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DEVELOPMENT OF METHODS FOR CALCULATION OF BUNCH RADIATION IN PRESENCE OF DIELECTRIC OBJECTS *

A. V. Tyukhtin†, E. S. Belonogaya, S. N. Galyamin, V. V. Vorobev,
Saint Petersburg State University, St. Petersburg, Russia

Abstract

Radiation of charged particles moving in presence of dielectric targets is of significant interests for various applications in accelerator and beam physics. Typically, the size of the target is much larger than the wavelengths under consideration. This fact gives us an obvious small parameter and allows developing approximate methods for analysis. We develop two methods: the "ray-optical method" and the "aperture method". We applied the aperture method to different dielectric objects such as a cone, a prism, and a concentrator of Cherenkov radiation. This paper is mainly devoted to the case of dielectric ball with vacuum channel. Using the aperture method, we obtain analytical results, demonstrate typical graphics for the radiation field, and give comparison with COMSOL Multiphysics simulations.

INTRODUCTION

Radiation of charged particles moving in the presence of dielectric objects ("targets") is of interest for various applications [1–3]. As an example, one can mention a new method of bunch diagnostics which requires calculation of Cherenkov radiation outside dielectric objects [2, 3]. Typically, the size of the target is much larger than the wavelengths under consideration, and this fact considerably complicates computer simulations. On the other hand, this fact gives us an obvious small parameter of the problem and allows developing approximate methods of analysis. We have offered two methods for the solution of such problems which can be called the "ray-optical method" [4, 5] and "aperture method" [6–11].

The essence of the developed methods described in detail in [8–11]. The main points are the following. The 1st and 2nd steps are the same for both methods. Firstly, we solve certain "etalon problem" which does not take into account "external" boundaries of the target, i.e. we consider the infinite medium with the boundary nearest to the charge trajectory and obtain the field inside the bulk of the target. This field can be called an "incident" field. At the second step, we select the part of the external surface of the object which is illuminated by Cherenkov radiation (this part can be called an "aperture"). Since the object size is much greater than wavelengths under consideration, we calculate the field on the external surface of the aperture using Snell's and Fresnel's laws.

The last steps of these methods are different. The ray-optical method uses the ray-optical laws for calculation of the wave field outside the object [4, 5]. However, this technique has essential additional limitations. The distance L from the aperture to the observation point should not be very large, i.e. the so called "wave parameter" D should be small: $D{\sim}\lambda L/\Sigma{\ll}1$, where Σ is the aperture area, and λ is the wavelength under consideration. The observation point also cannot be close to the focuses and caustics.

The aperture method is more general [6–11]. It is valid for observation points with arbitrary wave parameter D, including the Fraunhofer area (or far-field area) where $D{\gg}1$, as well as the neighborhoods of focuses and caustics. At the final step of this technique we calculate the field outside the target using the known Stratton-Chu formulas ("aperture integrals"). These formulas allow determining the field in the surrounding space if tangential components of electric and magnetic fields on the aperture are known. In papers [8, 11] we have verified the aperture method for the cone target using simulations in COMSOL Multiphysics. It has been showed that this technique can be applied even for the objects having the size of several wavelengths.

APERTURE INTEGRALS

Aperture integrals (or Stratton-Chu formulas) for Fourier transform of electric field can be written in the following general form (we use Gaussian system of units) [8–11]:

$$\vec{E}\left(\vec{R}\right) = \vec{E}^{(h)}\left(\vec{R}\right) + \vec{E}^{(e)}\left(\vec{R}\right),$$

$$\vec{E}^{(h)}\left(\vec{R}\right) = \frac{ik}{4\pi}\int_{\Sigma}\left\{\left[\vec{n}'\times\vec{H}\left(\vec{R}'\right)\right]G\left(\left|\vec{R}-\vec{R}'\right|\right) + \right.$$
$$\left. + \frac{1}{k^2}\left(\left[\vec{n}'\times\vec{H}\left(\vec{R}'\right)\right]\cdot\nabla'\right)\nabla' G\left(\left|\vec{R}-\vec{R}'\right|\right)\right\}d\Sigma', \quad (1)$$
$$\vec{E}^{(e)}\left(\vec{R}\right) = \frac{1}{4\pi}\int_{\Sigma}\left[\left[\vec{n}'\times\vec{E}\left(\vec{R}'\right)\right]\times\nabla' G\left(\left|\vec{R}-\vec{R}'\right|\right)\right]d\Sigma',$$

where Σ is the aperture area, $\vec{E}\left(\vec{R}'\right)$, $\vec{H}\left(\vec{R}'\right)$ is the field on the aperture, $k{=}\omega/c$ is the wave number of the outer space (vacuum), $\vec{n}'$ is the unit external normal to the aperture in the point $\vec{R}'$, $G(R) = \exp(ikR)/R$ is the Green function of Helmholtz equation, and ∇' is the gradient: $\nabla' = \vec{e}_x\partial/\partial x' + \vec{e}_y\partial/\partial y' + \vec{e}_z\partial/\partial z'$. Analogous formulas are known for the magnetic field as well. Note that $\left|\vec{E}\right| \approx \left|\vec{H}\right|$ in the points remote from the aperture for several wavelengths. It is often interesting to obtain the wave field in Fraunhofer area (or far-field area) where the "wave parameter" D is large: $D{\sim}\lambda R/\Sigma{\gg}1$. In this area, expressions (1) are simplified to the following form:

$$\left\{\begin{matrix}\vec{E}^{(h)}\\ \vec{E}^{(e)}\end{matrix}\right\} \approx \frac{ike^{ikR}}{4\pi R}\int_{\Sigma}\left\{\begin{matrix}\left[\vec{e}_R\times\left[\vec{e}_R\times\left[\vec{H}\left(\vec{R}'\right)\times\vec{n}'\right]\right]\right]\\ \left[\vec{e}_R\times\left[\vec{n}'\times\vec{E}\left(\vec{R}'\right)\right]\right]\end{matrix}\right\}\times$$
$$\times\exp(-ik\vec{e}_R\vec{R}')d\Sigma', \quad (2)$$

where $\vec{e}_R = \vec{R}/R$.

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† a.tyuhtin@spbu.ru

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THE BALL WITH CHANNEL

Here we consider the case of the dielectric ball with radius R_0 having the cylindrical vacuum channel with radius a. It is assumed that $kR_0 \gg 1$. The ball material is characterized by permittivity ε, permeability μ and the refractive index $n = \sqrt{\varepsilon\mu}$. The channel axis ($z$) coincides with the ball diameter. The point charge q moves along the z-axis with velocity $\vec{V} = c\vec{\beta}$ and intersects the ball center $x=y=z=0$ at the moment $t=0$. Further we use the spherical (R, θ, φ) and cylindrical (r, φ, z) coordinates.

Under additional condition $k(R-R_0) \gg 1$ one can obtain the following expression for Fourier-transform of the non-zero electric field components: $\vec{E} = \vec{E}^{(h1)} + \vec{E}^{(h2)} + \vec{E}^{(e)}$, where

$$\begin{Bmatrix} E_r^{(h1)} \\ E_z^{(h1)} \end{Bmatrix} = \frac{ikR_0^2}{4\pi} \int_{\theta_1}^{\theta_2} d\theta' \int_0^{2\pi} d\varphi' \sin\theta' \frac{e^{ik\tilde{R}}}{\tilde{R}} H_{\varphi'}^{(t)}(\theta') \times \\ \times \begin{Bmatrix} -\cos\theta' \cos\varphi' \\ \sin\theta' \end{Bmatrix}, \qquad (3)$$

$$\begin{Bmatrix} E_r^{(h2)} \\ E_z^{(h2)} \end{Bmatrix} = \frac{ikR_0^2 R}{4\pi} \int_{\theta_1}^{\theta_2} d\theta' \int_0^{2\pi} d\varphi' \sin\theta' \frac{e^{ik\tilde{R}}}{\tilde{R}^3} H_{\varphi'}^{(t)}(\theta') \times \\ \times \begin{Bmatrix} R_0 \sin\theta' \cos\varphi' - R\sin\theta \\ R_0 \cos\theta' - R\cos\theta \end{Bmatrix} \times \\ \times (\cos\theta \sin\theta' - \sin\theta \cos\theta' \cos\varphi'), \qquad (4)$$

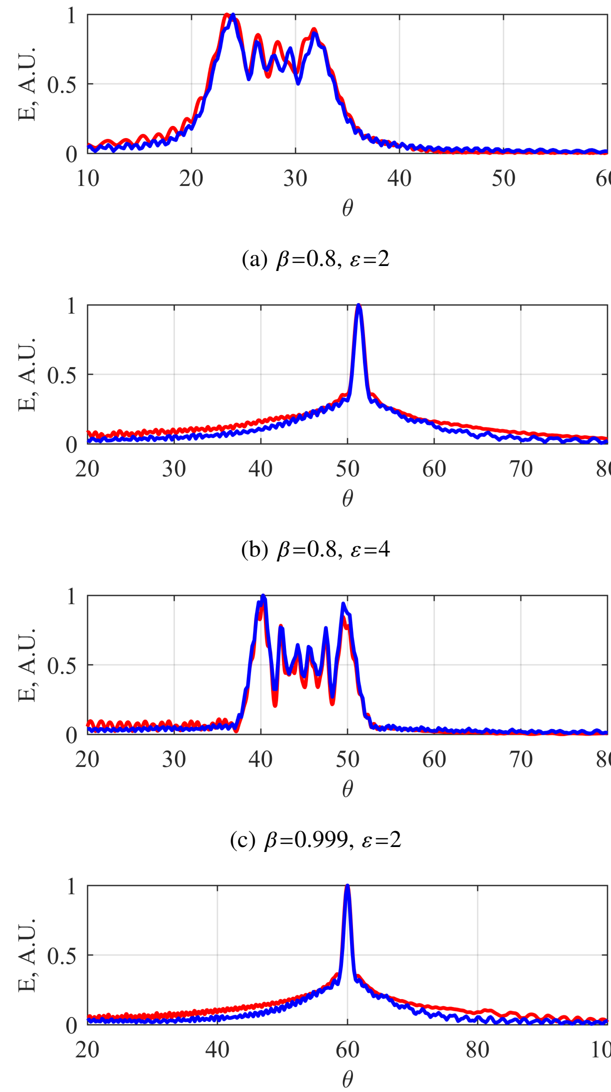
$$\begin{Bmatrix} E_r^{(e)} \\ E_z^{(e)} \end{Bmatrix} = \frac{ikR_0^2}{4\pi} \int_{\theta_1}^{\theta_2} d\theta' \int_0^{2\pi} d\varphi' \sin\theta' \frac{e^{ik\tilde{R}}}{\tilde{R}^2} E_{\theta'}^{(t)}(\theta') \times \\ \times \begin{Bmatrix} (R_0 \cos\theta' - R\cos\theta)\cos\varphi' \\ -R_0 \sin\theta' + R\sin\theta\cos\varphi' \end{Bmatrix}. \qquad (5)$$

Here $\tilde{R} = \sqrt{R_0^2 + R^2 - 2RR_0(\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\varphi')}$, $E_{\theta'}^{(t)}(\theta')$, $H_{\varphi'}^{(t)}(\theta')$ are tangential components of the transmitted field on the surface of the ball, and the angles θ_1, θ_2 determine boundaries of the area of the sphere where Cherenkov radiation penetrate from the ball:

$$\theta_1 = \max\{\theta_p - \theta_*, \arcsin(a/R_0)\}, \quad \theta_2 = \min\{\theta_p + \theta_*, 2\theta_p\}, \qquad (6)$$

where $\theta_* = \arcsin(1/n)$. The field components on the sphere are approximately equal to $H_{\varphi'}^{(t)} = T_v H_{\varphi'}^{(i)}$, $E_{\theta'}^{(t)} = H_{\varphi'}^{(t)} \cos\theta_t$, where $H_{\varphi'}^{(i)}(\theta')$ is the component of incident field [11, 12] on the ball surface, $T_v(\theta') = \frac{2\cos\theta_i(\theta')}{\cos\theta_i(\theta') + \sqrt{\varepsilon/\mu}\cos\theta_t(\theta')}$ is the transmission coefficient, $\theta_p = \arccos(1/(n\beta))$ is the angle of Cherenkov radiation, $\theta_i(\theta') = \theta' - \theta_p$ is the incidence angle, $\theta_t(\theta') = \arcsin(n\sin\theta_i(\theta'))$ is the refraction angle.

The comparison between analytical results and numerical simulations performed in COMSOL Multiphysics are shown in Fig. 1. One can see a good coincidence of these results. The best coincidence takes place in the most important area of large magnitudes of the field, because the firstly refracted wave plays the main role in this area.



(a) β=0.8, ε=2

(b) β=0.8, ε=4

(c) β=0.999, ε=2

(d) β=0.999, ε=4

Figure 1: Electric field amplitude (in normalized units) obtained by the aperture technique (red) and the COMSOL Multiphysics simulations (blue) depending on the angle θ; $R_0 = 300c/\omega$, $a = c/\omega$, $R = 600c/\omega$.

Radiation patterns in the Fraunhofer area are presented in Fig. 2. We see, that the angle width of the pattern increases with increase of permittivity of the ball material (due to the larger scatter of the refraction angles for the optically denser material).

In Fig. 3, the results of numerical simulations in COMSOL Multiphysics are presented. The plots show distribution of the electric field amplitude from the point charge outside the dielectric ball (vertical axis is coincides with direction of the charge movement). One can see that radiation is concentrated in certain areas. It is interesting that the field can increases with distance from the sphere for certain angle range (due to the intersection of the refracted rays). As the

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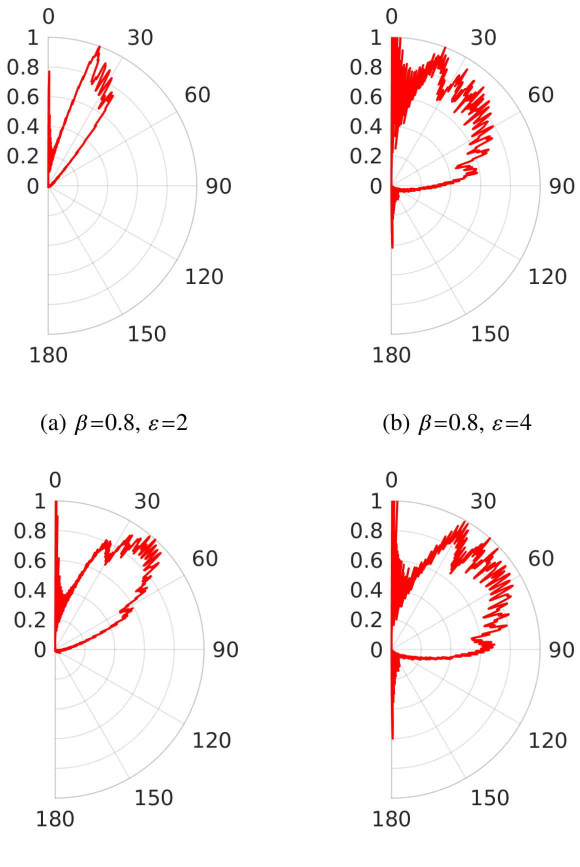
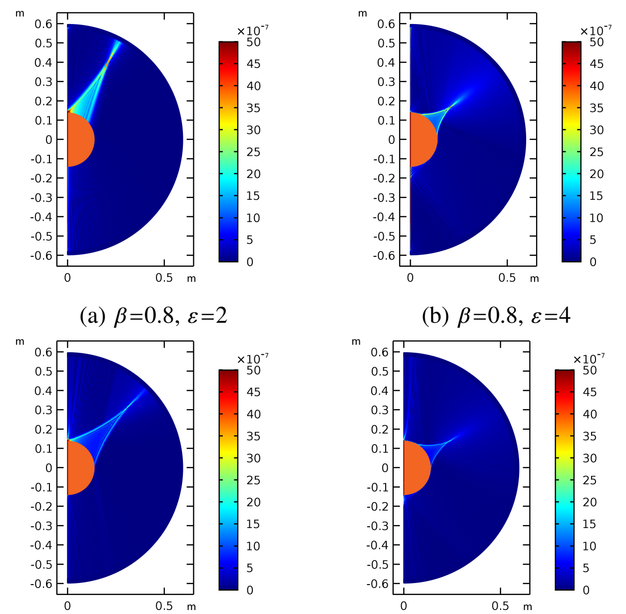


(a) β=0.8, ε=2 (b) β=0.8, ε=4

(c) β=0.999, ε=2 (d) β=0.999, ε=4

Figure 2: Radiation patterns for the electric field amlitude (in normalized units); calculation parameters are the same as in Fig. 1.

distance increases further, the field decreases due to the ray divergence and the diffraction spreading.



(a) β=0.8, ε=2 (b) β=0.8, ε=4

(c) β=0.999, ε=2 (d) β=0.999, ε=4

Figure 3: COMSOL Multiphysics results for the electric field amplitude (V/m); R_0=300c/ω, a=c/ω, q=1nC, ω=2π100 GHz

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