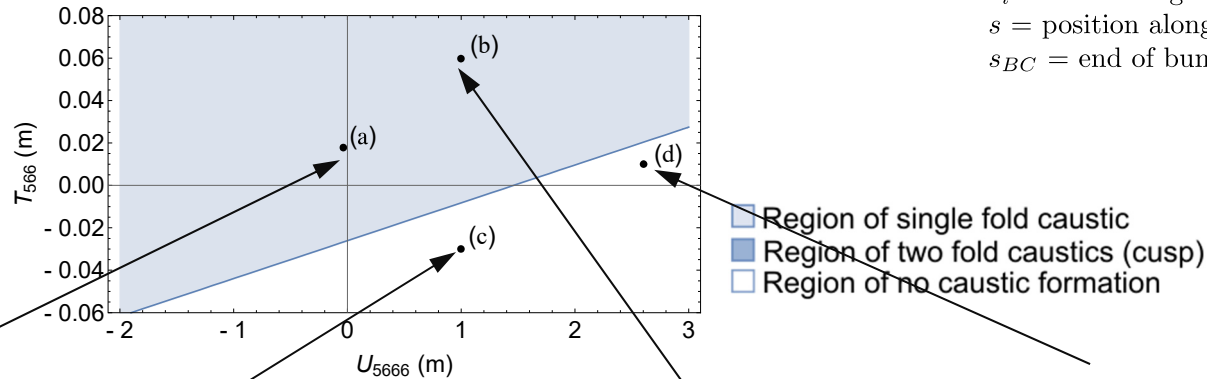
where,

δ = energy spread

z_i = initial longitudinal position

s = position along accelerator

s_{BC} = end of bunch compressor position



Caustic condition

$$R_{56}s \frac{d(\delta(z))}{dz} \Big|_{z=z_i} + T_{566}s \frac{d}{dz} (\delta^2(z)) \Big|_{z=z_i} + U_{5666}s \frac{d}{dz} (\delta^3(z)) \Big|_{z=z_i} + s_{BC} = 0.$$

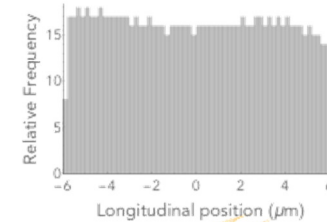
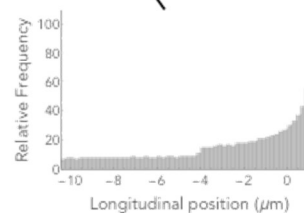
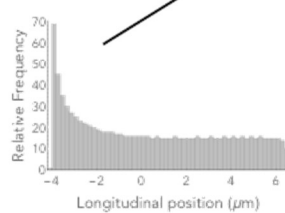
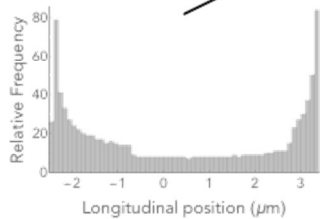
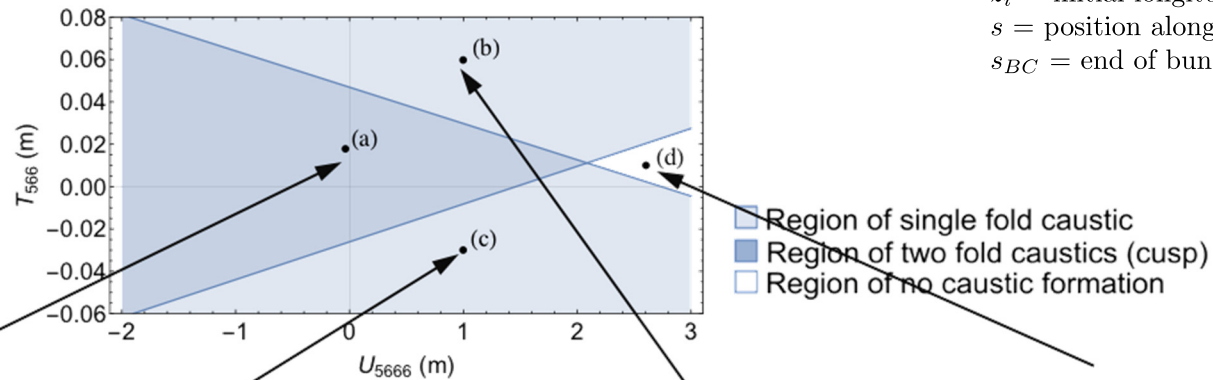
where,

δ = energy spread

z_i = initial longitudinal position

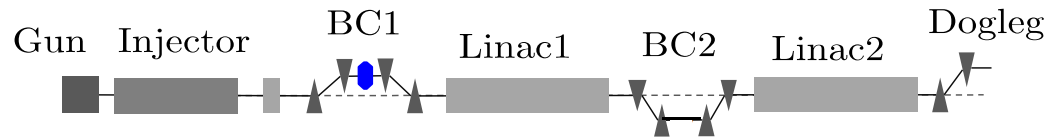
s = position along accelerator

s_{BC} = end of bunch compressor position

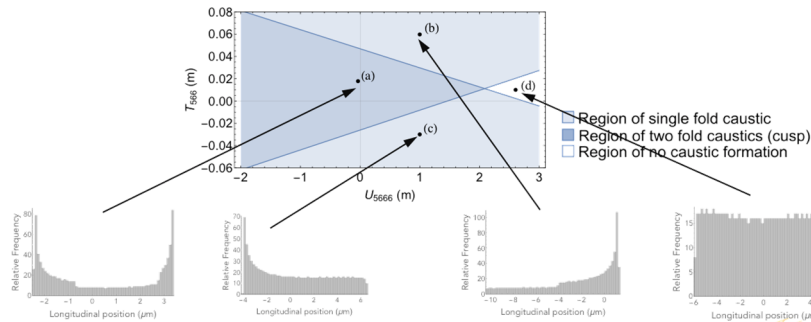


Caustic current horn suppression

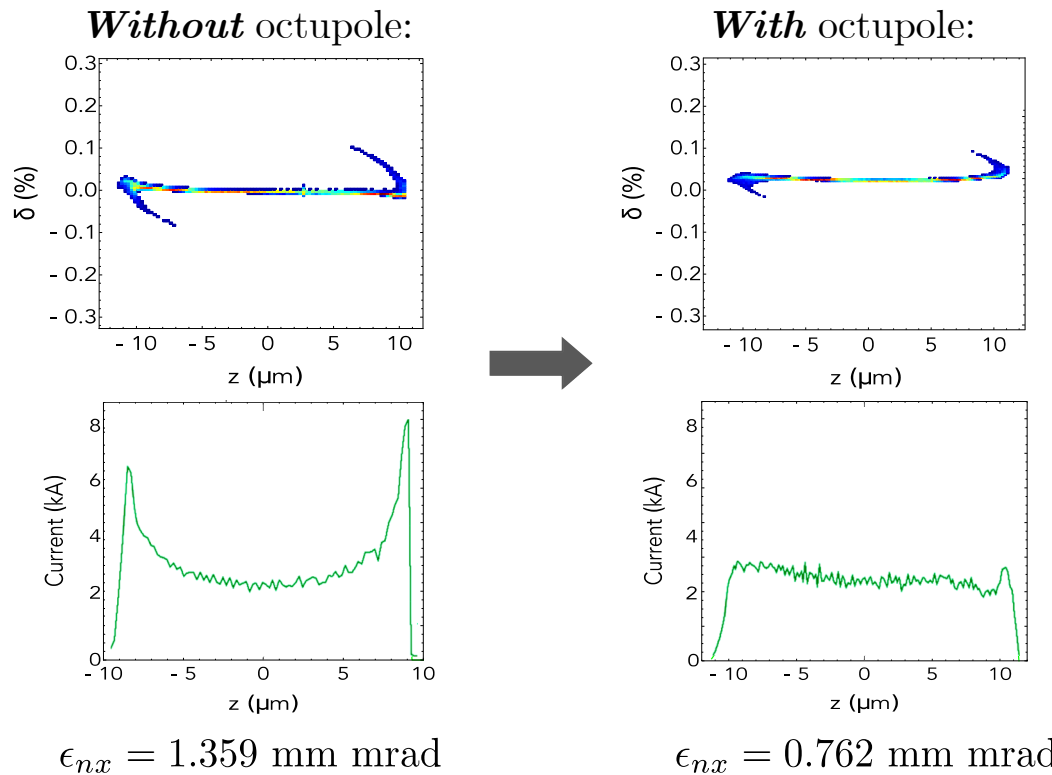
S-band FEL linac example



Octupole added to BC1 to vary U_{5666} .



Caustic current horn suppression

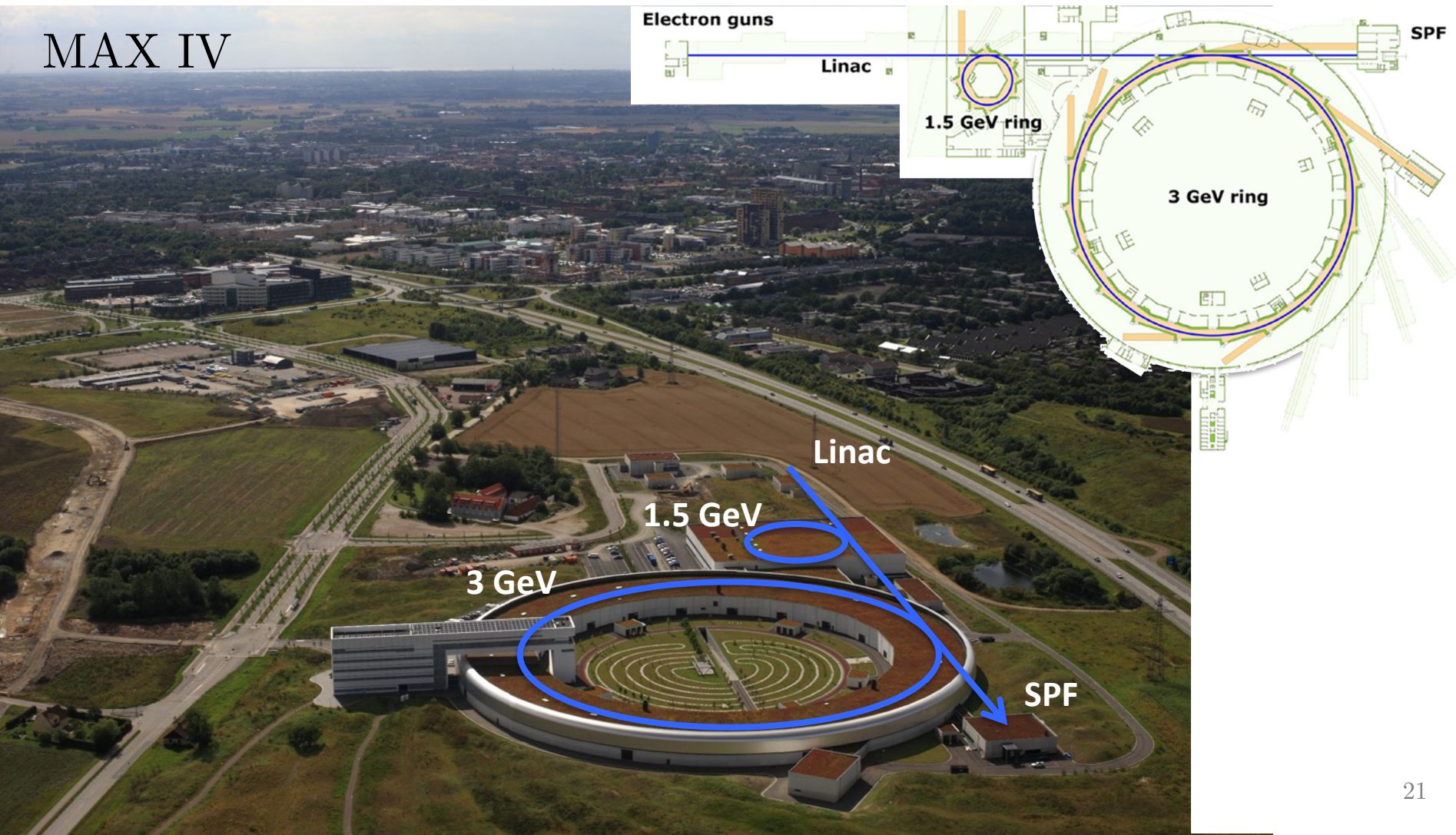


63% reduction in the
CSR-induced
emittance growth.

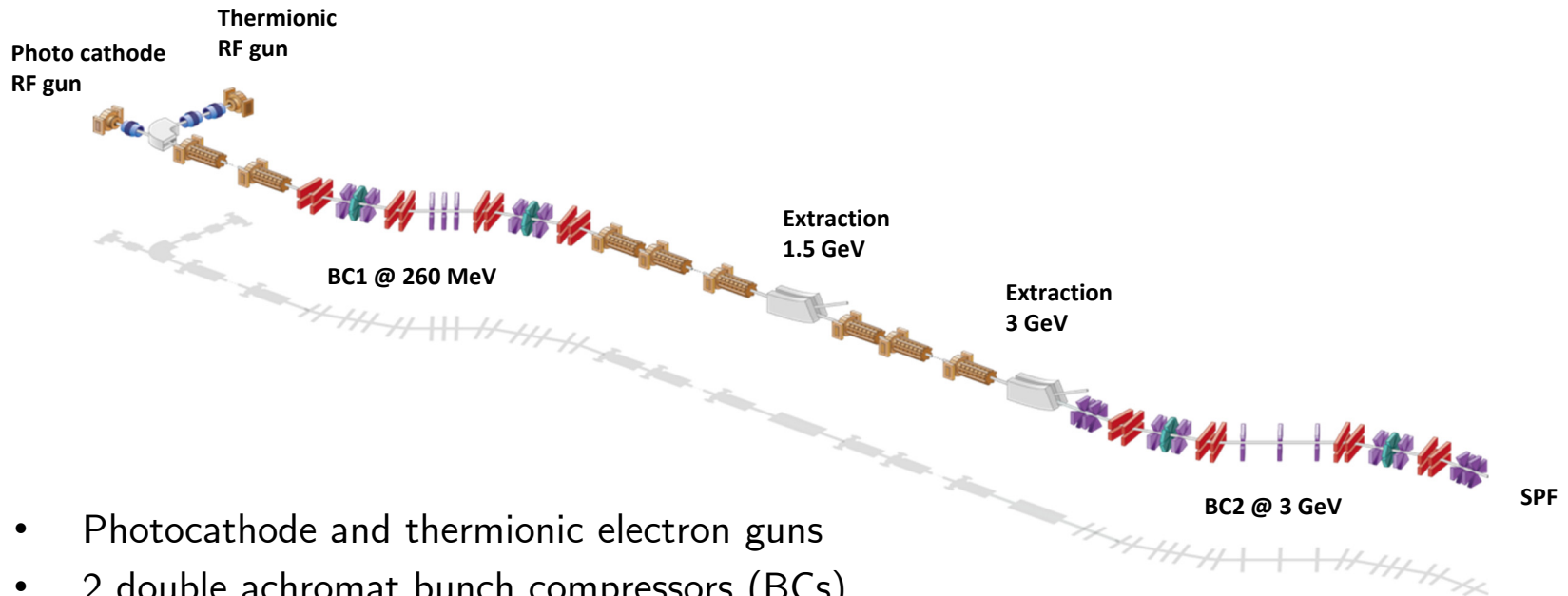
T. K. Charles *et al.* (2017) Phys.
Rev. Accel. Beams, **20**, 030705

Experimental results from MAX IV

MAX IV



MAX IV Linac

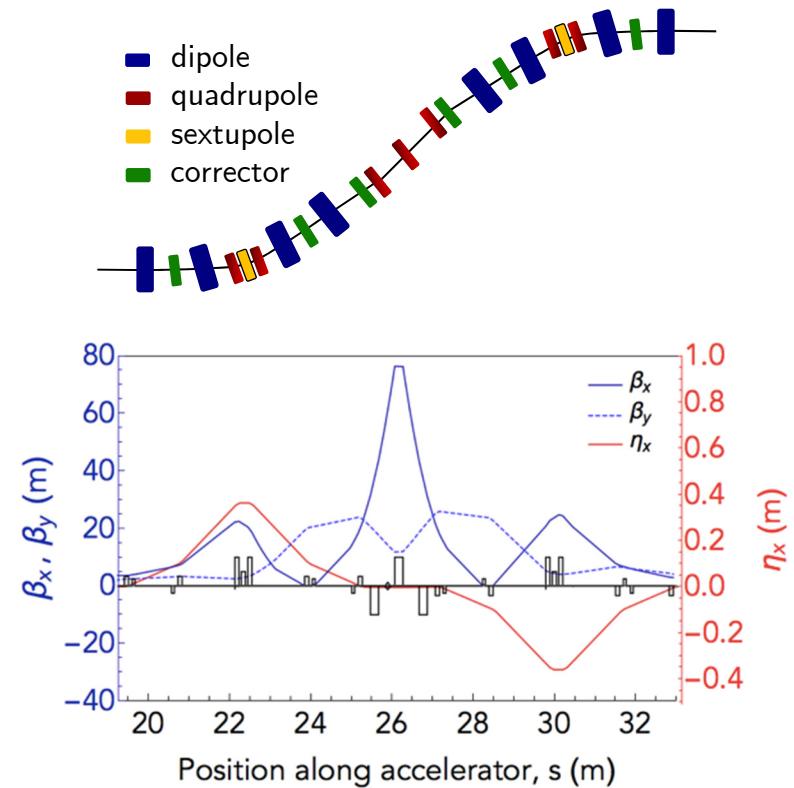


- Photocathode and thermionic electron guns
- 2 double achromat bunch compressors (BCs)
- RF frequency 2.998 GHz

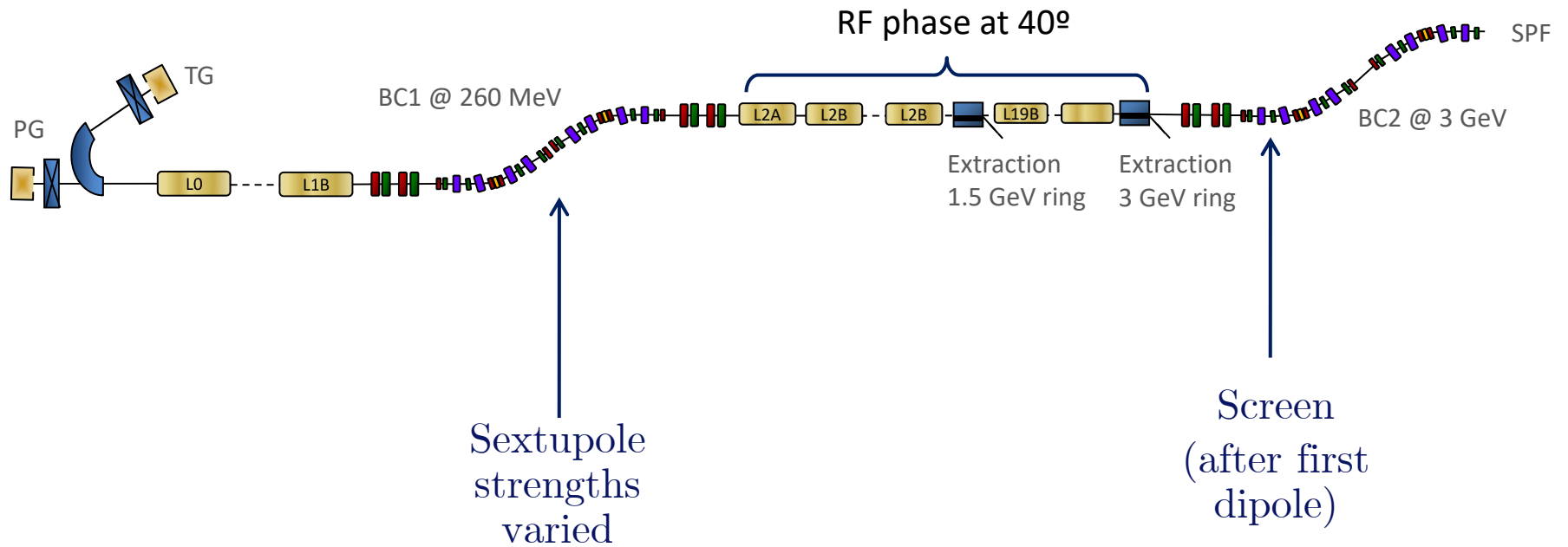
MAX IV Bunch Compressors

Double achromat bunch compressors:

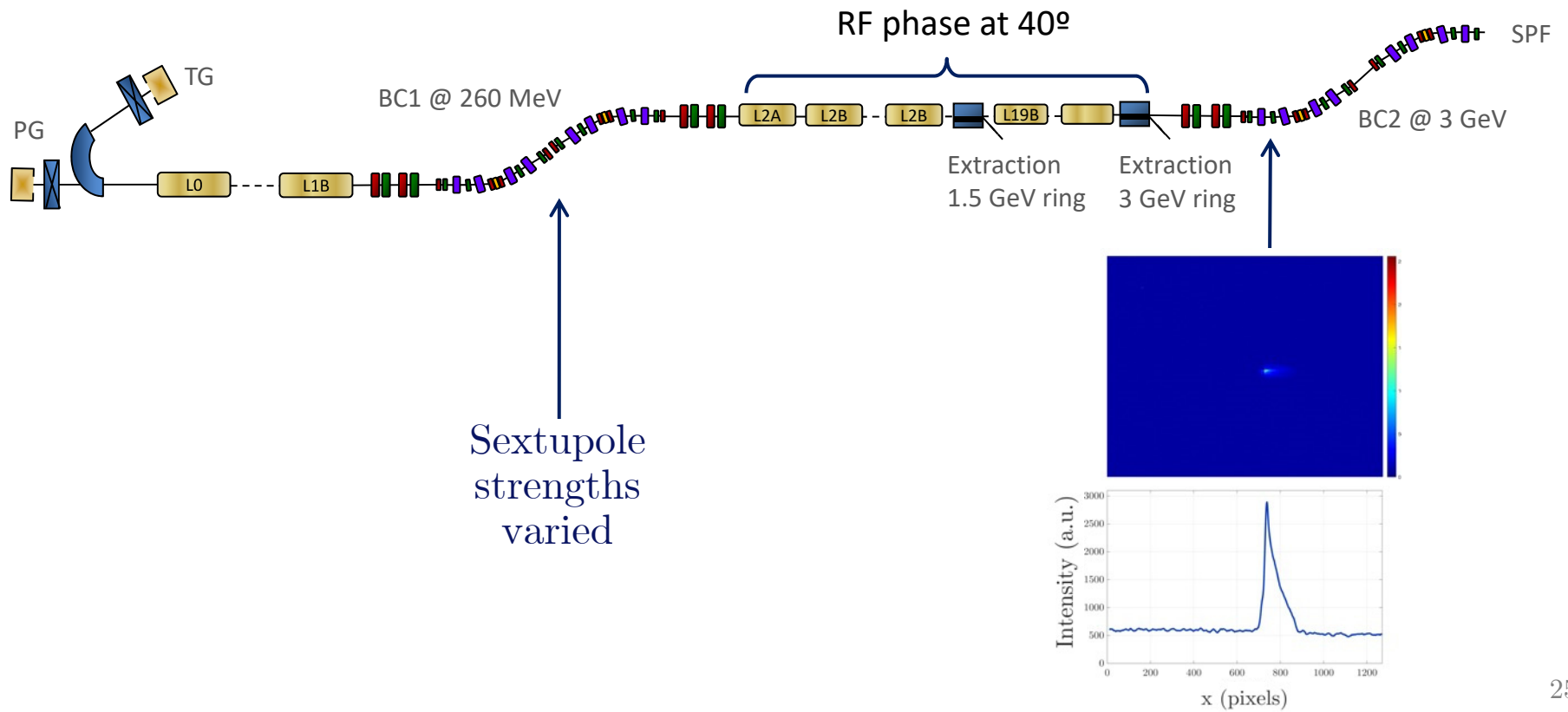
- positive fixed R56 (energy chirp from falling RF slope)
- natural T566 used for linearization, **self-linearizing**
- weak sextupoles at the center of each achromat used for fine tuning T566



Zero-crossing method

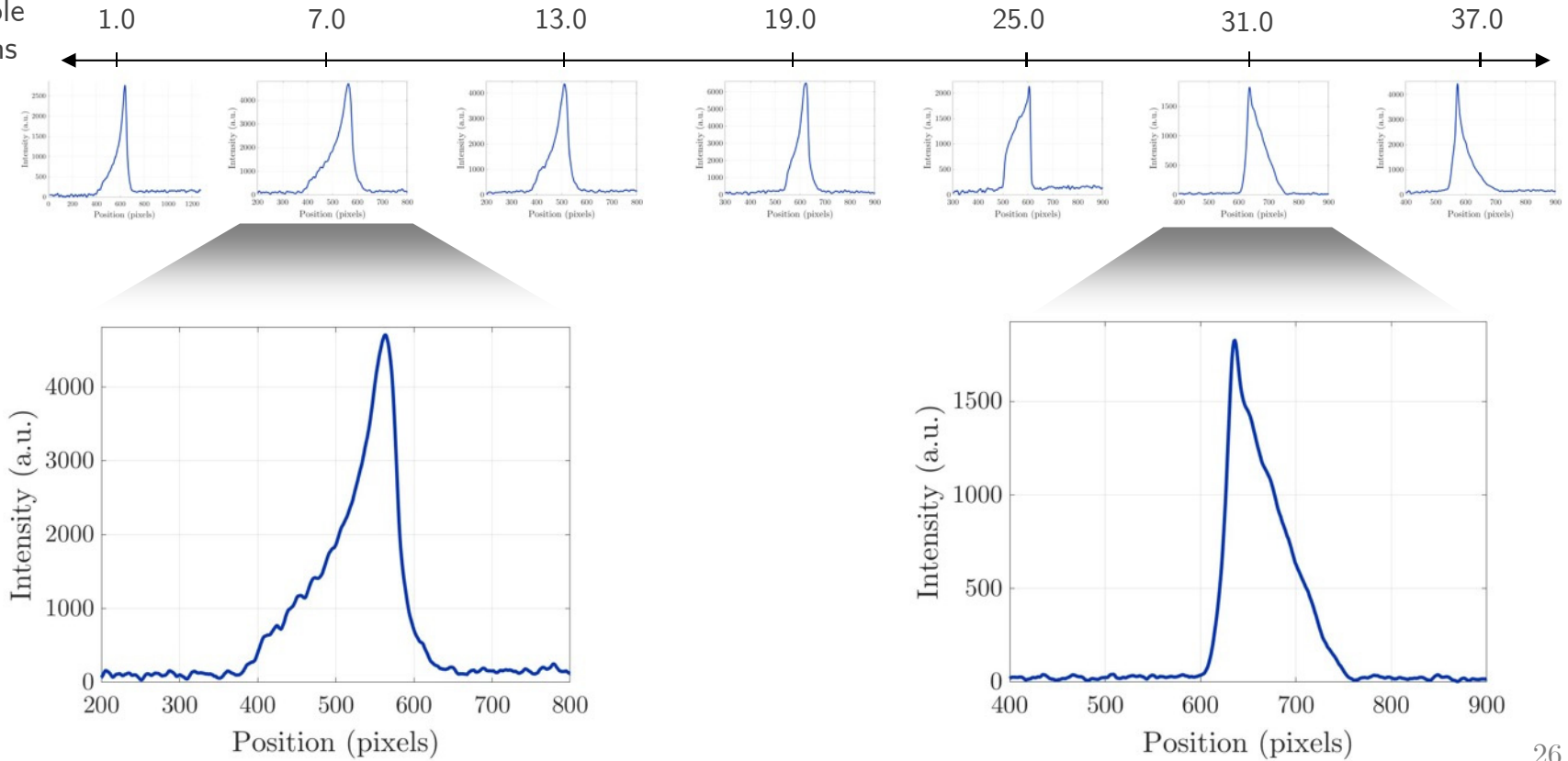


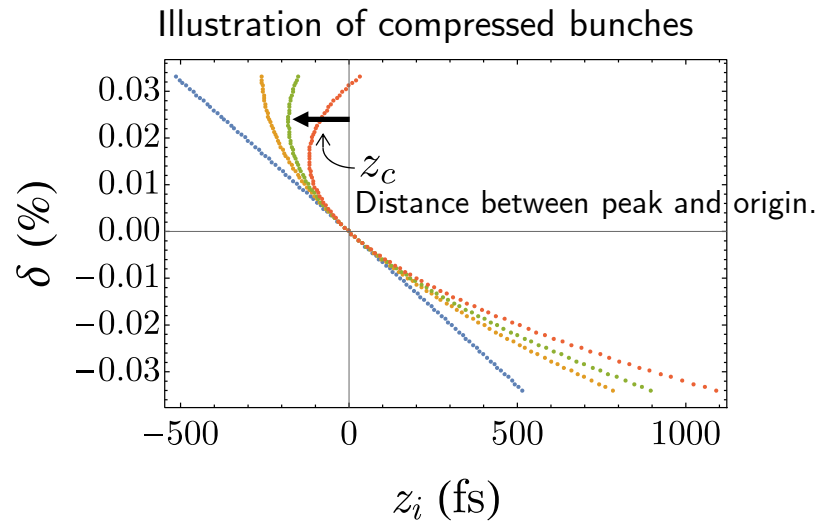
Zero-crossing method



Experimental results – linearly ramped current profiles

Sextupole strengths
[m⁻³]:





The origin is found as being the distance from one bunch edge to the position within the bunch where the accumulated integrated charge equals half of the total charge.

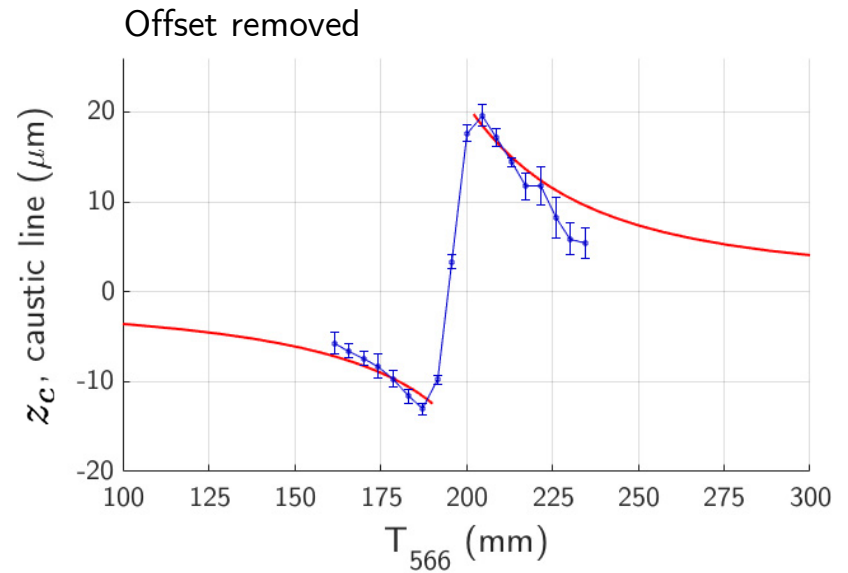
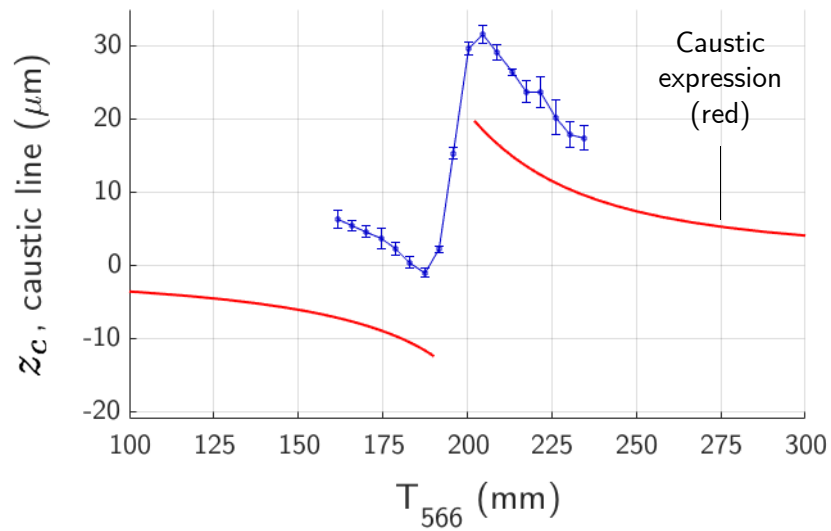
Caustic expression:

$$z_c(z_i) = -z_i - \frac{R_{56}\delta(z_i)}{2} + \frac{U_{5666}}{2} \frac{\delta^3(z_i)}{\delta'(z_i)} + \frac{\delta(z_i)}{2\delta'(z_i)}$$

$$\tilde{T}_{566}(z_i) = \frac{1}{2\delta(z_i)} \left(-R_{56} - \frac{1}{\delta'(z_i)} - 3U_{5666} \delta^2(z_i) \right)$$

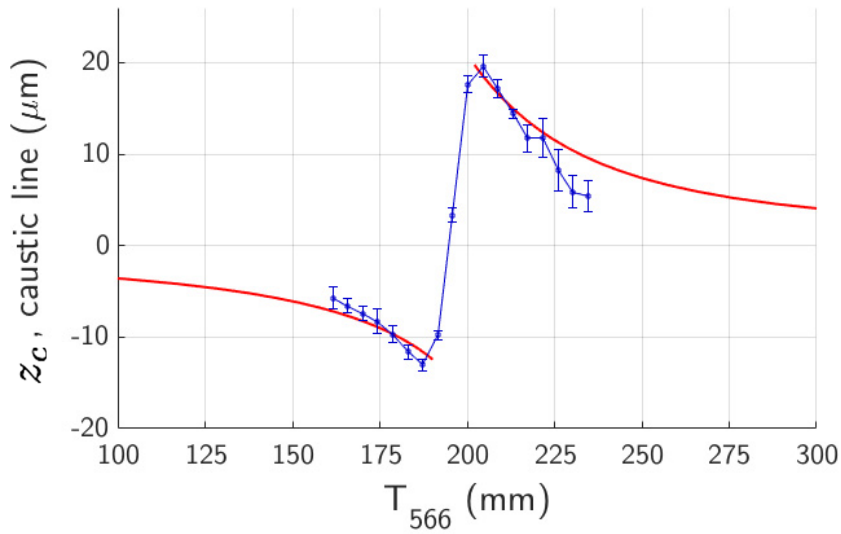
(Same expression as shown on slide 11, simply re-arranged)

Mapping out caustic lines



Offset due to the combined effects of wakefields and curvature of the phase space distribution.

Mapping out caustic lines



Offset of 14 micrometers removed

Where the caustic expression is undefined, the longitudinal phase space is considered well linearized, resulting in a short bunch with a symmetrical current profile.

$$z_c(z_i) = -z_i - \frac{R_{56}\delta(z_i)}{2} + \frac{U_{5666}}{2} \frac{\delta^3(z_i)}{\delta'(z_i)} + \frac{\delta(z_i)}{2\delta'(z_i)}$$

$$\tilde{T}_{566}(z_i) = \frac{1}{2\delta(z_i)} \left(-R_{56} - \frac{1}{\delta'(z_i)} - 3U_{5666} \delta^2(z_i) \right)$$

Conclusions

- Caustics are a useful way that we can look at various types of focusing.
- Demonstration of current horn suppression through manipulation of the higher-order dispersion.
- Preliminary experimental results show good agreement in shape with the theory.
- In addition to validating the caustic expressions, these measurements demonstrate the effectiveness of optical linearization of the MAX IV bunch compressors.

Acknowledgements

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- Sara Thorin, Jonas Björklund Svensson, and Joel Andersson (MAX IV)
- Andrea Latina and Frank Zimmermann (CERN)
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- David Paganin (Monash University)
- Mark Boland (Canadian Light Source)
- Peter Williams (Daresbury Laboratory)
- Dave Douglas (J-Lab)
- In memory of Greg LeBlanc (Australian Synchrotron)

Thank you