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RADIATION OF A CHARGE MOVING IN A WIRE STRUCTURE*

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Abstract

A theoretical approach based on theory of thin vibrator antenna describing the radiation produced by one-dimensional relativistic bunch propagating through the sparce lattice of PEC wires of finite length is presented. The validity of the method is verified by numerical simulations with COMSOL Multiphysics. Possible applications of interaction between charged particle bunches and artificial wire structures are discussed.

INTRODUCTION

Artificial wire structures are attractive to researchers over the past several decades. In the context of "left-handed" metamaterials, "wire medium" was used for providing negative "effective" dielectric permittivity [1]. Later on, electromagnetic (EM) properties of "wire medium" was studied in details [2, 3] under the assumption that considered wavelengths are much larger compared to the structure periods, therefore allowing usage of "effective" macroscopic parameters for it's description. In this case, nondivergent properties of Cherenkov radiation have been mentioned [4, 5].

For wavelengths comparable with the structure period, the description based on "effective" parameters fails and wire assembly should be considered as a "wire crystal". Corresponding "crystals" can be used for development of efficient radiation sources based on "volume free electron laser" (VFEL) principle [6–8]. Moreover, waveguides loaded with artificial metamaterials (including "wire crystals") are considered as promising candidates for high-power and high-gradient accelerators [9, 10].

In this report, we present the analytical approach for investigation of EM field produced by one-dimensional bunch moving through the wire structure composed of finite length PEC wires. This approach is free from limitations on ratio betweeen wavelength and wire radus or structure period. Moreover, COMSOL simulation and comparison between results are performed.

PROBLEM FORMULATION

Figure 1 shows geometry of the problem. One-dimensional Gaussian bunch

$$\rho(x,y,z,t) = \frac{q\delta(x)\delta(y)}{\sqrt{2\pi}\sigma}\exp\left(\frac{-(z-vt)^2}{2\sigma^2}\right) \qquad (1)$$

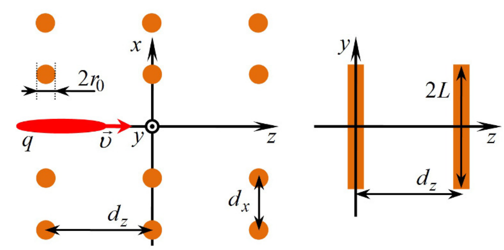

Figure 1: Geometry of the structure: PEC wires of length $2L$ and radius r_0 form a rectangular lattice with periods d_x, d_z traversed by a Gaussian bunch (1).

traverses with constant velocity $v = \beta c$ the periodic lattice of PEC cylinders distributed in vacuum. Wires are located in nodes of rectangular lattice with periods d_x, d_z excluding z-axis. Position of each cylinder's axis x_{lm}, z_{lm} is given by pair of integers (l, m): $x_{lm} = ld_x$, $z_{lm} = md_z$. Below, we will use the following approximation: each wire is excited by Coulomb field of the moving bunch but does not get affected by the field produced by each neighboring wire. This approximation is close to the "kinematic approach" of the PXR theory in real crystals and allows consideration of each wire excitation independently. The validity of this approach is verified by simulations in COMSOL (see Sec.). The approach used below for calculation of each wire response is related to Hallen's method [11] which is generalized here for the case of excitation by a charged particle bunch.

SINGLE WIRE EXCITATION

We will calculate the response of a single wire with "coordinates" (l, m). The geometry of this sub-problem is shown in Fig. 2. The problem is solved for amplitudes of Fourier harmonics $\vec{E}_\omega \exp(-i\omega t)$. "Incident" field for the case of relativistic motion, $\beta \to 1$ has the form :

$$E^{(i)}_{\omega y} \approx \frac{q}{\pi c}\frac{z'}{r^2_{lm}}\exp\left(i\frac{\omega z_{lm}}{c} - \frac{\omega^2}{\omega^2_\sigma}\right), \qquad (2)$$

where $r_{lm} = \sqrt{x^2_{lm} + (z')^2}$, $s_0 = i\sigma_0$, $\sigma_0 = \sqrt{\omega^2 v^{-2}(1-\beta^2)}$, $\mathrm{Re}\sqrt{} > 0$, $\omega_\sigma = \sqrt{2}c\beta/\sigma$.

The problem here is to find the surface current induced at the surface of the wire. We will suppose that wires are thin, i.e. $r_0 \ll L$, therefore wire flanges can be neglected and we can suppose that the surface current has only y-component does not depending on φ'. Therefore, surface current $\vec{j}^{surf}_e$ can be presented as $\vec{j}^{surf}_e = \vec{e}_y j^{surf}_e$, $j^{surf}_e = \frac{I(z')}{2\pi r_0}\delta(r' - r_0)$, where $I(z')$ is the total current of a wire

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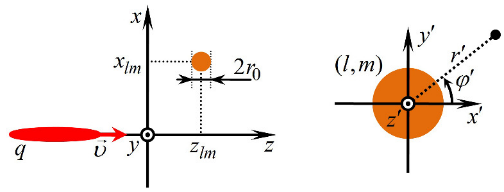

Figure 2: Single wire excitation and local coordinate frames (Cartesian and cylindrical) associated with the wire.

satisfying the boundary conditions at the ends of a wire,

$$I(L) = I(-L) = 0. \tag{3}$$

Hallen's integral equation for $I(z')$ is obtained as follows (see [12] for details). Based on boundary condition on the surface of PEC wire,

$$\left(E^{(i)}_{\omega y}+E_{\omega y}\right)\Big|_{r'=r_0} = 0, \tag{4}$$

we obtain for the "longitudinal" potential $U(y) \equiv 2\,A_{\omega y}(r', y)\big|_{r'=r_0}$:

$$d^2U(y)\big/ dy^2 + k_0^2 U(y) = 2ik_0 E^{(i)}_{\omega y}, \tag{5}$$

where $k_0 = \omega/c$, $A_{\omega y}$ is y-component of vector potential and Lorentz gauge condition is utilized. On the other hand, the connection between $U(y)$ and total current $I(y)$ can be calculated using Green's theorem as follows:

$$U(y) = \frac{2}{c}\int_{-L}^{L} dz' I(z') \exp\left(ik_0\left|y-z'\right|\right) K_1(y-z'), \tag{6}$$

$$K_1(y-z') = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{d\varphi'}{\sqrt{4r_0^2 \sin^2(\varphi'/2) + (y-z')^2}}. \tag{7}$$

Ordinary differential equation (5), integral equation (6) and boundary condition (3) formulates the Hallen's problem for the surface current $I^{(lm)}(z')$ on a wire (l, m).

In the symplest approximation ("quasistationary" approximation), a "local" term should be separated in (6) while the remaining integral term being inversely proportional to the large parameter of the problem $\Omega_0 = \Omega(0) \approx 2\ln(2L/r_0)$ should be neglected (see [12] for details). Here

$$\Omega(y) = \int_{-L}^{L} dz' K_1(y-z'). \tag{8}$$

Finally we obtain:

$$I^{(lm)}(z') = \frac{q\exp\left(ik_0 z_{lm} - \omega^2/\omega_\sigma^2\right)}{2\pi\Omega(z')}\left\{\frac{\sin(k_0 z')}{\sin(k_0 L)}\times\right.$$
$$\times\left[2iI_c(L)\cos(k_0L) - 2iI_s(L)\sin(k_0L)\right] + \tag{9}$$
$$\left. +2iI_s(z')\sin(k_0z') - 2iI_c(z')\cos(k_0z')\right\},$$

where

$$I_s(y)=\int\frac{\sin(k_0y')y'}{y'^2+x_{lm}^2}dy',\;\; I_c(y)=\int\frac{\cos(k_0y')y'}{y'^2+x_{lm}^2}dy'. \tag{10}$$

Vector potential of such a current is:

$$A^{(lm)}_{\omega y}(x,y,z)=\int_{-L}^{L} dz'\frac{I^{(lm)}(z')}{2\pi c}\int_{-\pi}^{\pi} d\varphi'\frac{\exp\left(ik_0R_{lm}\right)}{R_{lm}}, \tag{11}$$

$$R_{lm} = \sqrt{(r'_{lm})^2 + r_0^2 - 2r_0r'_{lm}\cos\varphi' + (y-z')^2}, \tag{12}$$

$r'_{lm} = \sqrt{(z-z_{lm})^2 + (x-x_{lm})^2}$, and field components can be calculated.

One important peculiarity of this approximate solution is the presence of the term $\sin(k_0L)$ in the denominator of (9) which equals zero for resonant frequencies:

$$\omega = \pm 2\pi f_m,\; f_m = c/(2L)m,\; m = 1, 2, \ldots. \tag{13}$$

For example, the first resonant wavelength $\lambda_1 = c/f_1 = 2L$ equals the total length of the wire. Taking into account the influence of radiation on surface current distribution resolves singularity for f_m, corresponding solution can be found in [12].

NUMERICAL RESULTS

For simulations, we have used frequency domain solver of COMSOL Multiphysics RF module. Corresponding simulated results can be directly compared with analytical ones. Structure of the model can be found in [12]. Simulations presented below are performed for $\sigma = 0$ (point charge). Man parameters of simulations are: $f = 10$GHz, $\lambda = 3$cm, $q = 1$nC, $d_x = d_z = 0.15c/\omega$, $L = 0.1\lambda$ ("short wire"), $2L = \lambda$ ("resonant wire"), $L/r_0 = 200$.

Figure 3 shows a comparison between COMSOL and analytical results for the case of charge flight near single resonant wire. Real part of $E_{\omega x}$ component is shown on a line parallel to z-axis. Real part of $E^{(i)}_{\omega x}$ is proportional to $\cos(\omega z/v)$, background in Fig. 3 corresponds to a half-cycle of this cosine function. An expressed peak for $z = 0$ corresponds to the response of a wire. As one can see, the curves are in very good agreement.

Figure 4 shows EM field for the case of a charge flight near a string of 4 "short" wires with coordinates $(1, m)$, $m = -3, -2, -1, 0$. As one can see, magnitudes of spikes are smaller compared to the resonant case and agreement between curves is a bit worse. The sign of each spike has a different sign compared to the case of "resonant wire". However, one can conclude that each wire gives an independent response. This fact proves the applicability of "kinematic approach" for wire structures used throughout the paper. It is worth noting that the presented approximate solution gives surprisingly good results for a single wire of "resonant" length. In the context of using wire structures for generation of EM radiation (in particular, in THz frequency range), resonant case is of most interest since radiated EM field is

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expected to be much larger in this case compared to the non-resonant wire. For potential THz source with $f = 1$THz the resonant length of a wire is equal to wavelength $\lambda = 0.3$mm.

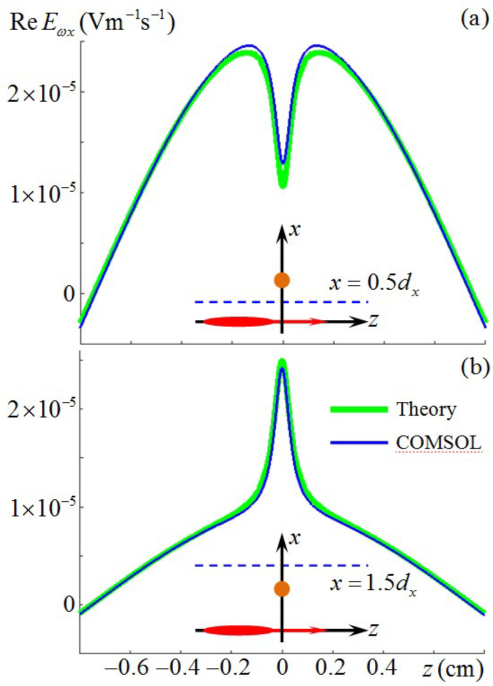

Figure 3: Real part of $E_{\omega x}$ component over z along the line $x = 0.5d_x$ (top) and $x = 1.5d_x$ (bottom). A spike for $z = 0$ corresponds to the response of a wire. Wire has the "resonant" length.

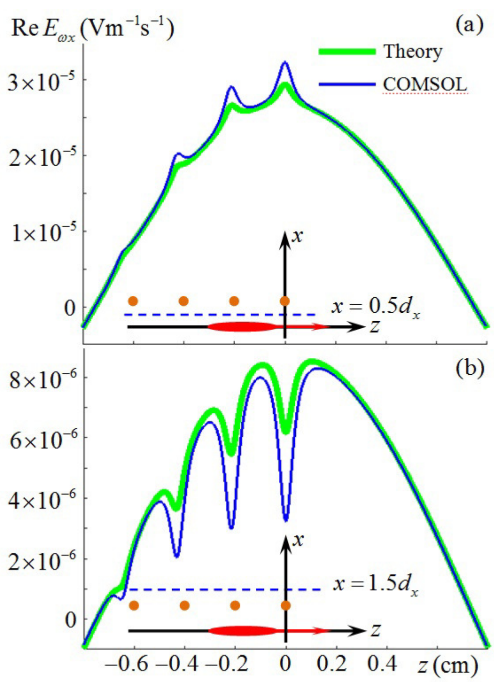

Figure 4: Real part of $E_{\omega x}$ component over z along the line $x = 0.5d_x$ (top) and $x = 1.5d_x$ (bottom) for the case of 4 "short" wires with coordinates $(1, m)$, $m = -3, -2, -1, 0$.

In this case, carbon nanotubes having their radii of order of nanometers and their lengths from tens of nm to several cm can be used. To generate this frequency, the bunch should be also short enough, $\sigma \lesssim 100\mu$m, which is accesible at SLAC.

REFERENCES

[1] J.B. Pendry, A.J. Holden, D.J. Robbins, and W.J. Stewart, *Low frequency plasmons in thin-wire structures*, J. Phys. Condens. Matter, V. 10, P. 4785 (1998).

[2] P. Belov, S. Tretyakov, and A. Viitanen, *Dispersion and reflection properties of artificial media formed by regular lattices of ideally conducting wires*, J. Electromagn. Waves Appl., V. 16, 1153 (2002).

[3] A.V. Tyukhtin and E.G. Doilnitsina, *Effective permittivity of a metamaterial from coated wires*, J. Phys. D, V. 44, 265401 (2011).

[4] V.V. Vorobev and A.V. Tyukhtin, *Nondivergent Cherenkov Radiation in a Wire Metamaterial*, Phys. Rev. Lett., V. 108, 184801 (2012).

[5] D.E. Fernandes, S.I. Maslovski, and M.G. Silveirinha, *Cherenkov emission in a nanowire material*, Phys. Rev. B, V. 85, 155107 (2012).

[6] V. Baryshevsky, K. Batrakov, A. Gurinovich, I. Ilienko, A. Lobko, V. Moroz, P. Sofronov, and V. Stolyarsky, *First lasing of a volume FEL (VFEL) at a wavelength range $\lambda \sim$ 4–6mm*, Nucl. Instrum. Meth. Phys. Res. A, V. 483, P. 21 (2002).

[7] V.G. Baryshevsky, K.G. Batrakov, A.A. Gurinovich, I.I. Ilienko, A.S. Lobko, P.V. Molchanov, V.I. Moroz, P.F. Sofronov, and V.I. Stolyarsky, *Progress of the volume FEL (VFEL) experiments in millimeter range*, Nucl. Instrum. Meth. Phys. Res. A, V. 507, P. 137 (2003).

[8] V.G. Baryshevsky and E.A. Gurnevich, *The electron beam instability in a one-dimensional cylindrical photonic crystal*, Nonlin. Phenom. Complex Syst., V. 15, P. 155 (2012).

[9] P.D. Hoang, G. Andonian, I. Gadjev, B. Naranjo, Y. Sakai, N. Sudar, O. Williams, M. Fedurin, K. Kusche, C. Swinson, P. Zhang, and J.B. Rosenzweig, *Experimental Characterization of Electron-Beam-Driven Wakefield Modes in a Dielectric-Woodpile Cartesian Symmetric Structure*, Phys. Rev. Lett., V. 120, 164801 (2018).

[10] X. Lu, M. A. Shapiro, I. Mastovsky, R.J. Temkin, M. Conde, J.G. Power, J. Shao, E.E. Wisniewski, and C. Jing, *Generation of High-Power, Reversed-Cherenkov Wakefield Radiation in a Metamaterial Structure*, Phys. Rev. Lett., V. 122, 014801 (2019).

[11] E. Hallen, *Theoretical Investigations into the Transmitting and Receiving Qualities of Antennae*, Nova Acta Regiae Soc. Sci. Upsaliensis, Ser. IV, Vol. 11, No. 4, P. 2 (1938).

[12] S.N. Galyamin, V.V. Vorobev and A. Benediktovitch, *Radiation field of an ideal thin Gaussian bunch moving in a periodic conducting wire structure*, Phys. Rev. Accel. Beams, V. 22, 043001 (2019).