TOWARDS A SEXTUPOLE-FREE ELECTRON STORAGE RING*
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Abstract
This paper studies if it is possible to build an electron storage ring with no or a small number of sextupole magnets. If it is possible, the electron storage ring will be greatly simplified. For the purpose, two methods are presented in the paper to handle head-tail instability: One is to use dielectric vacuum chamber made of such materials as ceramic or glass to reduce broadband impedance significantly. Then head-tail instability would be extremely weak. The other method is to install a bunch-by-bunch feedback system to suppress the already weak head-tail instability due to the dielectric vacuum chamber.

INTRODUCTION
The role of sextupole magnets in the magnet lattice of a storage ring (circular accelerator in general) is just to suppress the head-tail instability by resetting the ring chromaticity from a negative number to a number slightly positive. However, sextupole magnets give the unwanted effect of reducing the ring dynamic aperture in this process of chromaticity correction. In the case that the negative natural chromaticity is too large (negatively) and so the chromaticity correcting sextupole magnets are required to be too strong, the dynamic aperture would so small (or even zero) that injection into the ring may be impossible or extremely difficult. This difficulty is usually met, in the light source that injection into the ring may be impossible or extremely difficult. The role of sextupole magnets in the magnet lattice of a storage ring is just to suppress the head-tail instability by resetting the ring chromaticity from a negative number to a number slightly positive.

Bunch-by-bunch Transverse Feedback
Bunch-by-bunch TF hits the centroid of each electron bunch. Therefore, only the lowest rigid body dipole oscillations (l = 0 mode) of head-tail instability can be damped by the conventional bunch-by-bunch TF. According to the simple two particle model [2], electron bunches execute rigid body dipole oscillations with negative chromaticity, regardless of other parameters, and so head-tail instability can always be suppressed by bunch-by-bunch TF. However, simulation study shows that this is not always true [3], and the experiment performed at DIAMOND may seem confirm the simulation [4]. In this regard, it was argued that the electron bunch length scaled with respect to the broadband impedance causing head-tail instability is an important parameter determining whether or not bunch-by-bunch TF is applicable [5]. The parameter is defined as $x = \omega_r\sigma_t/Q_r$, where $\sigma_t$ is the bunch length measured in time, and $\omega_r$ and $Q_r$ are the impedance resonance frequency and quality factor, respectively. If $x$ is short enough (smaller than around 0.4), the bunch oscillations may be considered to be dipole mode enough. If $x$ is far above 0.4, the oscillations are considered to be of higher longitudinal modes ($l = 1..$ modes) (Fig. 1). These oscillations need more sophisticated feedback system for damping.

Head-tail Transverse Feedback
Higher modes of the head-tail instability can be suppressed by wide band intra-bunch that is technically difficult at the moment. But the next higher mode ($l = 1$ mode) can be suppressed by head-tail TF system that hits the head and tail of an electron bunch separately (head-tail kick) as in Fig. 2. In the figure, the kick of the usual bunch-by-

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* Work supported by Basic Science Research Program through the National Research Foundation of Korea (NRF-2016R1D1A1B03933884).
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bunch TF is termed as centre of mass (CM) kick. This type of feedback system is currently under study conceptually but not realized yet [6]. Combined use of CM TF and head-tail TF would greatly suppress head-tail instability.

Figure 2: Schematic figure of centre of mass (CM) kick of a bunch-by-bunch TF and head-tail kick. [Courtesy of Nakamura]

**High Chromaticity Effect**

The two-particle model evaluates the growth rate of head-tail instability at a negative chromaticity as [2]:

\[
\tau^{-1} = -\frac{\epsilon^2 N \xi_0}{2 \eta p_0} \left( W_0 / C \right),
\]

where \( \xi \) is the negative chromaticity, \( N \) is the number of electrons in a bunch, \( L \) is bunch length, \( \eta \) is the slippage factor, \( p_0 \) is the nominal momentum of electron beam, and \( W_0 / C \) is the wake per unit length along the storage ring. This formula may lead to a wrong conclusion that head-tail instability grows fast with negatively large chromaticity values for a given wake and a low emittance 4th generation storage ring would excite extremely strong head-tail instability without sextupole magnets. However, this formula is valid only for weak chromaticity. More complete recent study using not only the two-particle model but Vlasov analysis and computer simulation shows that the growth rate reaches extremum at reasonably low (negative) chromaticity strengths and then decreases as chromaticity strength grows further (see Fig. 16 of [7]). Therefore, in reality, a 4th generation light source would have weaker head-tail instability and better suited for realizing a sextupole-free accelerator. The same study shows that the growth rate keeps growing as \( W_0 \) grows [7]. This means that head-tail instability cannot be weakened significantly if the broadband impedance of the ring chamber is reduced substantially.

**Section Summary**

This section is summarized as follows:

1. Bunch-by-bunch TF can suppress head-tail instability if the scaled electron bunch length \( x \) is short enough.
2. Wide band intra-bunch TF can suppress higher modes that cannot be suppressed by bunch-by-bunch TF. But this type of TF is not realized yet for electron storage rings.
3. Head-tail TF that can suppress \( l = 1 \) mode is under study in Japan. Combined use of bunch-by-bunch TF and head-tail TF would be able to suppress head-tail instability significantly.

4. Head-tail instability may not be fully suppressed by bunch-by-bunch TF alone which is currently the only available TF.
5. However, head-tail instability may be suppressed by bunch-by-bunch TF alone, if vacuum chamber is made of dielectric materials and so the chamber broadband impedance is negligibly small.
6. Head-tail instability may be weaker at a negatively larger chromaticity and so 4th generation light sources may be better suited for sextupole-free accelerators.

**USE OF DIELECTRIC CHAMBER FOR LOW BROADBAND IMPEDANCE**

In particle accelerators, vacuum chamber is usually made of metal such as stainless steel, aluminium or copper. The primary role of the accelerator vacuum chamber is to maintain ultra-high vacuum and to withstand the atmospheric pressure. Metal is well suited to these roles. However, electrons moving inside the vacuum chamber create image charges on the chamber inner surface and the electromagnetic field excited by the moving electrons and the induced image charges perturb the motion of the electron bunches. This is the ‘wake field’ and its Fourier transform to the frequency space is defined as the impedance of the vacuum chamber. Fast growing perturbation of electron beam by impedance is ‘beam instability’. Particularly, head-tail instability is excited by broadband impedance that is mostly created by cross-sectional change of the metal chamber geometry. As an innovative idea to intrinsically prevent head-tail instability in an electron storage ring, this paper proposes to use glass, a dielectric material, for the vacuum chamber not to generate the image charges on the vacuum chamber inner surface. In this scheme, such unavoidable metal components as RF cavities and magnets, which should be connected to the dielectric chamber, should be grounded to localize and isolate the image charges created at each component and then move them out of the chamber constantly.

Dielectric materials such as ceramics and glass are capable of maintaining good vacuum state and have high melting points (> 3000 °C for ceramics and 1400~1600 °C for glass). However, the following technical problems should be solved to use these materials as accelerator vacuum chamber materials:

- Static charges induced by polarization of dielectric materials.
- Charging on the chamber wall due to hitting electron bunches.
- Resonant peaks of impedance coming from Cerenkov radiation within dielectric wave guide.

**Static Charge**

As electron bunches move constantly inside the dielectric vacuum chamber, static charges are induced on the inner surface of the dielectric chamber wall (Fig. 3). Intense static charges can affect the motion of electron bunches and may even damage the vacuum chamber. The amount of induced static charges depends upon the dielectric constant...
K of the chamber material. For smaller static charge effect, dielectric material with small K should be used. Dielectric constant of a representative ceramic material (Al₂O₃) is 9.1 whereas that of glass (SiO₂) is only 3.8. Therefore, glass has better electric property as the vacuum chamber material and even has the advantage of being easily processed. That is why this paper chooses glass as the vacuum chamber material. Glass is brittle but can be reinforced by wrapping with rubber type soft and strong materials.

Figure 3: Schematic figure of static charges induced on the dielectric vacuum chamber.

**Charging Effect by Hitting Electron Bunches**

If electron bunches happen to hit the dielectric chamber wall, the electrons kick the bound electrons out of wall atoms leaving excessive positive charges on the wall surface. This left-over positive charges deflect the incoming electron bunches by electrostatic force. This charging effect is evaluated by applying Rutherford scattering as shown in Fig. 4 with the left-over surface charges playing the role of scatterer.

Using the well know Rutherford scattering formula, the angle $\theta$ is given by

$$\theta = 2 \tan^{-1} \frac{W}{Ze},$$

where $W = -Z_1 Z_2 e^2 / b$ is the electrostatic energy between $-Ze$ (electron bunch) and $Z_2 e$ (scatter charge) separated by $b$. For a typical synchrotron light source, we may use $E = 3$ GeV, $b = 5$ mm and $Z_1 e \approx 10^{-9}$ C, and we also assume $Z_2 e \approx 10^{-9}$ C. This gives $\theta \approx -0.4$ μrad, which is a negligibly small deflection angle. This deflection can be easily corrected by corrector magnets installed along the accelerator circumference. In conclusion, the charging effect is negligible for high energy electron beam of a typical synchrotron light source.

**Cerenkov Effect**

The impedance of a dielectric vacuum chamber is negligibly low as it should be. However, high impedance resonance peaks may occur at high frequencies if Cerenkov radiation emitted when the radiation phase velocity is slower than the charged particle (electron in this case) velocity is combined with conditions of dielectric waveguide. For such conditions to be met, the chamber geometry should have cylindrical symmetry. In the case of circular cylindrical geometry, the wall thickness $t$ and the wavelength $\lambda$ should satisfy [8]

$$ (2p + 1) \frac{\lambda}{\lambda^2 - \beta^2 - 1} = t, $$

where $\beta = \nu / c$, $\varepsilon_r$ is relative permittivity. Note that this is approximately when an odd multiple of a quarter wavelength in the dielectric equals the wall thickness.

However, these resonances can be avoided if the circular or other cylindrical symmetries (elliptic or rectangular) are avoided for the vacuum chamber, which is actually the case for most light sources. Therefore, a dielectric vacuum chamber can have safely low impedance.

**Inclusion of Metal Components**

Metal components such as RF cavities and magnets should be connected to the glass vacuum chamber. Image charges are induced on these components. Magnet poles should be covered with metal sheets and image current flowing on cavities and metal sheets should be directed to the ground at the connection to the dielectric chamber to localize and isolate the resistive wall impedance. RF cavities generate narrow band impedance and the magnet pole covers generate resistive wall impedance. Note that the broadband impedance of the ring is not affected by these metal components.

**Section Summary**

Dielectric (particularly glass) vacuum chamber is feasible and able to reduce the chamber broadband impedance significantly.

**CONCLUSION**

An electron storage ring without sextupole magnets can be realized if the vacuum chamber is made of glass wrapped with rubber-like materials to have negligibly small broadband impedance. Head-tail instability still surviving can be suppressed by bunch-by-bunch transverse feedback.

**REFERENCES**


