

# APPLICATIONS OF DIMENSION-REDUCTION TO VARIOUS ACCELERATOR PHYSICS PROBLEMS

W. F. Bergan<sup>†</sup>, I. V. Bazarov, C. J. R. Duncan, and D. L. Rubin,  
Cornell University, Ithaca, NY, USA

## Abstract

Particle accelerators contain hundreds of magnets, making dimension-reduction techniques attractive when attempting to tune them. We apply this procedure to two different problems: correcting the orbit in the Cornell synchrotron and maximizing the dynamic aperture in the Cornell Electron Storage Ring (CESR). Cornell's rapid cycling synchrotron accepts a 200 MeV beam from the linac and accelerates it to 6 GeV for injection into the CESR. 'Kicker coils' (dipole correctors) are used to correct for residual fields which would otherwise cause beam loss at the low energies. In such cases, it is usually advisable to measure and correct the orbit. However, one cannot measure the orbit without first getting the beam to circulate a few hundred times, by which point the low-energy orbit would already be mostly corrected. In order to speed up the process of empirical orbit tuning, we form knobs which have the largest effect on the global orbit error, so that the dimensionality of the space which must be searched may be greatly reduced. A small dynamic aperture in CESR will have adverse effects on beam lifetime and injection efficiency, and so ought to be maximized by tuning sextupoles. However, it is often unclear which sextupoles one ought to tune to alleviate the problem. Moreover, once the chromaticity is properly adjusted, it should not be changed. Since we expect resonance driving terms (RDTs) to have a large impact on the dynamic aperture, we develop sextupole knobs which change the RDTs as much as possible while leaving the chromaticity fixed.

## INTRODUCTION

Particle accelerators are complicated instruments, consisting of hundreds if not thousands of magnets which must all be optimized to achieve the best machine performance. Moreover, regardless of the care with which one runs simulations to obtain a good starting point, inevitable misalignments and other errors force one to optimize directly on the real machine. High dimensional systems frequently pose difficulties for fast optimization, motivating methods to reduce the number of dimensions of the problem.

To further motivate this dimension reduction, we make use of the concept of sloppy models. This posits that, for many high-dimensional systems, low-dimensional approximations exist which capture most of the interesting behavior of the full system [1–4]. We have had success in applying this to minimize the vertical emittance at the Cornell Electron Storage Ring (CESR) [5–8]. However, since this is a broad concept, we propose to extend it to other aspects of machine tuning.

<sup>†</sup> wfb59@cornell.edu

We first investigate the use of sloppy models to efficiently tune the low-energy orbit in the 756-meter Cornell synchrotron, which accelerates 200 MeV positrons provided by the linac to 6 GeV for injection into CESR. The main issues arise at injection energy (200 MeV), where residual magnetic fields may have a large impact on the beam. 47 horizontal and 46 vertical kickers are used to provide low-energy orbit correction. However, beam-position measurements are not useful for orbit-correction due to the fact that once we have sufficient turns to obtain an orbit, it is already fairly well-corrected. This leads us to use the beam current accepted and accelerated by the synchrotron as our metric of performance and so we desire some systematic method to tune using this measurement. We therefore wish to determine the best groups of kickers to use to fix the synchrotron orbit for different anticipated failure modes, with the understanding that these may be quickly tuned by the operator in order to improve beam transmission.

We also wish to see how useful dimension-reduction can be when applied to the problem of maximizing the dynamic aperture in CESR. Sextupoles are used in storage rings in order to correct the horizontal and vertical chromaticity, but necessarily introduce nonlinear dynamics, resulting in a finite dynamic aperture. Since an insufficient dynamic aperture harms both lifetime and injection efficiency, the strengths of the sextupoles in a storage ring are optimized in simulations at the design stage in order to maximize it. However, unknown multipole moments also contribute to the resonance driving terms, and, especially when starting up the machine for the first time, there is no guarantee that the sextupole strengths truly match the design. When poor injection efficiency or lifetime is observed, it is desirable to adjust sextupoles to improve the dynamic aperture. However, CESR has 76 independently-powered sextupoles, so that even after removing two degrees of freedom to prevent the chromaticities from changing, the space is still too large to search efficiently. We therefore investigate the use of sextupole knobs which preserve the chromaticity, but give the largest expected improvement to the dynamic aperture.

## SYNCHROTRON ORBIT TUNING

For minimizing orbit errors in the synchrotron, we only deal with the horizontal case here, since vertical tuning will proceed similarly. We first consider the case of distributed random dipole errors. To attempt to correct these, we take the singular value decomposition (SVD) of the response matrix  $J$ , where  $J_{ij}$  is the change in orbit at location  $i$  due to a unit change in the strength of steering  $j$ . The locations to evaluate the orbit were placed every 10 cm in the synchrotron

in simulation. The right singular vectors obtained through this process are the groups of kickers which give the largest effect on the orbit error. To test these in simulation, we introduced random strength errors in the synchrotron dipoles and used each singular vector in sequence, starting with the most important, to try to minimize the maximum orbit error. This metric was chosen because the beam loss will not be due to the cumulative effect of small orbit errors, but rather will occur where the beam is steered into or near the beam pipe. We see the results of this optimization in Fig. 1. We find that the maximum orbit deviation has been roughly halved by the eight highest-eigenvalue knobs.

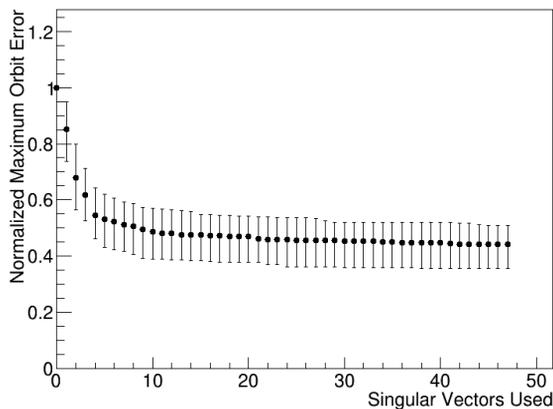


Figure 1: The maximum orbit error remaining after tuning using a given number of knobs, normalized to the initial maximum orbit error. We take the median and interquartile range over a distribution of random dipole kicks. The maximum orbit may be reduced by roughly a factor of two using the eight knobs with the highest eigenvalues.

We also consider the case where one of the kickers becomes disabled. In this case, the orbit error is due a local kick at the location of the bad kicker. Moreover, in trying to design a knob to address this condition, we must take into account the fact that the problematic kicker will not respond to whatever command it is given. The optimal knob for a particular kicker, call it kicker  $n$ , may be determined as follows. As above, we start with the orbit response matrix  $J$ . The vector of orbit errors due to a unit deviation in this kicker will be given by  $x_i = J_{in}$ . If we then construct the response matrix  $\tilde{J}$  in the space of kickers excluding the unresponsive one and find its pseudo-inverse,  $\tilde{J}^+$ , we may write the vector of necessary kicker changes  $k_j$  as  $k_j = -\tilde{J}_{ji}^+ x_i = -\tilde{J}_{ji}^+ J_{in}$ . Since the problematic kicker can usually be easily identified, this approach should be sufficient, and in simulation typically reduces the maximum orbit error by a factor of 3. However, it would be convenient if we could identify a single or a few knobs that will remedy the problem, regardless of which kicker is broken. We therefore construct the matrix  $B$ , where the element  $B_{jn}$  is the strength of kicker  $j$  when one wishes to fix the orbit error caused by steering  $n$  being unresponsive and off by one unit. The SVD of this matrix

gives us kicker combinations for mitigating typical errors due to an unresponsive kicker.

To test these knobs in simulation, for each horizontal steering in the synchrotron, we introduce some fixed error disable it, so that it will not be adjusted in tuning. We then use the knobs obtained from the SVD of  $B$  one at a time to reduce the maximum orbit deviation. The results are shown in Fig. 2. We see that the highest-eigenvalue singular vector alone is able to reduce the maximum orbit by more than a factor of 2, while subsequent knobs achieve relatively little. This suggests that this knob captures most of the ability to reduce orbit errors due to disabled synchrotron kickers, and provides a simple and effective way to perform this tuning.

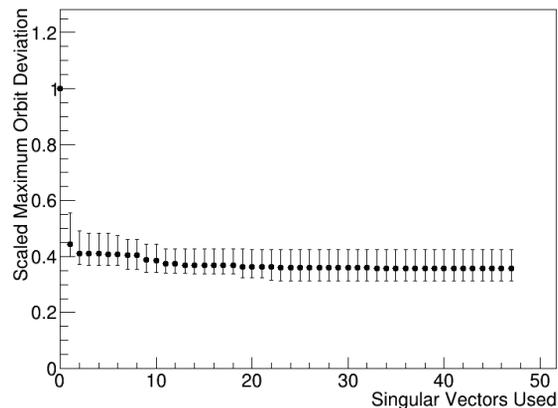


Figure 2: The maximum orbit error remaining after tuning using a given number of knobs, normalized to the initial maximum orbit error. We take the median and interquartile range over a distribution of each kicker in turn being disabled, and the knobs are from the SVD of the matrix  $B$  (see text). In most cases, the highest-eigenvalue knob alone is able to reduce the maximum orbit error by more than a factor of 2.

## DYNAMIC APERTURE TUNING

For maximizing the dynamic aperture in CESR, we wish to find combinations of our 76 sextupoles which significantly affect the dynamic aperture while preserving the chromaticities. The amplitude-dependent tune shift (ADTS), which describes how the tune changes with particle amplitude, and resonance driving terms (RDTs), which describe the strengths of the various resonances in an accelerator [9], are used to characterize the dynamic aperture. We therefore assume that the most important knobs for tuning the dynamic aperture are those which have the largest impact on the RDTs and ADTS. We construct a response matrix for the effect of the sextupoles on the chromaticities, first and second order resonance driving terms, and amplitude-dependent tune shifts, with weights reflecting the relative importance of these parameters. We may then work in the null space of the chromaticity response matrix and take the SVD of the RDT-ADTS response matrix. This gives us knobs which are expected to have a large impact on the dynamic aperture without altering the chromaticity.

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In order to test these knobs, we run simulations to introduce Gaussian random errors into the strengths of the sextupoles in CESR with a standard deviation of  $0.5 \text{ m}^{-3}$ . The original dynamic aperture and the dynamic aperture for the lattice with altered sextupole distribution are shown as Fig. 3 and 4, respectively. We use optimizers built into Tao [10] to first fix the horizontal and vertical chromaticities with existing chromaticity knobs. We then use the eight RDT-derived sextupole knobs with the largest eigenvalues to maximize the horizontal dynamic aperture for on-energy and off-energy particles. The results of this corrected sextupole distribution are shown in Fig. 5. We see a significant recovery in the dynamic aperture. Note that we do not attempt to recover the vertical dynamic aperture, since it is already larger than the physical vertical aperture of the storage ring.

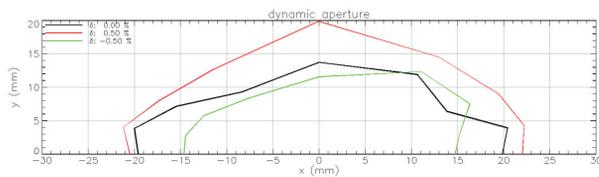


Figure 3: The dynamic aperture for CESR with the ideal lattice and sextupole distribution.

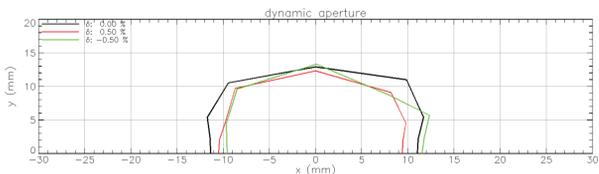


Figure 4: The dynamic aperture for CESR with random errors introduced in the sextupole distribution.

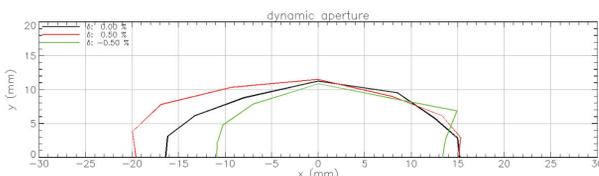


Figure 5: The dynamic aperture for CESR after chromaticity correction and dynamic aperture optimization with the eight best sextupole knobs.

## CONCLUSIONS

We have demonstrated that dimension-reduction techniques are compatible with dynamic aperture optimization, with two chromaticity knobs and eight dynamic-aperture knobs being sufficient to recover the original chromaticities and much of the dynamic aperture. We have also shown that such techniques can be used to perform some orbit correction in the absence of reliable beam position data.

## FUTURE WORK

The optimization of dynamic aperture is a nonlinear problem, and, as such, our linearized methods may not hold

generally. We therefore wish to explore methods which will allow a fuller interaction with the nonlinear picture. We also wish to test the knobs described here on the synchrotron and CESR.

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