BEAM DYNAMICS SIMULATIONS WITH CRAB CAVITIES IN THE SPS MACHINE*

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Abstract

The LHC Upgrade, called High Luminosity LHC, aims to increase the integrated luminosity by a factor of 10. To achieve this, the project relies on a number of key innovative technologies, including the use of superconducting Crab Cavities with ultra-precise phase control for beam rotation. A set of prototype Crab Cavities has been recently installed in the second largest machine of CERN, the Super Proton Synchrotron (SPS), that operated as a test-bed from May to November of 2018. The tight LHC constraints call for axially non-symmetric cavity designs that introduce high order multipole components. Furthermore, the Crab Cavities in the presence of SPS non-linearities can affect the long term stability of the beam. This paper presents how the SPS dynamic aperture is affected for different cavity, machine and beam configurations.

INTRODUCTION

The High Luminosity LHC (HL-LHC) aims to increase the instantaneous luminosity of LHC to $L \sim 5 \cdot 10^{34} \text{ cm}^{-2} \cdot \text{s}^{-1}$ and the integrated luminosity from 300 fb⁻¹ to 3000 fb⁻¹. Among other upgrades, this will be achieved by reducing the beta function at the Interaction Point (IP); however, the stronger focusing unavoidably increases the crossing angle, which leads to a reduction of the geometrical reduction factor and in turn of the luminosity. To counteract this effect a Crab Crab Cavity (CC) scheme will be employed that will rotate the bunches just before arriving at the IP, allowing the full length of each bunch to be seen by the colliding bunch.

Until recently, the CCs have only been tested with leptons (KEK, Japan). Although they can introduce noise in the accelerator through phase and amplitude jitter, which could lead to beam degradation, this is not an important problem for leptons as they undergo a natural beam-size dumping through synchrotron radiation. However, this beam degradation could be a problem for hadrons and it was therefore of paramount importance to test the validity of the scheme before its installation in the LHC. With this in mind, a prototype set of vertical HL-LHC CCs was installed in SPS, the second largest accelerator of CERN, which served as a test-bed for the first CC experiments in the presence of may protons, from April to November 2018. The main results of work these experiments are described in [1].

Several effects can affect the long term stability of the prothis ton beam (Dynamic Aperture, DA), including the high order from 1 CC multipole components that result from the asymmetric

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cavity designs (imposed by the tight HL-LHC space constraints), and the non-linearities of the SPS machine. This paper presents the DA simulations performed for different cavity, machine and beam configurations.

DA IN THE PRESENCE OF CC **MULTIPOLES**

The DA in the SPS was studied for different CC multipole configurations, with the multipoles applied only on the first CC. Note that other than the chromatic sextupoles (for chromaticity correction), no other SPS non-linearities and no aperture constraints were included. Given that the SPS experiments were performed with different CC phase configurations, the simulations were done for a phase-cancelling mode, where $\phi_1 = 0^\circ$, $\phi_2 = 180^\circ$, and an in-phase mode: $\phi_1 = \phi_2 = 0^\circ$. In the first case the effect of CC kicks are cancelled out whereas in the latter they are added.

The SPS optics and beam parameters at the location of the CCs are given in Table 1 and the CC multipole values [2] are shown in Table 2. The simulations were performed for the SPS injection energy, E = 26 GeV, as at this energy we have the largest CC kick, with V_{CC} = 2 MV, $\Delta p/p$ = 10^{-3} and $Q'_{x,y} = 0.0$. The indices 1, 2 indicate the first and second CC respectively. The tracking simulations were performed using MAD-X [3] and SixDesk [4] for 10⁶ turns. Since the CCs are vertical, quadrupolar and octupolar errors are normal multipoles (b_2, b_4) , whereas the sextupolar errors are skew multipoles (a₃).

Table 1: Parameter Table

Parameter	Value	
s-location [m]	6312.7213, 6313.3213	
f [MHz]	400.528	
β_{x1},β_{y1} [m]	29.24, 76.07	
$\beta_{x2}, \beta_{y2} [m]$	30.31, 73.82	
Q_x, Q_y	26.13, 26.28	
E _{inj} [GeV]	26.00	
$\gamma_{\rm rel}$	27.71	
$\epsilon_{n,x}, \epsilon_{n,y}$ [µm· rad]	2.50, 2.50	
V _{RF} [MV]	2	
$\epsilon_{\rm s} [{\rm eV} \cdot {\rm s}]$	0.5	

In Fig. 1 (top) it can be seen that there is a complete overlap between the cases in the absence and presence of phase-cancelling CCs, both in the absence of CC multipoles (black and orange lines respectively; note that the black line

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Table 2: Values of CC Multipoles in Units of mTm/mⁿ⁻¹

Multipole	Value	
b ₂ (Q)	-0.06	
a ₃ (S)	1159	
b ₄ (O)	-4	

is behind the orange one). It can be clearly seen that the quadrupolar and octupolar multipoles (red and blue lines respectively) do not affect the DA, as they overlap with the black and orange lines. There are only two cases where the DA is affected, and that is when the sextupolar multipole is included, either on its own or together with the other multipoles (green and purple lines respectively); even then though, the DA is as high as $35-40\sigma$. On the other hand, when the CCs are in-phase (bottom plot of Fig. 1) the DA reduction is dominated by the cavities themselves, and not by the CC multipoles; this is clear from the fact that the orange line (CCs present, without CC multipoles) overlaps with all other coloured lines that include CC multipoles. In other words these simulations showed that the effect of the CC multipoles on the DA, when used with the values given in the literature, is minimal.



Figure 1: DA in σ with respect to angle in transverse phase space for the cases where the CCs are in a phase-cancelling mode (top), i.e. $\phi_1 = 0^\circ$ and $\phi_2 = 180^\circ$, and an in-phase mode (bottom), i.e. $\phi_{1,2} = 0^\circ$; Q: quadrupolar, S: sextupolar, O: octupolar multipoles.

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Dynamic Aperture with Respect to Skew Sextupolar Component

Given that the strongest higher order oscillating multipole is the skew sextupole, an additional study took place calculating the minimum DA for a large range of the sextupolar strength values. Six different cases were studied for two different momentum deviation values ($\Delta p/p = 0, 10^{-3}$) and for three different CC voltages ($V_{CC} = 0 \text{ MV}$, 1 MV and 2 MV); the CCs were in phase: $\phi_1 = \phi_2 = 0^\circ$. Figure 2 shows the minimum DA for a₃ values between 0 and 140,000 10^{-3} Tm/m²; the blue vertical line shows the literature value of $a_3=1159 \ 10^{-3}$ Tm/m² and the horizontal lines show the physical aperture for each case.

Two trends can be seen in this plot, one for each momentum deviation: the green, black and red points, all with $\Delta p/p = 0$, are grouped together even if they have a different V_{CC} value, as are the blue, grey and pink points, with $\Delta p/p = 10^{-3}$. Since the particles with larger momentum deviation perform larger longitudinal oscillations, they are exposed to a wider range of the a₃ non-linearity. This result therefore demonstrates that the non-linearity of the oscillating multipolar field has a much stronger effect than the strength of the CC voltage.

For the $V_{CC} = 2 \text{ MV}, \Delta p/p = 10^{-3}$ case (pink data points), it can be seen that the minimum DA reduces by a factor of two, from 30σ to $\sim 15\sigma$ for a₃ values that are 50 times larger than the one found in the literature. Note that even for a₃ that is as high as $140,000 \ 10^{-3} \text{Tm/m}^2$, i.e. two orders of magnitude larger than the literature value, we are still limited by the physical aperture and not by DA. During the SPS tests of May-November 2018, an experimental effort took place to characterise with two different techniques the CC a₃ component [5].



Figure 2: Minimum DA in σ with respect to a_3 ; the vertical blue line shows the a_3 literature value and the horizontal lines the physical aperture for each study (the green, blue and black lines are behind the red dotted line).

DA IN THE PRESENCE OF SPS NON-LINEARITIES

The aforementioned results do not include any non-linear fields other than the chromatic sextupoles used for chromaticity correction. The realistic SPS model though includes other sources of non-linearities among which the most important ones are the odd multipoles produced by the error harmonics of the main dipole magnets and remanent fields in sextupoles and octupoles due to magnetic hysteresis; the latter are relevant only at low energies.

In order to establish the SPS non-linear optics model with beam-based measurements at injection energy (26 GeV) [6], chromaticity measurements were repeated exhibiting different betatron and dispersion functions; in this way the contribution of the different non-linear errors was disentangled. An effective optics model has been built by fitting the strength of the multipolar errors in order to reproduce the experimental observations with the 3 different optics. The procedure has been repeated 5 times for different machine configurations, allowing to establish an average model and to evaluate the statistical uncertainties. To confirm the validity of the effective non-linear model at higher energy, a single chromaticity measurement of the Q26 optics at 270 GeV was acquired and used to fit a model containing the odd multipoles produced by dipoles only. Independent parameters for each multipolar error have been allowed for each of the two different kinds of SPS dipoles, MBA and MBB, that have different aperture but same length and integrated field.

 Table 3: Multipole Errors from SPS Nonlinear Model

$\begin{array}{c cccc} Multipole & 26GeV & 270GeV \\ \hline b_{3a}[m^{-2}] & (-2.8\pm0.6)\cdot10^{-3} & 8.1\cdot10^{-4} \\ b_{3b}[m^{-2}] & (1.6\pm0.3)\cdot10^{-3} & 1.1\cdot10^{-3} \\ b_{5a}[m^{-4}] & -7.9\pm0.5 & 9.2 \\ b_{5b}[m^{-4}] & -6.8\pm1.5 & -10 \\ b_{7a}[m^{-6}] & (8.8\pm2.6)\cdot10^4 & 1.3\cdot10^5 \\ b_{7b}[m^{-6}] & (1.7\pm0.8)\cdot10^5 & 1.4\cdot10^5 \\ \end{array}$			
$ \begin{array}{cccc} b_{3b}[m^{-2}] & (1.6\pm0.3)\cdot10^{-3} & 1.1\cdot10^{-3} \\ b_{5a}[m^{-4}] & -7.9\pm0.5 & 9.2 \\ b_{5b}[m^{-4}] & -6.8\pm1.5 & -10 \\ b_{7a}[m^{-6}] & (8.8\pm2.6)\cdot10^4 & 1.3\cdot10^5 \end{array} $	Multipole	26 GeV	270 GeV
$ \begin{array}{cccc} b_{5a}[m^{-4}] & -7.9 \pm 0.5 & 9.2 \\ b_{5b}[m^{-4}] & -6.8 \pm 1.5 & -10 \\ b_{7a}[m^{-6}] & (8.8 \pm 2.6) \cdot 10^4 & 1.3 \cdot 10^5 \end{array} $	$b_{3a}[m^{-2}]$	$(-2.8 \pm 0.6) \cdot 10^{-3}$	$8.1\cdot 10^{-4}$
	$b_{3b}[m^{-2}]$	$(1.6 \pm 0.3) \cdot 10^{-3}$	$1.1 \cdot 10^{-3}$
$b_{7a}[m^{-6}]$ (8.8 ± 2.6) · 10 ⁴ 1.3 · 10 ⁵	$b_{5a}[m^{-4}]$	-7.9 ± 0.5	9.2
	$b_{5b}[m^{-4}]$	-6.8 ± 1.5	-10
$b_{7b}[m^{-6}]$ $(1.7 \pm 0.8) \cdot 10^5$ $1.4 \cdot 10^5$	$b_{7a}[m^{-6}]$	$(8.8 \pm 2.6) \cdot 10^4$	
	$b_{7b}[m^{-6}]$	$(1.7 \pm 0.8) \cdot 10^5$	$1.4 \cdot 10^{5}$

Table 3 shows a comparison of the simplified model measured at 270 GeV against what was measured at injection energy. The two models are found to be compatible, except for the sextupolar component of the MBA dipoles (b_{3a}). However such a discrepancy is likely to be attributed to a calibration error of the sextupoles used to correct chromaticity. The overall good agreement extends the validity of the effective model measured at injection energy to the conditions used for the CC simulations.

Due to the synchrotron oscillations, a particle with a specific initial $\Delta p/p$ or longitudinal action, *z*, will see a variation of the CC field; the larger the initial longitudinal action is, the larger the range of the non-linear CC field the particle will experience. To study the effect this has on DA the following simulations were performed for $\Delta p/p = 0$ but with different initial *z* and were repeated for voltages ranging from 0 to

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2.5 MV per CC (Fig. 3). These simulations did not include the CC multipoles, as it was shown they have no effect on DA for the values found in the literature, but instead took into account the non-linear SPS fields, up to b₇. The study was performed for 10⁶ turns with the CCs being in-phase ($\phi_1 = \phi_2 = 0^\circ$). The horizontal lines of Fig. 3 represent the SPS physical aperture. These simulations demonstrate a very positive result: even for the largest *z* and the highest operational voltage of 2.5 MV per CC, our particles are only limited by the physical aperture that drops from 7.6 σ to 3.2 σ , and not by DA.



Figure 3: Minimum DA for different initial longitudinal actions, *z*, and CC voltage; the physical aperture is shown by the horizontal lines.

CONCLUSIONS

One of the ways the High-Luminosity LHC project will increase the LHC luminosity is by using a CC scheme that will recover the head-on collisions. The CC SPS experiments took place from May to November 2018 to examine, for the first time ever, the behaviour of CCs in the presence of protons. This paper presented the DA simulation studies for CCs in the presence of CC multipoles and machine nonlinearities. It was concluded that the CC multipoles, for values found in the literature, play no significant role in the DA. In order to see an important effect multipoles that are two orders of magnitude larger than the literature values need to be employed, but even then we are limited by the physical aperture and not by the DA. Furthermore, it was demonstrated that the DA reduction comes from the nonlinearity of the oscillating multipolar field rather than the CC voltage. Finally, it was shown that in the presence of machine non-linearities (up to b_7) our beam is only limited by the physical aperture, and not by DA, even for high CC voltages and large initial longitudinal actions.

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