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3D THEORY OF MICROBUNCHED ELECTRON COOLING FOR ELECTRON-ION COLLIDERS*

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Abstract

The Microbunched Electron Cooling (MBEC) [1] is a promising cooling technique that can find applications in future hadron and electron-ion colliders. A 1D model of MBEC has been recently developed in Ref. [2, 3]. This model predicts the cooling time below two hours for eRHIC 255 GeV proton beams, when two amplification sections are used in the cooling system. In this work, we go beyond the 1D model of Ref. [2] and develop a realistic 3D theory of MBEC. We derive an analytical expression for the cooling rate. Our analytical results are in reasonable agreement with computer simulations.

INTRODUCTION

The idea of coherent electron cooling has been originally proposed by Ya. Derbenev [4] as a way to achieve cooling rates higher than those provided by the traditional electron cooling technique [5, 6]. The mechanism of the coherent cooling can be understood in a simple setup shown in Fig. 1. An electron beam with the same relativistic γ-

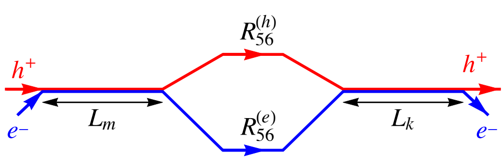

Figure 1: Schematic of the microbunched electron cooling system. Blue lines show the path of the electron beam, and the red lines indicate the trajectory of the hadron beam.

factor as the hadron beam co-propagates with the hadrons in a section of length L_m called the "modulator". In this section, the hadrons imprint microscopic energy perturbations onto the electrons via the Coulomb force. After the modulation, the electron beam passes through a dispersive chicane section, $R_{56}^{(e)}$, where the energy modulation of the electrons is transformed into a density fluctuation referred to as "microbunching"[1]. Meanwhile, the hadron beam passes through its dispersive section, $R_{56}^{(h)}$, in which more energetic particles move in the forward direction with respect to their original positions in the beam, while the less energetic particles trail behind. When the beams are combined again in a section of length L_k called the "kicker", the electric field of the induced density fluctuations in the electron beam acts back on the hadrons. With a proper choice of the chicane strengths, the energy change of the hadrons in the kicker leads, over many passages through the cooling section, to a gradual decrease of the energy spread of the hadron beam. The transverse cooling is achieved in the same scheme by introducing dispersion in the kicker for the hadron beam.

In most cases, the cooling rate in the simple setup shown in Fig. 1 is not fast enough for practical applications. It can be considerably increased if the fluctuations in the electron beam are amplified on the way from the modulator to the kicker. Following an earlier study by Schneidmiller and Yurkov [8] of microbunching dynamics for generation of coherent radiation, Ratner proposed a broadband amplification mechanism [1] in which the amplification is achieved through a sequence of drifts and chicanes such that the density perturbations in the drifts execute a quarter-wavelength plasma oscillation. A detailed theory of the amplification in MBEC is recently published in Ref. [3].

Previous analysis of MBEC in Refs. [1–3] used a 1D model of particle interaction in which particles are replaced by thin slices in the beam. In this paper we will develop a 3D theory for the Microbunched Electron Cooling (MBEC) where particles are treated as point charges and compare its results with the 1D model.

MBEC COOLING IN 3D

In the 1D model of MBEC in Ref. [2] we replaced point charges of the hadron and electron beams by charged sheets with a Gaussian charge distribution in the transverse direction. The force f_z between a hadron with charge Ze and an electron with charge $-e$ in this model is given by

$$f_z = -\frac{Ze^2}{\Sigma^2}\Phi\left(\frac{z\gamma}{\Sigma}\right), \tag{1}$$

where $z = z_e - z_h$ is the distance between the charges, Σ is the rms transverse size of the beams (which are assumed axisymmetric and of the same cross section), and the expression for the function Φ can be found in [2]. The coordinates z_e for the electron and z_h for the hadron are measured along the longitudinal axis of the beams. As was shown in Ref. [2], the cooling rate in the system shown in Fig. 1 is expressed in terms of the imaginary part of the effective impedance $\mathcal{Z}_{1D}(\varkappa)$,

$$\mathrm{Im}\mathcal{Z}_{1D}(\varkappa) = -\frac{4I_e L_m L_k}{c\Sigma^2\gamma^3 I_A\sigma_e} q_e\varkappa e^{-\varkappa^2 q_e^2/2} H_{1D}^2(\varkappa), \tag{2}$$

where $\varkappa = k\Sigma/\gamma$ is the dimensionless wave number, I_e is the electron beam current, $I_A = m_e c^3/e = 17$ kA is the

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[1] In a long modulator section the microbunching can be generated directly in the modulator when the energy modulation is converted into a density fluctuation through plasma oscillations [7].

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Alfvén current, σ_e is the rms electron relative energy spread, $q_e = R_{56}^{(e)}\sigma_e\gamma/\Sigma$, with the function H defined as

$$H_{1D}(\varkappa) = \int_0^\infty d\xi\,\Phi(\xi)\sin(\varkappa\xi). \quad (3)$$

The cooling rate of hadrons is

$$N_c^{-1} = -\frac{2q_h r_h c}{\pi\Sigma\sigma_h}\int_0^\infty \varkappa\,d\varkappa\,\mathrm{Im}\mathcal{Z}_{1D}(\varkappa)\,e^{-\varkappa^2 q_h^2/2}, \quad (4)$$

where N_c is the cooling time in the number of the revolution periods, $r_h = (Ze)^2/m_h c^2$, σ_h is the rms relative energy spread of hadrons and $q_h = R_{56}^{(h)}\sigma_h\gamma/\Sigma$.

It is now straightforward to generalize the 1D approach to a 3D one. We just need to replace the interaction force of slices (1) with the longitudinal Coulomb force of relativistic point charges moving in z-direction,

$$f_z = -Ze^2\frac{z\gamma}{(x^2+y^2+\gamma^2 z^2)^{3/2}}, \quad (5)$$

where $x = x_e - x_h$ and $y = y_e - y_h$ with x_e, y_e the electron coordinates and x_h, y_h the hadron coordinates in the Cartesian coordinate system. This equation can be also written as

$$f_z = -\frac{Ze^2}{\Sigma^2}\Phi_{3D}\left(\frac{r}{\Sigma},\frac{z\gamma}{\Sigma}\right)$$

where $r = \sqrt{x^2+y^2}$ and $\Phi_{3D}(\rho,\zeta) = \zeta(\rho^2+\zeta^2)^{-3/2}$. Respectively, function H_{1D} in Eq. (3) is now replaced by

$$H_{3D}(\rho,\varkappa) = \int_0^\infty d\zeta\,\Phi_{3D}(\rho,\zeta)\sin(\varkappa\zeta). \quad (6)$$

Note that using Eq. (5) for the interaction force we ignore the relative particle motion during the interaction which becomes more important at small distances. We also ignore the polarization effects that tend to shield the interaction (the Debye shielding) at large distances.

The square of the function H_{1D} in Eq. (2) comes from two events of the particle interaction: the first one is in the modulator and the second one in the kicker. In the derivation of the cooling rate (4), it was assumed that the particles do not change their relative longitudinal positions when they propagate from the modulator to the kicker, and hence their interaction force is the same in these two places. This assumption cannot be justified in 3D, where the transverse particles' coordinates in the modulator, x_e^M, y_e^M and x_h^M, y_h^M will likely change to different values x_e^K, y_e^K and x_h^K, y_h^K in the kicker due to the betatron oscillations in the path connecting these two regions[2]. With account of this fact, the function $H_{1D}^2(\varkappa)$ in Eq. (2) should be replaced by

$$\langle H_{3D}\left(\frac{1}{\Sigma}\sqrt{(x_e^M-x_h^M)^2+(y_e^M-y_h^M)^2},\varkappa\right) \times H_{3D}\left(\frac{1}{\Sigma}\sqrt{(x_e^K-x_h^K)^2+(y_e^K-y_h^K)^2},\varkappa\right)\rangle \quad (7)$$

[2] Here we assume that x_e^M, y_e^M and x_h^M, y_h^M do not change through the length of the modulator, which is true if the modulator length is much smaller than the period of the betatron oscillations. The same assumption is made with regard to the kicker.

where the angular brackets denote averaging over the Gaussian transverse distribution functions of the hadrons and electrons in the modulator. In this formula the kicker coordinates are functions of the modulator ones: $x_e^K(x_e^M, y_e^M)$, $y_e^K(x_e^M, y_e^M)$, $x_h^K(x_h^M, y_h^M)$, $y_h^K(x_h^M, y_h^M)$ which are determined by the optics between the modulator and the kicker. With this replacement of the H_{1D}^2 by Eq. (7) we obtain the cooling rate in the 3D geometry.

SIMPLIFIED MODEL OF ZERO BETATRON PHASE ADVANCE

Using Eq. (7) requires the knowledge of the beam optics in the MBEC cooler which is not currently available. It makes sense then to consider a special case of the betatron phase advance between the modulator and the kicker equal to $2\pi n$ (for both electrons and hadrons), where n is an integer, and with the equal beta functions in the modulator and the kicker. This means that the transverse coordinates of electrons and hadrons in the modulator and the kicker are the same. Dropping the superscripts ^{M} and ^{K} in the transverse coordinates we then need to calculate the function $\langle H_{3D}\left(\frac{1}{\Sigma}\sqrt{(x_e-x_h)^2+(y_e-y_h)^2},\varkappa\right)^2\rangle$ and substitute it into Eq. (2). The angular bracket in this expression denote averaging over the distribution functions of hadrons and electrons.

Using Eq. (6) and the formula for $\Phi_{3D}(\rho,\zeta)$ it is easy to find that $H_{3D}(\rho,\varkappa) = \varkappa K_0(\rho\varkappa)$. For averaging of H_{3D}^2 we will use the Gaussian distribution functions for electrons, $f_e(x_e,y_e) = (2\pi\Sigma^2)^{-1}e^{-(x_e^2+y_e^2)/2\Sigma^2}$, and the same function for the hadrons, $f_h(x_h,y_h) = (2\pi\Sigma^2)^{-1}e^{-(x_h^2+y_h^2)/2\Sigma^2}$, which means that both beams have the same transverse size Σ in x and y directions. Then the averaged value is given by the following integral:

$$\langle H_{3D}^2\rangle = \frac{\varkappa^2}{(2\pi)^2}\int d\hat{x}_e d\hat{y}_e d\hat{x}_h d\hat{y}_h e^{-(\hat{x}_e^2+\hat{y}_e^2)/2}e^{-(\hat{x}_h^2+\hat{y}_h^2)/2} \times K_0^2\left(\varkappa\left((\hat{x}_e-\hat{x}_h)^2+(\hat{y}_e-\hat{y}_h)^2\right)^{1/2}\right), \quad (8)$$

where the hat variables are the coordinates normalized by Σ. The plot of this function computed numerically is shown in Fig. 2. For comparison we also show the plot of the function $H_{1D}^2(\varkappa)$ from 1D theory.

Replacing $H_{1D}^2(\varkappa)$ in Eq. (2) by $\langle H_{3D}^2\rangle$ and substituting the result in Eq. (4) we obtain the following formula for the cooling rate in 3D,

$$N_c^{-1} = \frac{8I_e L_m L_k r_h q_h q_e}{\pi\Sigma^3\gamma^3 I_A\sigma_e\sigma_h}\int_0^\infty \varkappa^2 d\varkappa e^{-\varkappa^2(q_e^2+q_h^2)/2}\langle H_{3D}^2\rangle. \quad (9)$$

Note first, that because the 3D H-function is larger than the 1D one the cooling rate in 3D is faster that the one estimated in Ref. [2]. Also note a remarkable difference in the behavior of this two functions for $\varkappa \gg 1$: when H_{1D}^2 decreases (as $1/\varkappa^2$) in this limit, its 3D analog tends to a constant

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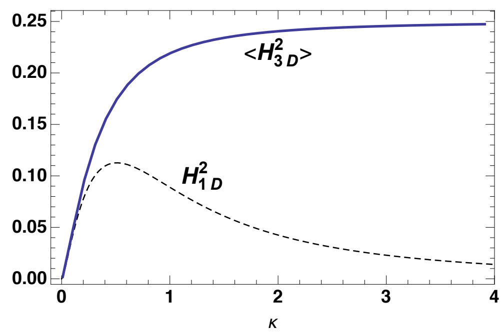

Figure 2: Plot of $\langle H^2_{3D}\rangle$ and H^2_{1D} as functions of $\varkappa$.

value (one can show that this value is equal to $\frac{1}{4}$). The finite value of $\langle H^2_{3D}\rangle$ in the limit $\varkappa \to \infty$ qualitatively changes the behavior of the cooling rate as a function of the dimensionless chicane strengths q_e and q_h: in 1D there is an optimal value for these parameters, but in 3D, as follows from the analysis of Eq. (9), the cooling rate increases without limit when $q_h, q_e \to 0$ (because the integral $\int_0^\infty \varkappa^2\, d\varkappa\, \langle H^2_{3D}\rangle$ diverges). This seemingly strange dependence is explained in the following way. Large values of $\varkappa$ correspond to close interactions of the particles, and due to the singular behavior of the force (5) at small distances these near collisions contribute considerable amount to the cooling rate. In reality, this effect would disappear when one would take into account that the particles shift during the interaction (and also when the assumption of the phase advance $2\pi n$ is dropped).

COMPARISON WITH COMPUTER SIMULATIONS

We carried out computer simulations of the cooling rate to compare it with our theoretical analysis. The computer model is described in Ref. [2] — it was modified to add the transverse coordinates and the force (5). The transverse positions of the particles in both beams were randomly assigned from a Gaussian distribution with the rms width Σ. We also added to the code a control parameter ϵ to eliminate the effect of strong interaction of particles at small distances: an interaction between a hadrons and an electron was turned off if the distance between them was too small, $(x_h - x_e)^2 + (y_h - y_e)^2 + (z_h - z_e)^2\gamma^2 < \epsilon\Sigma^2$. The results of the simulations for different values of the parameters ϵ are shown in Fig. 3 as a function of the dimensionless chicane strength $q_h = q_e = q$. The simulations were carried out for zero (or, equivalently, $2\pi n$) phase advance between the modulator and the kicker, that is for $x_e^K = x_e^M$ $y_e^K = y_e^M$ and $x_h^K = x_h^M$ and $y_h^K = y_h^M$. Three values of the parameter ϵ were used: 0.01, 0.005 and zero. The smaller values of ϵ required more averaging in the runs. Note that the cooling time monotonically decreases together with q in contrast to the 1D model of Ref. [2], as predicted by Eq. (9).

We also did simulations for the case of the relative phase advance (between the hadrons and electrons) $\mu = \pi$. In this case the interacting particles that happen to be close in the modulator, go to opposite transverse positions in the kicker. More specifically, we assumed that $x_e^K = -x_e^M$ $y_e^K = -y_e^M$ and $x_h^K = x_h^M$ and $y_h^K = y_h^M$. Remarkably, after calculating the average given by Eq. (7) we found that numerically it coincides with H^2_{1D}. This conclusion is also confirmed by numerical simulations of the 3D case shown in Fig. 4. In contrast to the case $\mu = 2\pi n$, the cooling time now has a minimum (at $q = 0.6$), exactly as in the case of the 1D model.

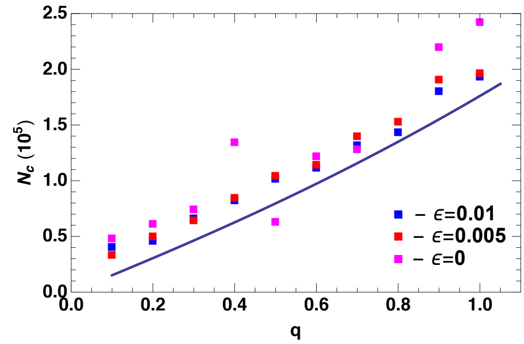

Figure 3: Comparison of the cooling time N_c given by Eq. (9) (solid curve) with simulations (color symbols).

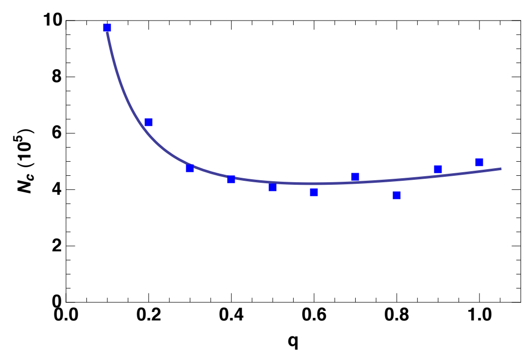

Figure 4: Cooling time N_c for the case $\mu = \pi$ (solid curve) in comparison with simulations (color symbols).

SUMMARY

In this work, we extended the 1D model of Ref. [2] to three dimensions. We showed that the cooling rate derived in 1D can also be used in 3D if the spectral form of the interaction function between charged slices is replaced by the Fourier transformed Coulomb force between point charges. In addition, the 3D functions need to be properly averaged over the transverse cross section of the beam. We found that for the case of $2\pi n$ phase advance the cooling time approaching zero in the limit $R_{56} \to 0$. We also discovered that in the case of $\mu = \pi$ the 3D model gives the same cooling rate as the 1D one. Our analytical results are confirmed by computer simulations.

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