

# A FAST METHOD FOR MULTI-OBJECTIVE NONLINEAR DYNAMICS OPTIMIZATION OF A STORAGE RING

Jiajie Tan, Zhenghe Bai\*, Penghui Yang, Weimin Li, Lin Wang†  
 National Synchrotron Radiation Laboratory, USTC, Hefei 230029, China

## Abstract

Multi-objective evolutionary algorithms (MOEAs), including multi-objective genetic algorithm and particle swarm optimization algorithm, have been widely applied in the nonlinear dynamics optimization of storage ring light sources. In the optimization, the direct tracking of objectives, which are, for example, dynamic aperture (DA) and momentum aperture, is very time-consuming. We noticed that there is some positive correlation between on- and off-momentum nonlinear dynamics performances, which can be used to reduce the computation time when applying MOEAs. In this paper, a fast method is proposed, in which a strategy is introduced to speed up the process of optimizing nonlinear dynamics using MOEAs. Taking the SSRF storage ring as an example, on- and off-momentum DAs are optimized using MOEAs with and without the fast strategy, and then a comparison is made to demonstrate the fast method.

## INTRODUCTION

Dynamic aperture (DA) is one of the key nonlinear properties in storage ring design. Large on-momentum DA helps the injection, while off-momentum DAs affect Touschek lifetime. In precise DA calculation, direct tracking method is often used. However, calculating DA by tracking is time-consuming. Generally, calculating nonlinear properties costs a disproportionate amount of time compared to linear properties, which make it inefficient to optimize both linear and nonlinear properties simultaneously.

As a fast and flexible method, multi-objective evolutionary algorithms (MOEAs) have been widely used in DA optimization [1, 2]. MOEAs give out a global vision of solutions, but the diversity of solutions is not so meaningful in some cases. For example, when selecting candidates with good nonlinear performance after preliminary linear design, large amount of candidates need to be filtered. Efficiency is more important than solution diversity. By observing the DA optimization objective space, we noticed an interesting characteristic of solution distribution. This characteristic can be used to develop a fast optimization method which is more suitable for the physical background.

In this paper, we applied two different methods to DA optimization. One is a normal multi-objective particle swarm optimization (MOPSO), and the other is a classical particle swarm optimization which maximize the weighted sum of objective function values. The Shanghai Synchrotron Radiation Facility (SSRF) was taken as an example to

illustrate the two methods. And we compared the methods and discussed their relative merits.

## FAST METHOD FOR DA OPTIMIZATION

A common optimization strategy to optimize DA with MOEAs is choosing two objective functions representing DA areas of on- and off-momentum particles:

$$f_1 = S(\delta = 0), f_2 = [S(\delta = -3\%) + S(\delta = 3\%)]/2, \quad (1)$$

where  $S(\delta)$  is the DA area of the momentum deviation  $\delta$ .

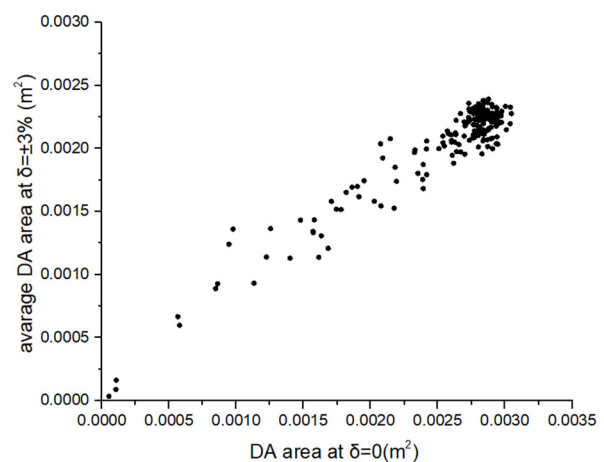


Figure 1: The objective space of DA optimization: area of on-momentum DA and average area of off-momentum DA ( $\delta = \pm 3\%$ ).

We found a characteristic of solution distribution in many cases of DA optimization [1, 2]. Unlike optimization for two independent objective functions, the Pareto optimal solutions of DA optimization always only contain a very small part of solutions. For example, in Fig. 1, the Pareto optimal solutions only consist of 4 solutions on the upper right of figure. The edge of the points in the figure is similar to a right angle. In DA optimization, both on- and off-momentum DAs are important. It is obviously that a solution on the side of the angle, which has bad on-momentum DA and good off-momentum DA, is always worse than another solution on the corner which has better on-momentum DA and similar off-momentum DA. It is the same to those solutions with good off-momentum DA and bad on-momentum DA. We inferred there is a positive correlation between on- and off-momentum DA areas to a certain extent. The relevance of the two objective functions can explain the reason of the solution distribution and it can be used to improve optimization method.

\* baizhe@ustc.edu.cn  
 † wanglin@ustc.edu.cn

The aim of DA optimization is to search for solutions with both large on-momentum DA and large off-momentum DA. MOEAs can provide a global vision of solutions and keep the diversity, but it is not meaningful in some situation. Sometimes we hope solutions converge on the corner as fast as possible. A classic method of maximizing a weighted sum  $f(\mathbf{X}) = \sum_i \lambda_i f_i(\mathbf{X})$  is an appropriate option. Generally, single objective PSO has a faster convergence speed compared to MOPSO. We can easily prove that a solution with both large on-momentum DA and large off-momentum DA has a large  $f(\mathbf{X})$ . The value of  $\lambda_i$  depends on solution distribution and the specific need of the problem. In this paper, we set an easy weighted sum  $f(\mathbf{X}) = f_1 + f_2$  for DA optimization. In consideration of the solution distribution characteristic, this easy objective function can lead to similar Pareto optimal solutions.

### APPLICATION TO SSRF

SSRF is a medium energy (3.5 GeV) third-generation light source with a circumference of 432 m. The lattice of SSRF storage ring is a four-fold symmetry structure, and one super-period is shown in Fig. 2. Each super period includes five DBA cells [3]. There are 8 families of sextupoles in the lattice. Two families of sextupoles (SF and SD) are chromatic sextupoles which work to adjust the chromaticity to  $\xi_x = 2$ ,  $\xi_y = 2$ . To enlarge the DA, six families of harmonic sextupoles (S1, S2, S3, S4, S5 and S6) are introduced in the lattice.

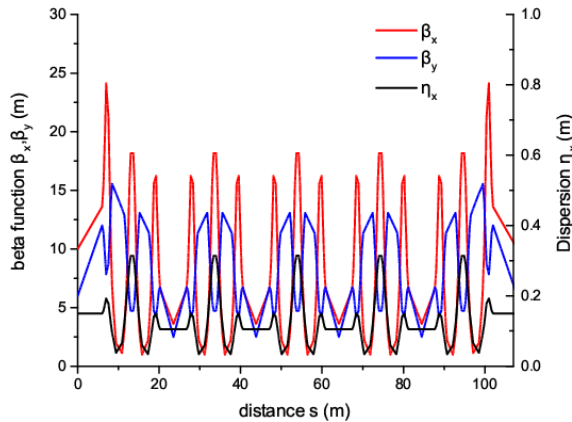


Figure 2: Lattice functions of one super-period of the SSRF storage ring.

The strengths of the six families of harmonic sextupoles are chosen to be the free parameters in DA optimization. Then chromaticity correction will be done by adjusting chromatic sextupoles then DA can be calculated by tracking method. Simulation is performed with ELEGANT [4]. To reduce the amount of computation in DA optimization, we chose the “n-line” mode for tracking in which  $n$  is set to 9 and we tracked only 100 turns for each solution.

We applied MOPSO and weighted sum PSO to SSRF DA optimization to compare the two methods. The two programs have the same population of 200 and run for 50

generations. Based on our experience, running for 50 generations is enough to observe the convergence of the solutions. Each program takes about one week to get a final population with a normal personal computer.

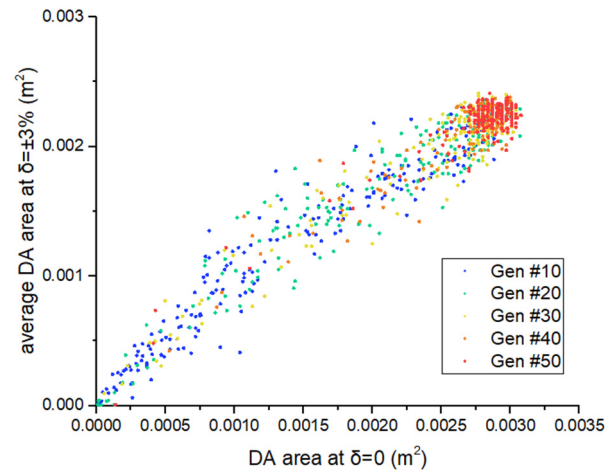


Figure 3: On- and off-momentum DA areas of solutions at different generations of MOPSO.

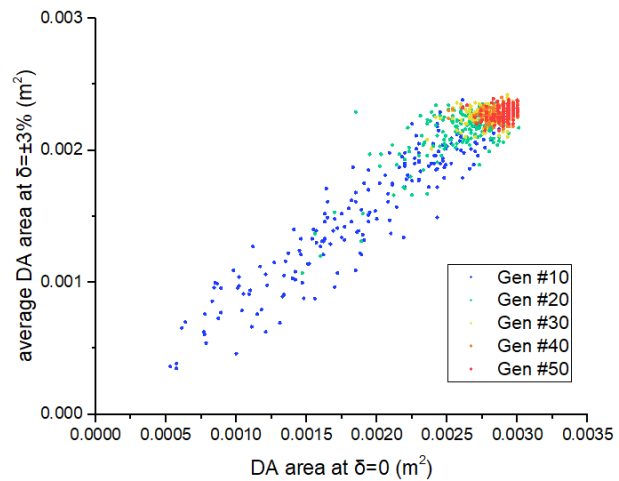


Figure 4: On- and off-momentum DA areas of solutions at different generations of the weighted sum PSO.

Figures 3 and 4 show the solutions at different generations in objective space of the two programs. At the 10<sup>th</sup> generation, solutions of both of the programs distributed as a strip. This implies there is a positive correlation in on- and off-momentum DA areas. At the 20<sup>th</sup> generation, solutions in Fig. 4 already gathered in a small area while solutions in Fig. 3 were still scattered. It can be seen that convergence speed in Fig. 4 is faster than that in Fig. 3. Fig. 5 shows the change of the average DA area. The convergence speed of the weighted sum PSO is faster than MOPSO obviously.

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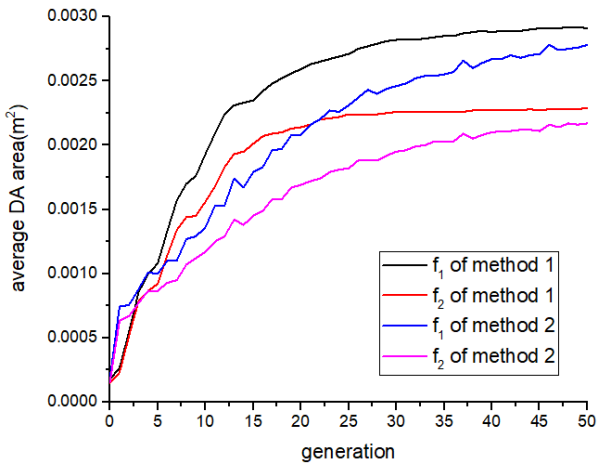


Figure 5: Average DA area of two methods at different generations. Method 1 is the weighted sum PSO and method 2 is MOPSO.  $f_1$  and  $f_2$  are objective functions in Eq. 1.

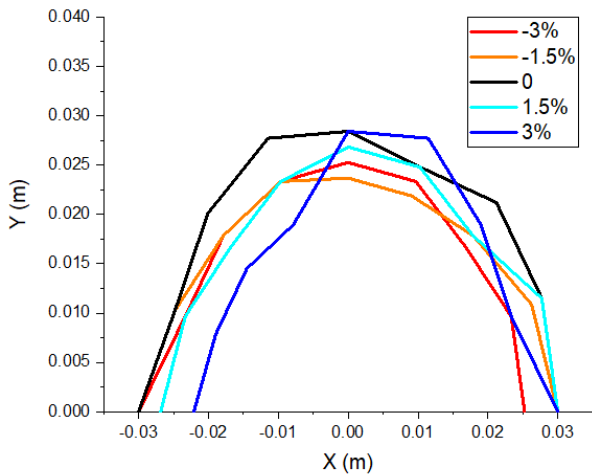


Figure 6: Dynamic apertures optimized with the weighted sum PSO.

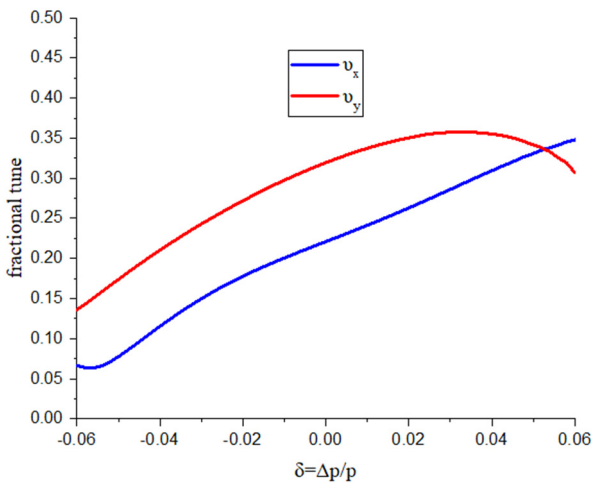


Figure 7: Momentum dependent tunes with chromaticities corrected to (2, 2).

The optimized DA with the weighted sum method is showed in Fig. 6. Compared to the designed DA [3], the new DA has a similar large on-momentum DA area while the off-momentum DA is evidently enlarged. The tune shifts with momentum are shown in Fig. 7.

## CONCLUSION

In this paper, we observed the characteristic of solution distribution of DA optimization and found a positive correlation between on- and off-momentum DAs. We applied it to develop a weighted sum single-objective optimization method. SSRF was taken as an example to check the efficiency and correctness of this method. In the conclusion the weighted sum method is a fast and reliable method to optimize DA. Besides DA area, some other properties also performed a correlation such as Touschek lifetime and DA area. In the future, we will try to expand this method to simultaneously optimize other properties such as DA and Touschek lifetime.

## ACKNOWLEDGEMENTS

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