THYGBD3

BEAM-BEAM STUDIES FOR SUPER PROTON-PROTON COLLIDER

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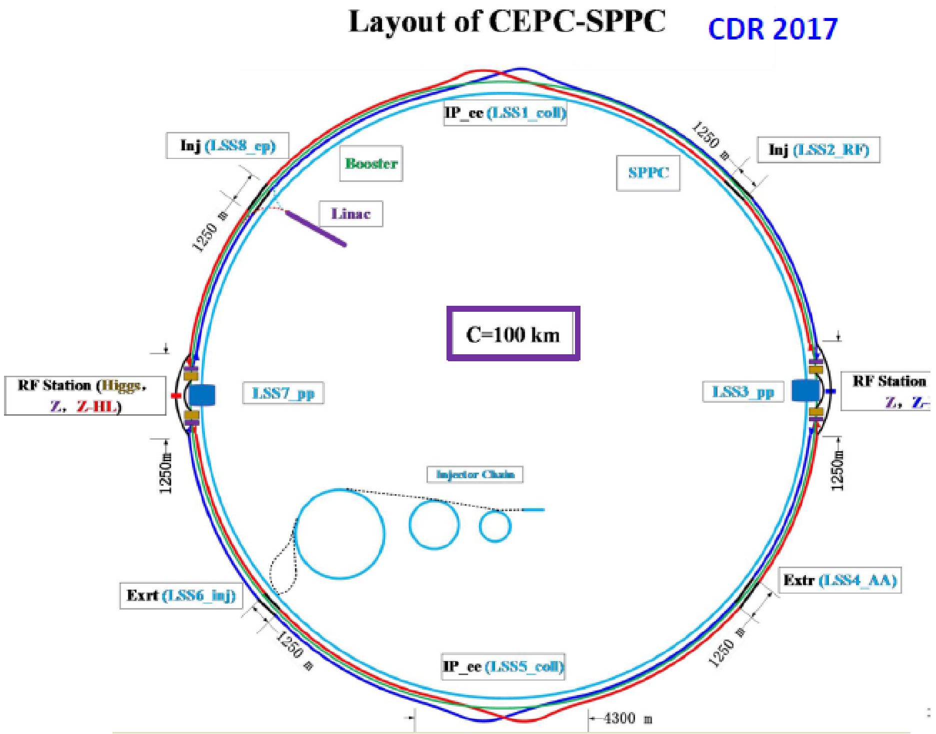
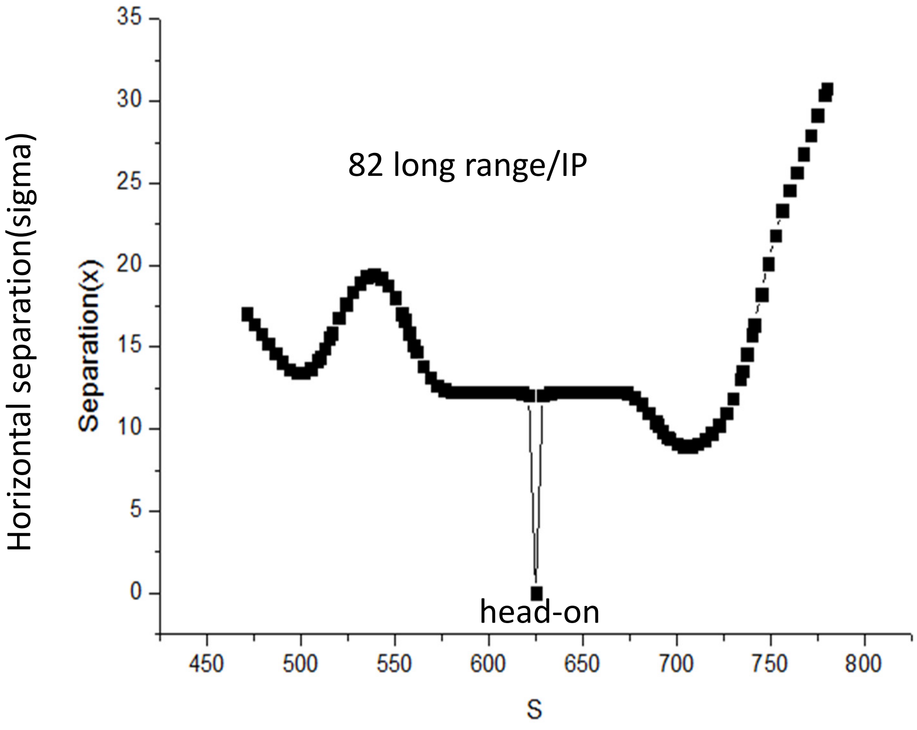
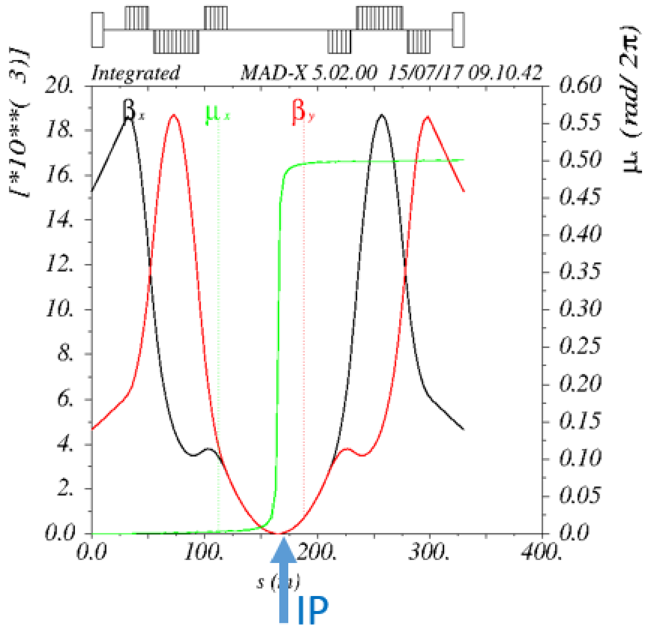
SPPC



SPPC design		
Circumference	C	100 km
Beam energy	E	37.5 GeV
Normalized Emittance	$\gamma\varepsilon$	2.4 μm
beta function at IP	b*	0.75 m
Bunch population	Np	1.5×10^{11}
Full crossing angle	θc	110 μrad
Number of IP	NIP	2
Number of bunch	Nb	10,080
Pea Luminosity	L	1.01×10^{35} cm^{-2}s^{-1}

X-separation





SPPC twiss parameters in IR

Long range interaction: every 3.75m.

Study of the beam-beam effects in SPPC

- Beam-beam simulation using weak-strong model
  - HH/HV crossing without long range
  - HH/HV crossing with long rage
- Resonances caused by the beam-beam interactions
  - HH crossing (single collision) with long rage
  - HV crossing with long range

Weak-strong simulation

- Round beam collision

$$\Delta p_r(s_i) = \frac{2N_{p,i}r_p}{\gamma}\frac{1}{r}\left[1 - \exp\left(-\frac{r^2}{2\sigma_r(s_i)^2}\right)\right]$$

$$\Delta p_z(s_i) = \frac{N_{p,i}r_p}{\gamma}\frac{1}{\sigma_r(s_i)^2}\exp\left(-\frac{r^2}{2\sigma_r(s_i)^2}\right)\frac{d\sigma_r^2(s_i)}{dz}$$

- Integrate along bunch length, Long range, assume round beam

Operated left to right

$$\mathcal{M}(s^*) = \prod_{i=0}^{N-1} e^{-U_{bb}(x,s_i)} M(s_i, s_{i+1})$$

$$= \left[\prod_{i=0}^{N-1} M^{-1}(s_i, s^*) e^{-U_{bb}(x,s_i)} M(s_i, s^*)\right] M(s^*)$$

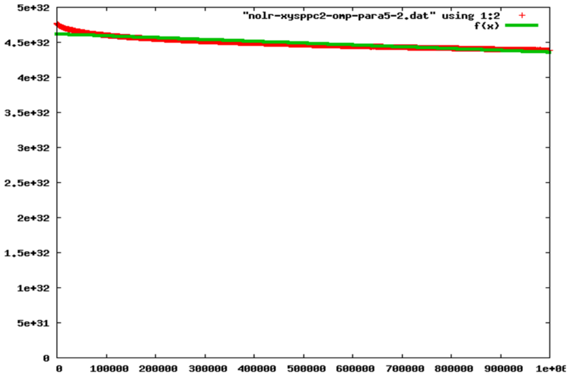
Transfer IP to LR (long range interaction point) | bb | LR to IP

Revolution matrix
Simplified model

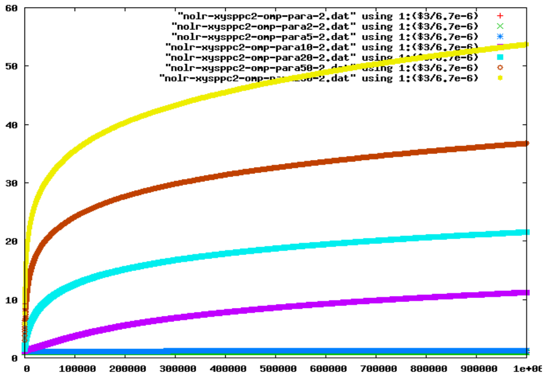
Simulation results

- Np=10^5, 10^6 turns
- Fit luminosity decay rate, emittance growth rate

Luminosity decreases



Beam size increases



Luminosity decrement, beam lifetime in the simulation

HH crossing, w or wo crossing angle

$Np(10^{11})$	x/IP	dL/L0 (0mrad)	dL/L0 (110mrad)
1.5	0.007633	0	0
3.0	0.015266	0	0
7.5	0.038165	0	0
15	0.076330	-0.26	-6.66
30	0.152660	-2.36	-210
75	0.381650	-380	-494
150	0.763300	-552	-436

HV crossing

$Np(10^{11})$	x/IP	dL/L0 (per day)
1.5	0.007633	0
3.0	0.015266	0
7.5	0.038165	-14.5
15	0.076330	-203
30	0.152660	-309
75	0.381650	-231
150	0.763300	-215

HH crossing, with long range bb interactions.

HV crossing w LRbb.

HH cross $Np(10^{11})$		Beam life [h] 7s	Beam life [h] 5s	dL/L0	HV cross	Beam life [h] 7s	Beam life [h] 5s	dL/L0
1.5	0.0153	No lost	148	0		No lost	221	0
3.0	0.0305	No lost	21.5	-0.12		264	17.5	-0.12
4.5	0.0457	2.14	1.35	-2.26		8.76	3.15	-2.26

Red is disaster.

Resonances induced by the beam-beam interactions

- Beam-beam force (round beam)

$$F_r(r) = 2C_p \frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{r}, \qquad C_p = \frac{N_p r_p}{\gamma}$$

- Beam-beam potential

$$U_{bb}(r) = -\int^r F(r')dr' = -2C_p \int^r \frac{1 - e^{-\frac{r'^2}{2\sigma^2}}}{r'} dr'$$

- Integral over bunch length, long range, multi-IP

$$\begin{aligned}
\mathcal{M}(s^*) &= \prod_{i=0}^{N-1} e^{-U_{bb}(x,s_i)} M(s_i, s_{i+1}) \\
&= \left[\prod_{i=0}^{N-1} M^{-1}(s_i, s^*) e^{-U_{bb}(x,s_i)} M(s_i, s^*) \right] M(s^*) \\
&= \left[e^{-\oint U_{bb}(M(s^*,s')x,s')ds'} \right] M(s)
\end{aligned}$$

One turn map, resonance driving term for beam-beam interactions

- Integrated beam-beam interaction

$$\oint U_{bb}(M(s^*, s')x, s')ds' \Rightarrow U_{bb}$$

- Hamiltonian describing one turn map

$$H = \boldsymbol{\mu} \cdot \boldsymbol{J} + \delta_P(s)U_{bb}$$

U_{bb} is represented by x, p_x, y, p_y, z.

- Fourier expansion of Hamiltonian, resonance driving term

$$U_{\boldsymbol{m}} = \frac{1}{(2\pi)^2} \int d\phi_x d\phi_y U_{bb} e^{i\boldsymbol{m}\boldsymbol{\phi}}$$

- Analytic integration for $\phi_{x,y}$ is complex for a long range and crossing collisions.
- Direct integration for $\phi_{x,y}$ is used now.

Integration for ϕ_{xy}

- Choose $\Delta\phi_{xy}=2\pi/100-2\pi/400$

$$U_{\boldsymbol{m}} = \frac{1}{(2\pi)^2}\int d\phi_x d\phi_y U_{bb} e^{i\boldsymbol{m}\boldsymbol{\phi}}$$

$$\frac{\partial U_0}{\partial J_x} = \frac{1}{(2\pi)^2}\int d\phi_x d\phi_y \frac{\partial U_{bb}(r)}{\partial x}\frac{\partial x}{\partial J_x}$$

$$= -\frac{1}{(2\pi)^2}\int d\boldsymbol{\phi}\frac{x}{2J_x}F_x(x-x_{LR}, y-y_{RL})$$

$$\frac{\partial U_0}{\partial J_y} = -\frac{1}{(2\pi)^2}\int d\boldsymbol{\phi}\frac{y}{2J_y}F_y(x-x_{LR}, y-y_{RL})$$

$$F_x(x,y) = 2C_p\frac{1-e^{-\frac{r^2}{2\sigma^2}}}{r^2}x,$$

$$2\pi\Delta\nu_x = \frac{\partial U_{\boldsymbol{0}}}{\partial J_x} \qquad 2\pi\Delta\nu_y = \frac{\partial U_{\boldsymbol{0}}}{\partial J_y}$$

$$x(s) = \sqrt{2\beta_x(s)J_x}\cos(\varphi_x(s)+\phi_x)$$

$$y(s) = \sqrt{2\beta_y(s)J_y}\cos(\varphi_y(s)+\phi_y).$$

Betatron function β(s) and phase $\varphi_x(s)$ at the long range interaction (s)

$$\frac{\partial^2 U_0}{\partial J_x^2} = \frac{1}{(2\pi)^2}\int d\boldsymbol{\phi}$$
$$\left[\frac{1}{2}\sqrt{\frac{\beta_x}{2J_x^3}}\cos(\varphi_x+\phi_x)F_x(x-x_{LR}, y-y_{RL})\right.$$
$$\left.-\frac{\beta_x}{2J_x}\cos^2(\varphi_x+\phi_x)\frac{\partial F_x}{\partial x}\right]$$

$$\frac{\partial^2 U_0}{\partial J_x \partial J_y} = -\frac{1}{(2\pi)^2}\int d\boldsymbol{\phi}$$
$$\sqrt{\frac{\beta_x\beta_y}{4J_xJ_y}}\cos(\varphi_x+\phi_x)\cos(\varphi_y+\phi_y)\frac{\partial F_x}{\partial y}$$

$$\frac{\partial^2 U_0}{\partial J_y^2} = \frac{1}{(2\pi)^2}\int d\boldsymbol{\phi}$$
$$\left[\frac{1}{2}\sqrt{\frac{\beta_y}{2J_y^3}}\cos(\varphi_y+\phi_y)F_y(x-x_{LR}, y-y_{RL})\right.$$
$$\left.-\frac{\beta_y}{2J_y}\cos^2(\varphi_y+\phi_y)\frac{\partial F_y}{\partial y}\right]$$

$$2\pi\frac{\partial\nu_i}{\partial J_j} = 2\pi\frac{\partial\nu_j}{\partial J_i} = \frac{\partial^2 U_{\boldsymbol{0}}}{\partial J_i \partial J_j}$$

Integration (summation) along s

$$\oint U_{bb}(M(s^*, s')x, s')ds'$$

- Integrate along s with taking account of bunch length and long range

$$U_{\boldsymbol{m}}(\boldsymbol{J}, z) = \frac{1}{(2\pi)^2}\int \lambda_p(z')ds \int d\phi_x d\phi_y e^{i\boldsymbol{m}\boldsymbol{\phi}} U_{bb}(\boldsymbol{r})$$

$$s = (z - z')/2 \qquad \lambda_p(z') = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z'^2}{2\sigma_z^2}\right)$$

$$U_{\boldsymbol{m}} = \frac{1}{(2\pi)^2}\sum_{LR}\int d\phi_x d\phi_y U_{LR} e^{i\boldsymbol{m}\cdot\boldsymbol{\phi}}$$

$$x(s) = \sqrt{2\beta_x(s)J_x}\cos(\varphi_x(s) + \phi_x)$$

$$y(s) = \sqrt{2\beta_y(s)J_y}\cos(\varphi_y(s) + \phi_y).$$

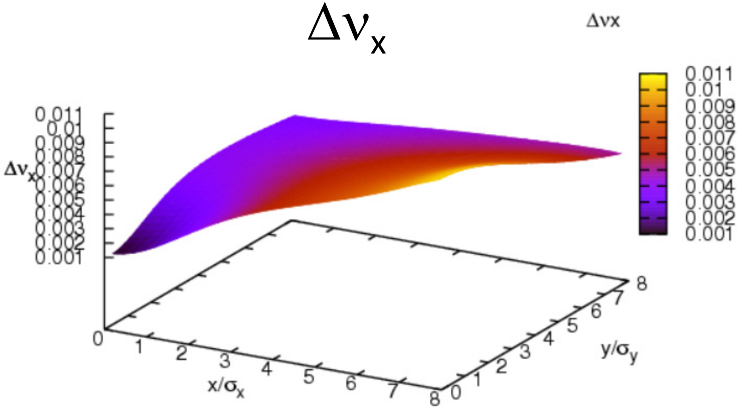
$$x(s_{LR}) = \sqrt{2\beta_{x,lr}J_x}\cos(\varphi_{x,LR} + \phi_x)$$

$$y(s_{LR}) = \sqrt{2\beta_{y,lr}J_y}\cos(\varphi_{y,LR} + \phi_y).$$

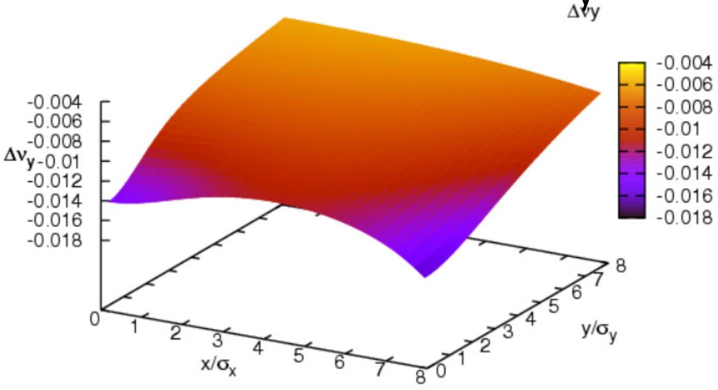
- Tune shift and its slope (2nd derivatives) are calculated by U_0.

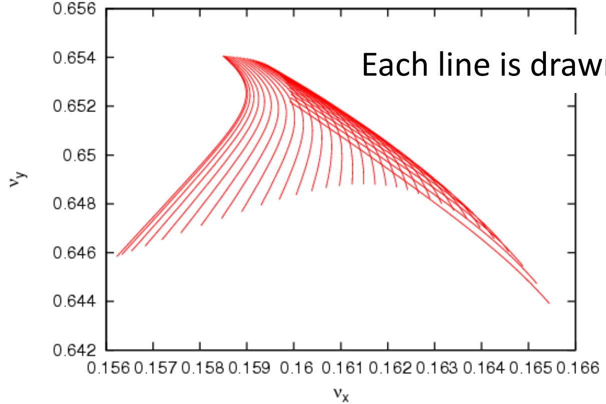
Tune shift (direct integral for Udϕ)

$\Delta\nu_x$

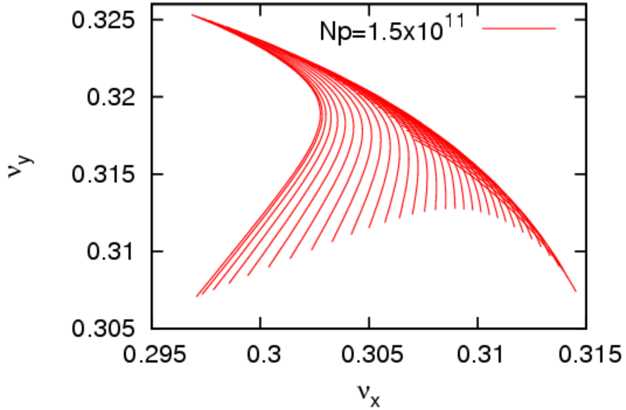


$\Delta\nu_y$

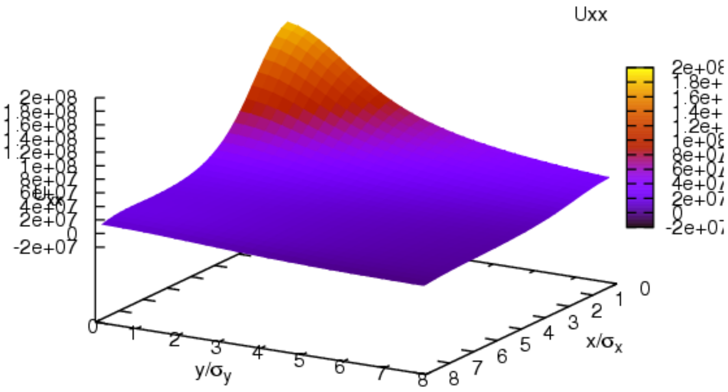


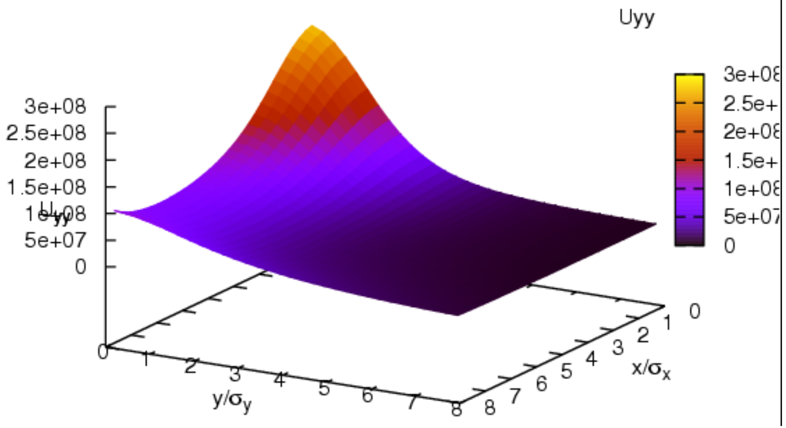


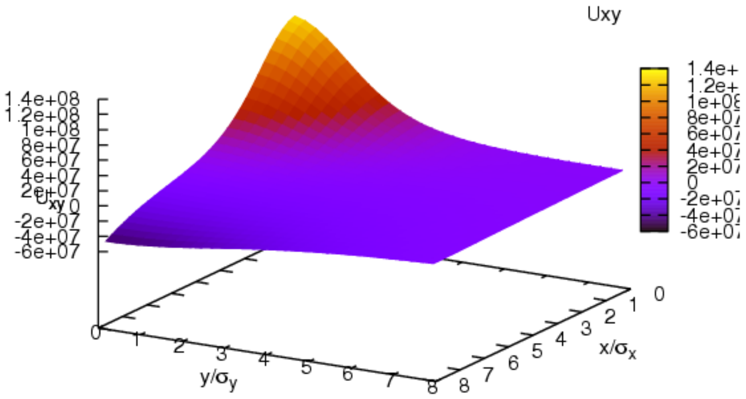
Each line is drawn for given Jx, 0-32ε_x.



Uxx, Uxy, Uyy







Resonances

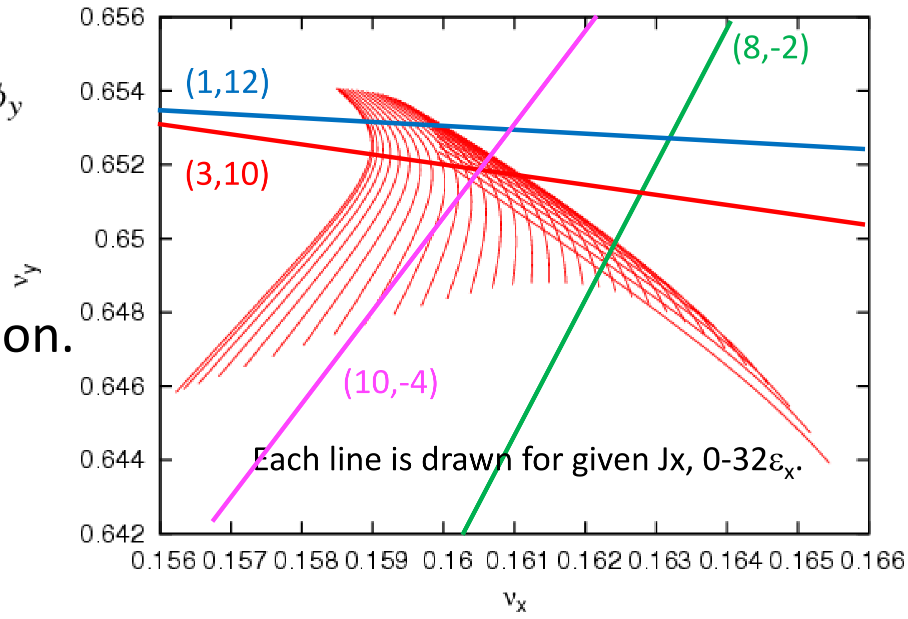
- A resonance occurs when the betatron amplitude satisfying

$$m_x \nu_x(\boldsymbol{J}_R) + m_y \nu_y(\boldsymbol{J}_R) = n$$

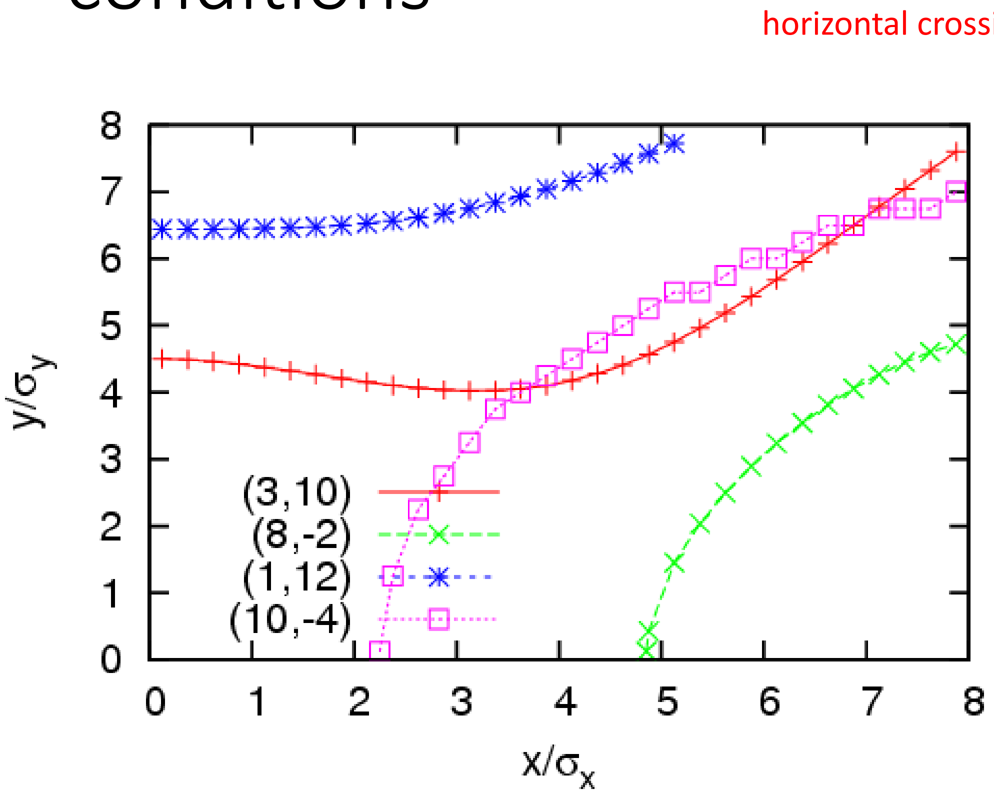
- The resonance line crosses the tune spread area.
- Resonance base

$$P_1 = \frac{J_x - J_{x,R}}{m_x} \qquad \psi_1 = m_x \phi_x + m_y \phi_y$$

- Fixed point P_1=0.
- For larger P_1 (J_x), ψ_1 moves due to detuning from the resonance condition.
- Motion in P_1 (J_x around J_{xR}), ψ_1 space depicts separatrix.
- Keep $P_2=(J_x-J_{xR})/m_x+(J_y-J_{yR})/m_y$



Betatron amplitude satisfying resonance conditions



- The beam-beam force is symmetric for y in the horizontal crossing.
- Only even m_y appears (4,-1)=>(8,-2).

Resonance width

- Characterize emittance growth

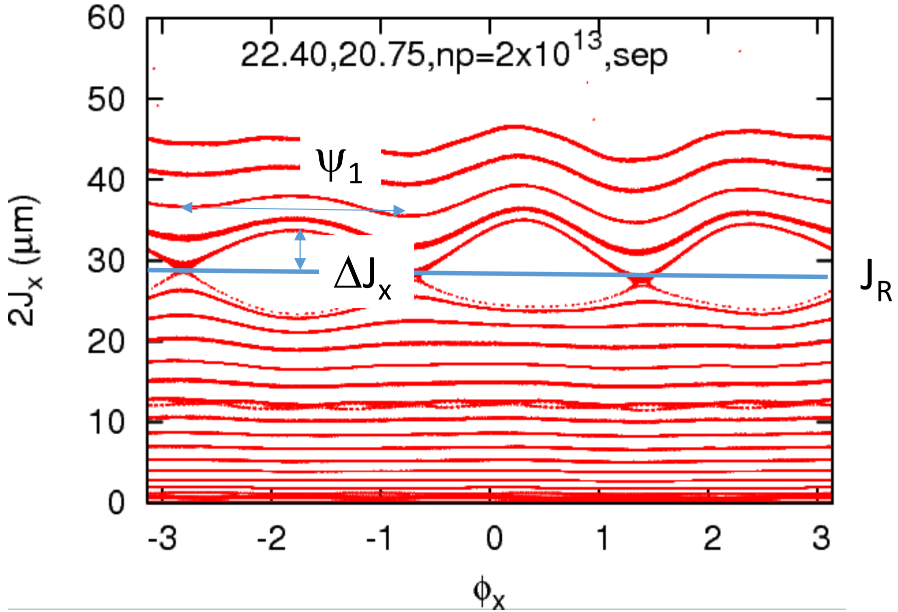
$$P_1 = \frac{J_x - J_{x,R}}{m_x} \qquad \psi_1 = m_x\phi_x + m_y\phi_y$$

$$H = \frac{\Lambda}{2}P_1^2 + U_{\boldsymbol{m}}(\boldsymbol{J}_R)\cos\psi_1.$$

$$\Lambda \equiv m_x^2\frac{\partial^2 U_{00}}{\partial J_x^2} + 2m_x m_y\frac{\partial^2 U_{00}}{\partial J_x \partial J_y} + m_y^2\frac{\partial^2 U_{00}}{\partial J_y^2}$$

$$\Delta P_1 = 2\sqrt{\frac{U_{\boldsymbol{m}}}{\Lambda}} \qquad \Delta J_x = 2m_x\sqrt{\frac{U_{\boldsymbol{m}}}{\Lambda}}.$$

Half width



Standard map

- Transfer (revolution) map for H

$$H = \frac{\Lambda}{2} P_1^2 + U_{\boldsymbol{m}}(\boldsymbol{J}_R)\cos\psi_1.$$

$$I = \Lambda P_1,\ \theta = \psi_1.\ t = s/L$$

- Standard map $\quad K = \Lambda U_{\boldsymbol{m}}$

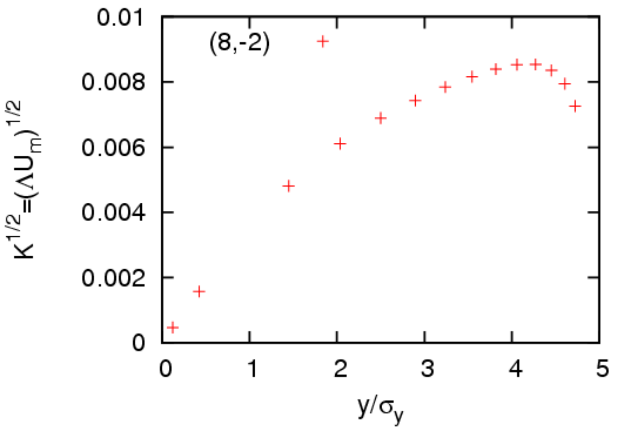
$$I_{t+1} = I_t + K\sin\theta_t$$
$$\theta_{t+1} = \theta_t + I_{t+1}$$

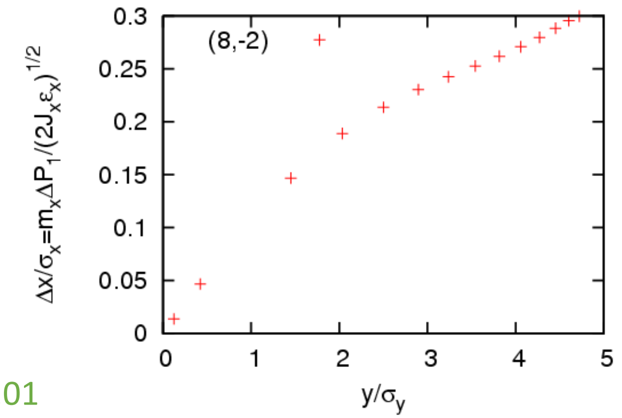
- Resonance half width

$$\Delta I = 2\sqrt{K}. \qquad \Delta P_1 = 2\sqrt{\frac{U_{\boldsymbol{m}}}{\Lambda}} \qquad \Delta J_x = 2m_x\sqrt{\frac{U_{\boldsymbol{m}}}{\Lambda}}.$$

- K, which is called "stochasticity parameter" is very small, $K<10^{-4}$. Strong chaotic system is K>1.

K and half resonance width for (8,-2) resonance

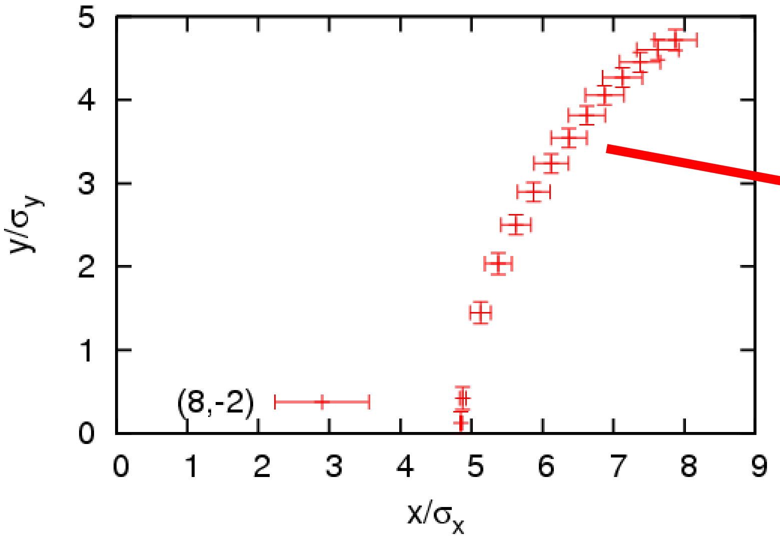




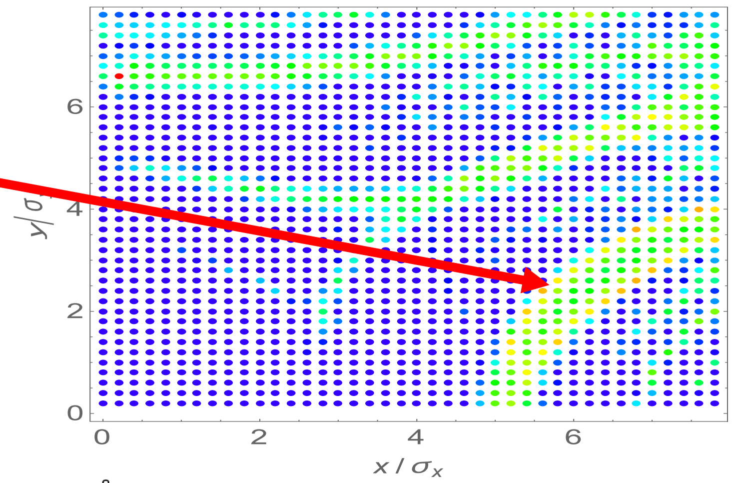
J-PARC Space charge K~0.01

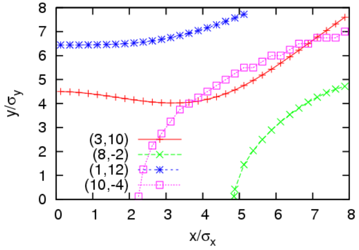
Resonance width in amplitude space

- SPPC, Np=1.5x10^{11}, 1-IP H crossing



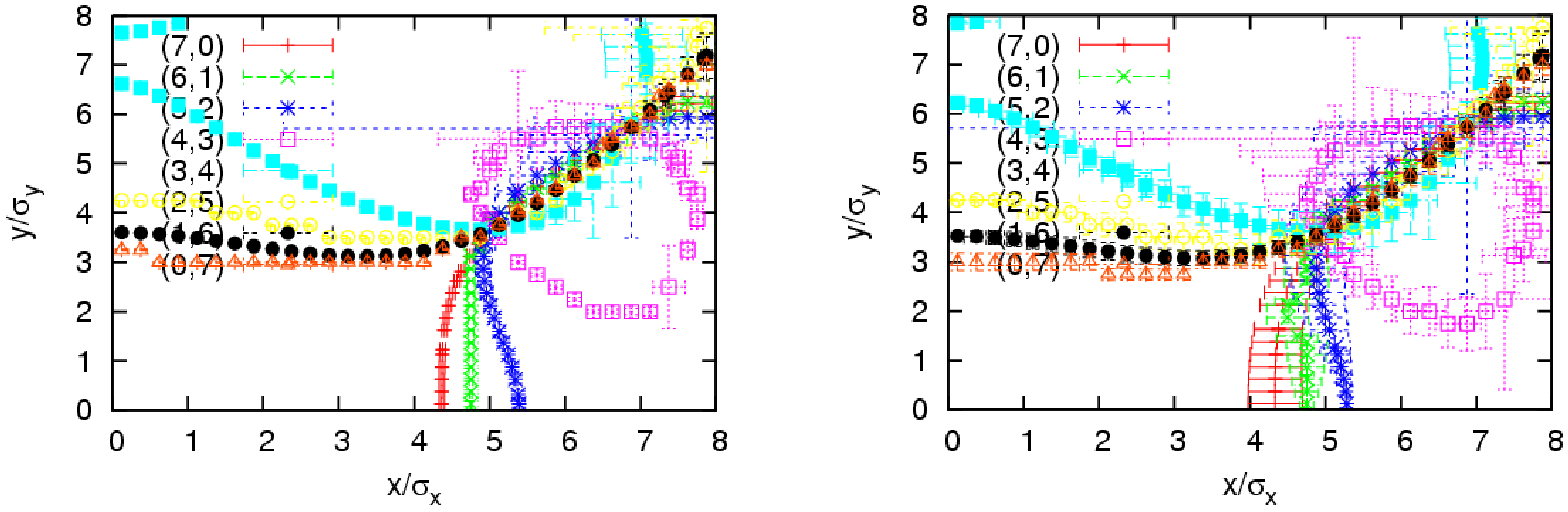
FMA analysis





Resonance width in amplitude space

- SPPC, Np=3x10^{11}, 2-IP, (ν_x, ν_y)=(0.29,1.30)
- 7-th order resonances, Studied numerically in K. Ohmi, F. Zimmermann, PRST-AB18,121003 (2015)
- The width is larger for finite z. Odd order resonances can be enhanced by finite z.

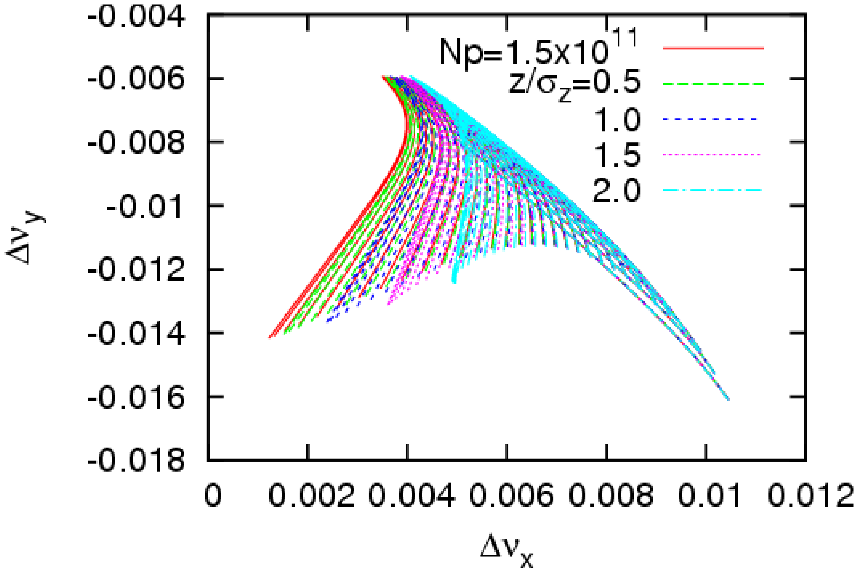


Synchrotron motion

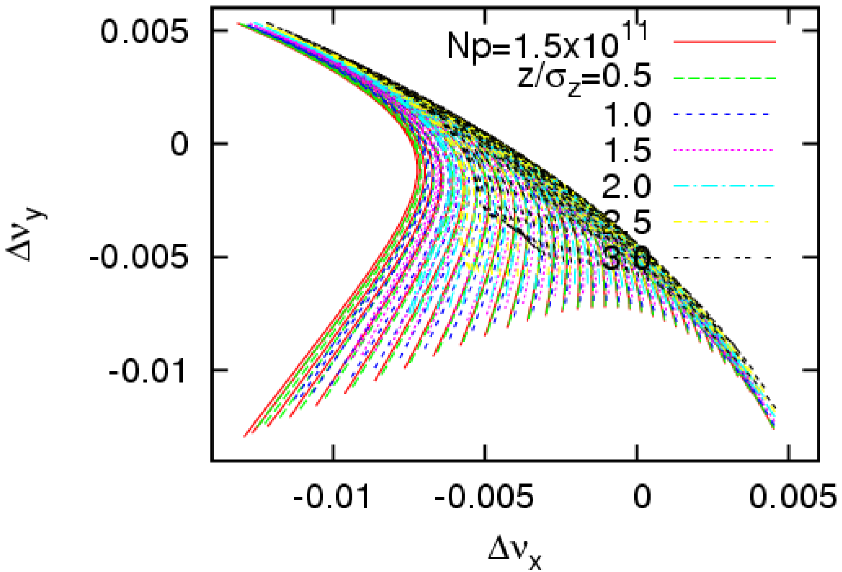
$$z = \sqrt{2\beta_z J_z}\cos\phi_z$$
$$\delta = \sqrt{2J_z/\beta_z}\sin\phi_z$$

- The beam-beam potential is calculated as function of z.
- Tune shift dependence on z

Horizontal crossing



Horizontal/Vertical crossing



Beam-beam potential

- Synchrotron motion is very slow compare with betatron motion.
- U is separated to average, synchrotron, synchro-betatron terms

$$U_{bb} = U_{\boldsymbol{0},0} + \sum_{m_z \neq 0} U_{\boldsymbol{0},m_z} e^{-im_z\phi_z}$$
$$+ \sum_{\boldsymbol{m}\neq 0, m_z} U_{\boldsymbol{m},m_z} e^{-i\boldsymbol{m}\cdot\boldsymbol{\phi} - im_z\phi_z}$$

$$z = \sqrt{2\beta_z J_z}\cos\phi_z$$
$$\delta = \sqrt{2J_z/\beta_z}\sin\phi_z$$

- Fourier component for the synchrotron motion

$$U_{\boldsymbol{m},m_z}(\boldsymbol{J}, J_z) = \frac{1}{2\pi}\int U_{\boldsymbol{m}}(\boldsymbol{J}, z) e^{im_z\phi_z} d\phi_z$$

$$U_{\boldsymbol{m}}(\boldsymbol{J}, z) = \int \lambda_p(z')ds \int \frac{d\boldsymbol{\phi}}{(2\pi)^2} e^{i\boldsymbol{m}\boldsymbol{\phi}} U_{bb}(r, z)$$

- Resonance condition for synchro-beta resonances

$$m_x\nu_x(\boldsymbol{J}, J_z) + m_y\nu_y(\boldsymbol{J}, J_z) + m_z\nu_z = n$$

Resonance width for the synchro-betatron resonances

- Resonance with (**m**,0) and its sideband (**m**,m$_z$)

$$\bar{U}(\boldsymbol{J}, J_z) = U_{\boldsymbol{o},0}(\boldsymbol{J}, J_z) + \sum_{\boldsymbol{m}\neq 0, m_z} U_{\boldsymbol{m},0}(\boldsymbol{J}, J_z) e^{-i\boldsymbol{m}\cdot\boldsymbol{\phi} - i m_z \phi_z}$$

$$\bar{H} = \bar{U} = \frac{\Lambda_{\boldsymbol{m}}}{2} P_1^2 + U_{\boldsymbol{m},m_z}(\boldsymbol{J}_R, J_z) \cos\psi_1$$

$$\Lambda_{\boldsymbol{m}} \equiv m_x^2 \frac{\partial^2 U_{\boldsymbol{o},0}}{\partial J_x^2} + 2 m_x m_y \frac{\partial^2 U_{\boldsymbol{o},0}}{\partial J_x \partial J_y} + m_y^2 \frac{\partial^2 U_{\boldsymbol{o},0}}{\partial J_y^2}$$

Resonance half with

$$\Delta P_1 = 2\sqrt{\frac{U_{\boldsymbol{m},m_z}}{\Lambda_{\boldsymbol{m}}}}$$

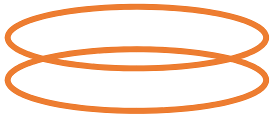
- Separation of the sideband

$$\delta P_1 = \frac{\mu_z}{\Lambda_{\boldsymbol{m}}}$$

- Overlapping condition

Empirical factor

$$\Delta P_1 > \frac{2}{3}\delta P_1 \qquad 3\sqrt{\Lambda_{\boldsymbol{m}} U_{\boldsymbol{m},m_z}} > 2\mu_z$$



  - Condition: The resonance width is larger than their separation.

Modulation due to the synchrotron motion

- Synchrotron motion should be considered even if the resonance condition is not satisfied, because it is very slow.

$$\hat{U}(\boldsymbol{J}, J_z, t) = \sum_{m_z \neq 0} U_{\boldsymbol{0},m_z} e^{-i m_z \mu_z t}$$

$$\begin{aligned}\hat{U}(\boldsymbol{J}, J_z, t) &= \hat{U}(\boldsymbol{J}_R) + \left.\frac{\partial \hat{U}}{\partial \boldsymbol{J}}\right|_{\boldsymbol{J}_R} (\boldsymbol{J} - \boldsymbol{J}_R) \\ &= \sum_{m_z \neq 0} \frac{\partial U_{\boldsymbol{0},m_z}}{\partial \boldsymbol{J}} (\boldsymbol{J} - \boldsymbol{J}_R) e^{-i m_z \mu_z t} \\ &= \sum_{m_z \neq 0} \frac{\partial U_{\boldsymbol{0},m_z}}{\partial \boldsymbol{J}} \cdot \boldsymbol{m} P_1 e^{-i m_z \mu_z t}\end{aligned}$$

- Standard map for a synchro-beta resonance with the modulation

$$\begin{aligned} I_{t+1} &= I_t + K_{\boldsymbol{m},m_z} \sin\psi_1 \\ \theta_{t+1} &= \theta_t + I_{t+1} + \sum_{m_z \neq 0} \frac{\partial U_{\boldsymbol{0},m_z}}{\partial \boldsymbol{J}} \cdot \boldsymbol{m} \cos(m_z \mu_z t) \end{aligned}$$

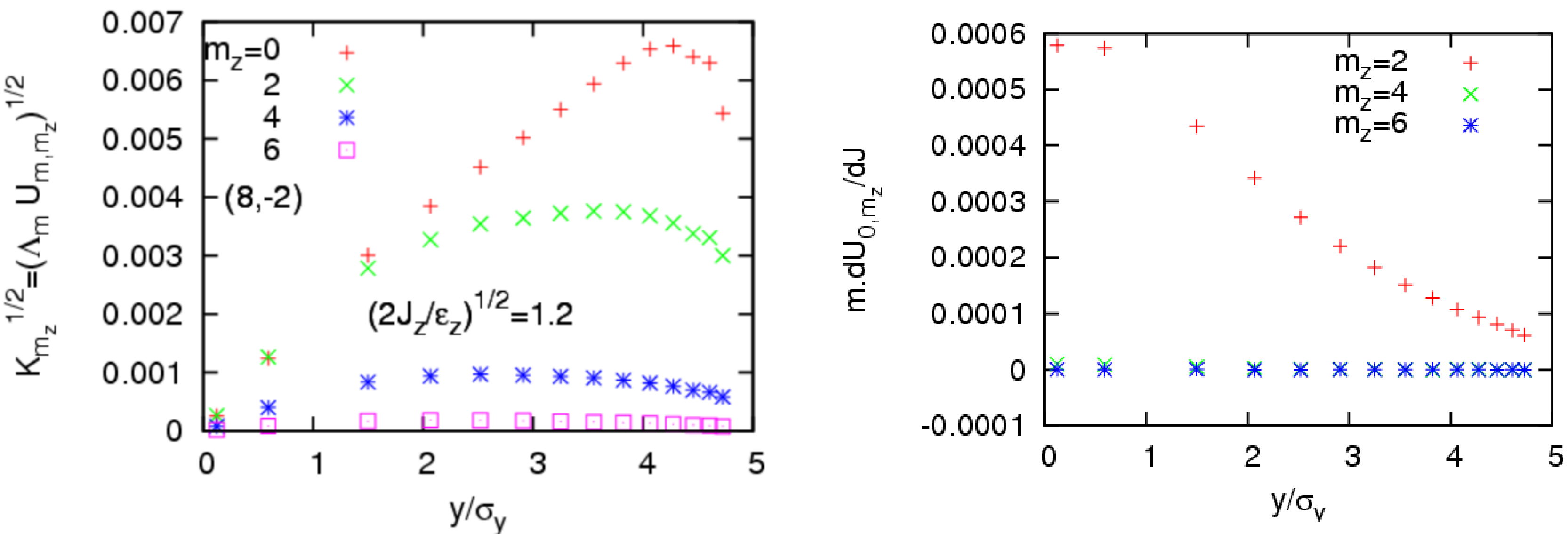
$$I = \Lambda P_1, \ \theta = \psi_1. \ t = s/L$$

- Chaotic area due to the modulation

$$\Delta P_1 = \mathrm{Max}_{m_z} \left(\frac{1}{\Lambda} \frac{\partial U_{\boldsymbol{0},m_z}}{\partial \boldsymbol{J}} \cdot \boldsymbol{m} \right)$$

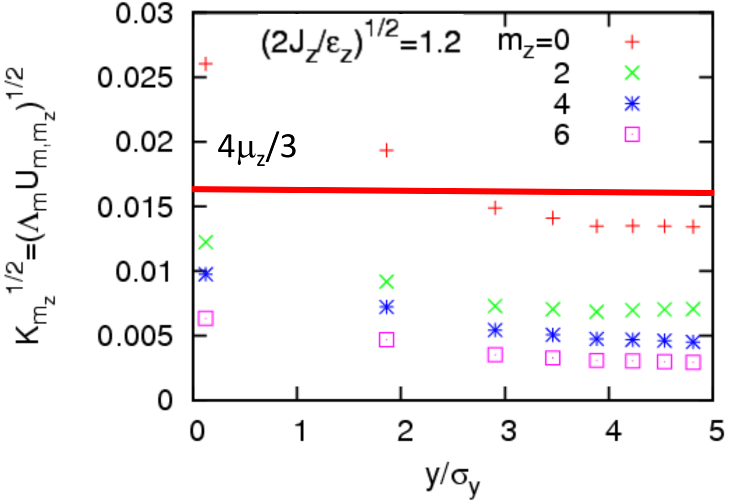
No overlap or weak modulation diffusion

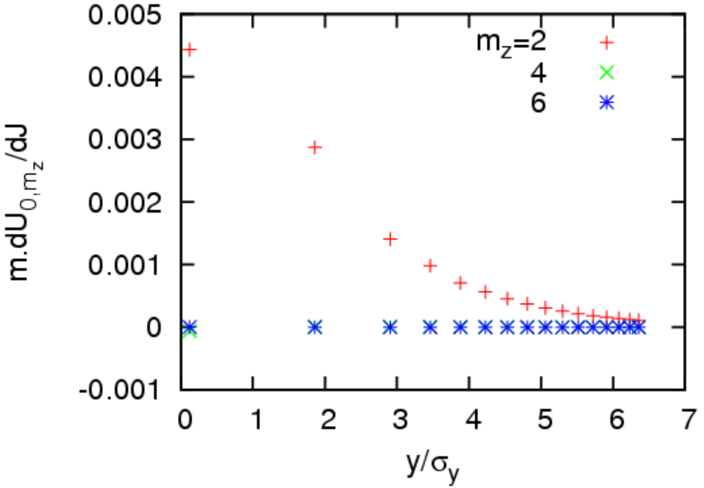
- SPPC, Np=1.5x10^{11}, 1-IP, $\Delta m_z=2$, μ_z->2μ_z
- $K^{1/2}<4\mu_z/3=0.008$ no overlap between synchrotron sidebands
- **m.**dU/d**J**<<$K^{1/2}$ stochastic area is narrower than the resonance width.



Higher intensity

- SPPC, Np=3×10^{11}, 2-IP, (ν_x, ν_y)=(0.29,1.30), (7,0) resonance
- Resonances m_z=0 and 2 can overlap. $\Delta m_z=2$, $\mu_z \to 2\mu_z$
- **m.**dU/d**J**<$K^{1/2}$ stochastic area is narrower than the resonance width, but contributes overlap of sidebands with m_z=2.
- Many resonances overlap (7,0),(6,1)…(0,7) and their 2nd synchrotron sidebands. It is disaster in this parameter .





Chromaticity

- Hamiltonian

$$H = \boldsymbol{\mu}_0 \cdot \boldsymbol{J} + 2\pi\delta\boldsymbol{\xi} \cdot \boldsymbol{J} + \delta_P(s)U_{bb}$$

- Modulation term

$$U_\xi = 2\pi\boldsymbol{\xi} \cdot \boldsymbol{m}P_1\sqrt{2J/\beta_z}\sin\mu_s t$$

- Stochastic area due to chromaticity

$$\Delta P_1 = \frac{2\pi\sigma_\delta\boldsymbol{\xi} \cdot \boldsymbol{m}}{\Lambda\boldsymbol{m}}\sqrt{\frac{2J}{\varepsilon_z}}$$

Summary

- Incoherent beam-beam effects have been studied for SPPC.
- Beam-beam simulation using weak-strong model showed the beam-beam limit between ξ=0.03-0.045, where 82 long range interactions were considered.
- Nonlinear resonances which cause the emittance growth have been studied considering head-on and long range interaction.
- Weak resonance effect in the design parameter at (ν_x, ν_y)=(0.31,1.32).
- Synchrotron sideband effect was seen, but not very strong. The long range interaction is independent of z.
- Higher intensity than the design and tune near a strong resonance showed disaster, as is consistent with the weak-strong simulation.

Thank you for your attention