Muon Beam Dynamics and Spin Dynamics in the g-2 Storage Ring

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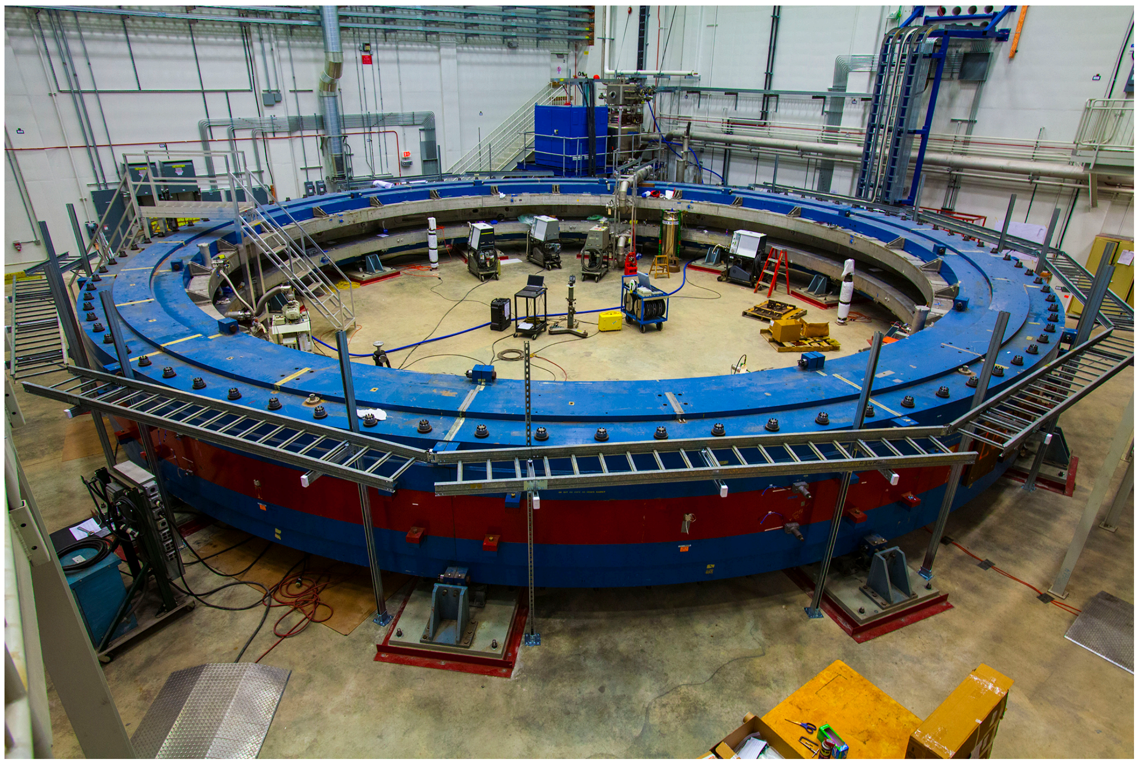
Muon g-2 experiment

- Goal
  - measure the anomalous magnetic moment of the muon (g-2) with 0.14 ppm uncertainty (*BNL 0.54 ppm*)
- Method
  - Circulate polarized muons at the 'magic' momentu, in uniform magnetic field and measure precession frequency
- Polarized muons
  - select of highest energy muons in $\pi^+ \to \mu^+ \nu_\mu$
- Measure muon polarization
  - Correlation with energy of decay positrons

Status

- Commissioning run summer 2017
- Production run is just beginning
- Statistics in excess of BNL by summer 2018



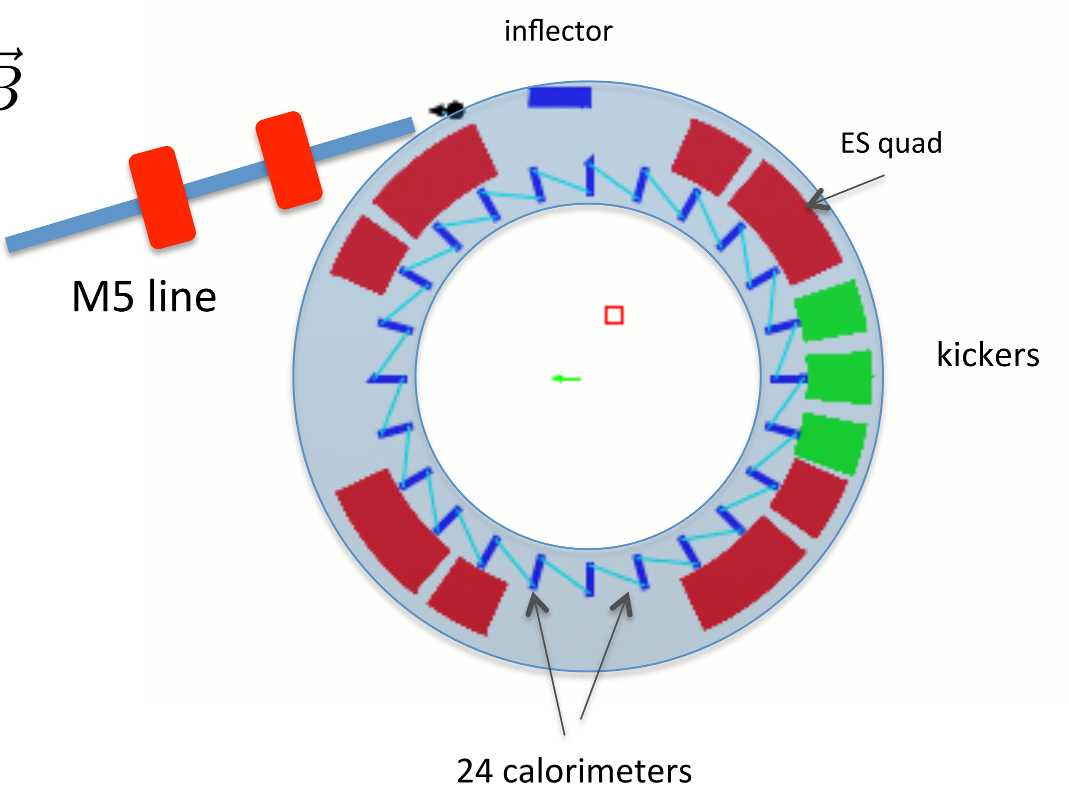
- *Upgrades of several systems planned for summer 2018*

Muon storage ring

$$a_\mu = \frac{g-2}{2}$$

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{q}{m} a_\mu \vec{B}$$

(ideal field and trajectory)



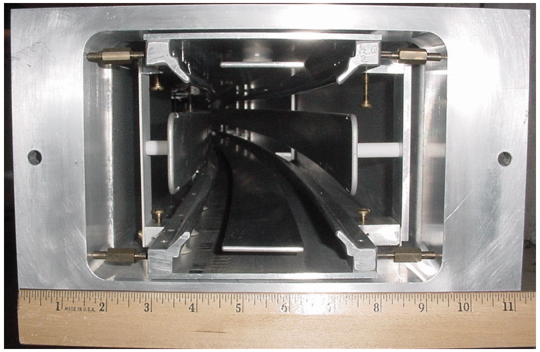
Requirement of uniform B-field
=> electrostatic focusing

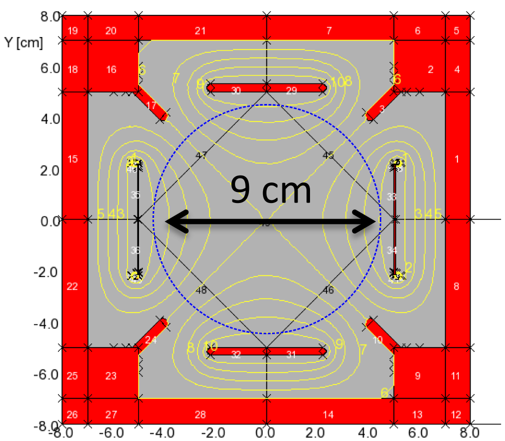
Circumference = 44.69 m

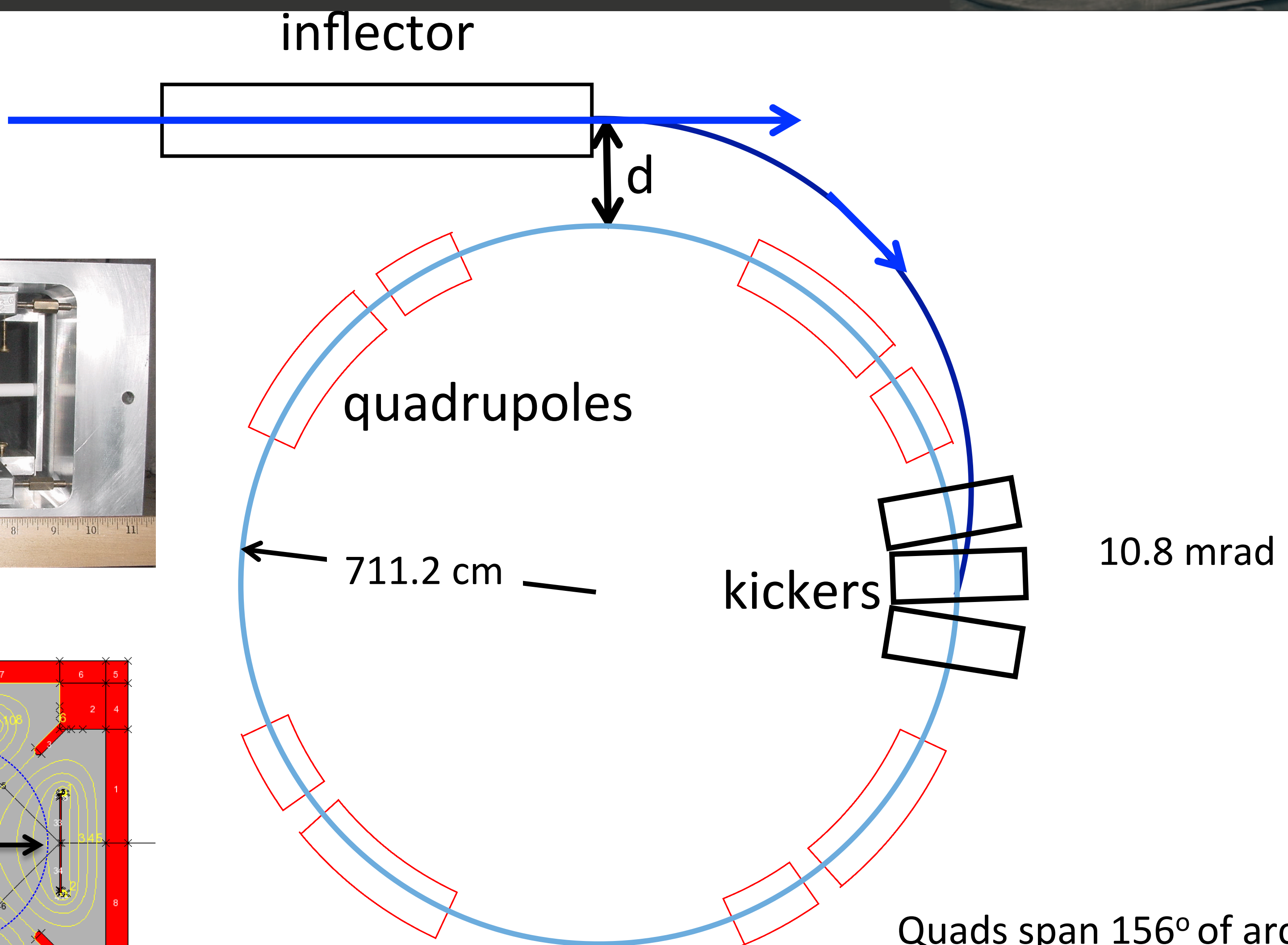
Muon storage ring

- Circumference = 44.69 m
- Revolution period = 149.1 ns
- B_y = 1.45 T
- $P_\mu = 3.09$ GeV/c
- $\gamma = 29.3 = (\frac{1}{a_\mu} - 1)^{1/2}$
- Aperture = 9 cm
- $\tau_\mu = 64.4$ μs (at 3.09 GeV/c) = 432 turns

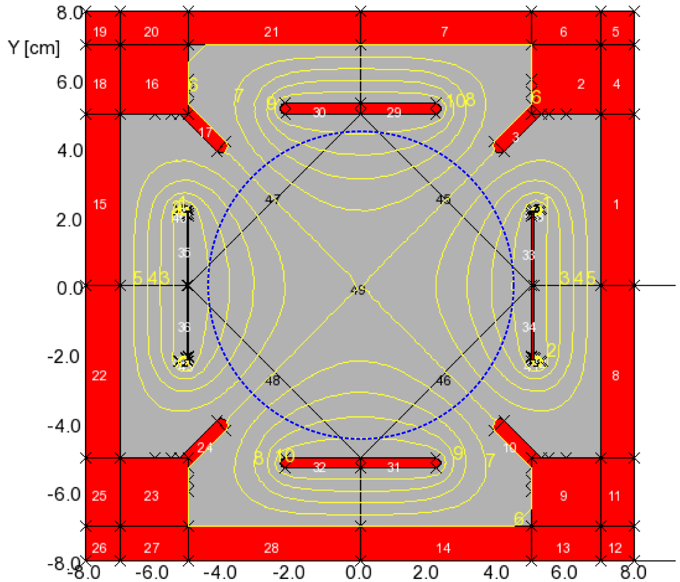
Electrostatic quadrupoles

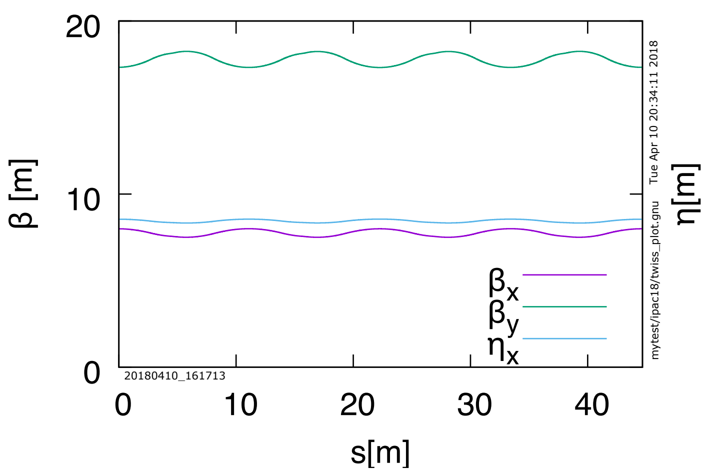


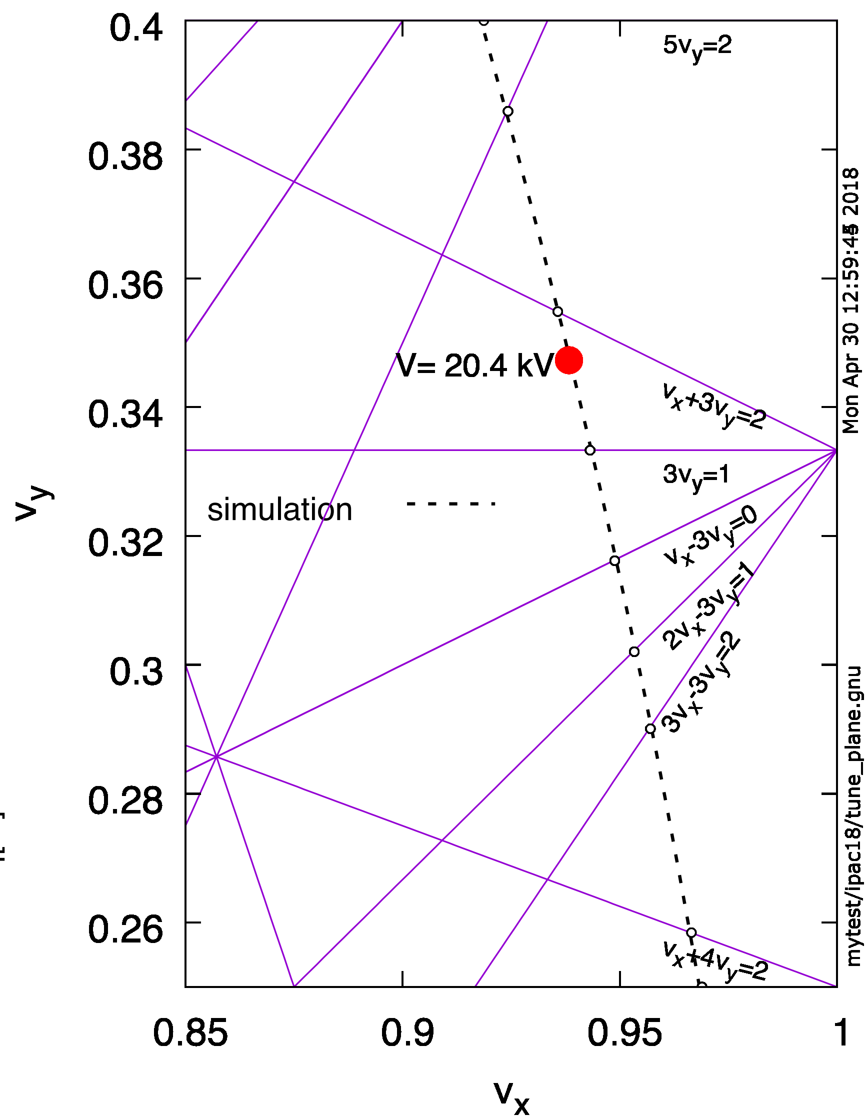




Weak focusing lattice

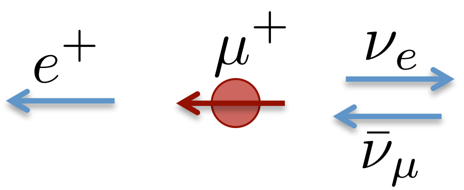






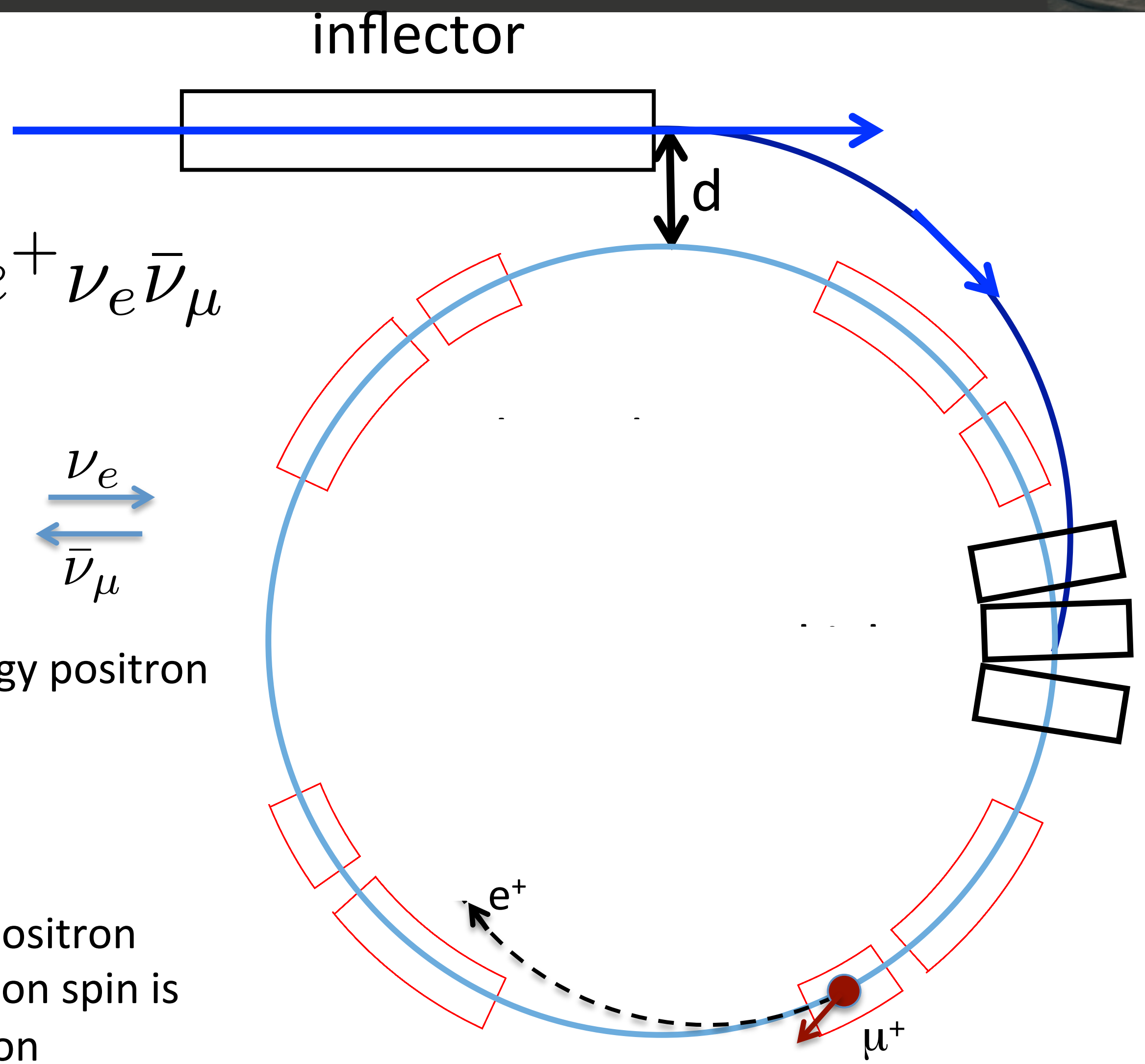
Muon decay

$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$



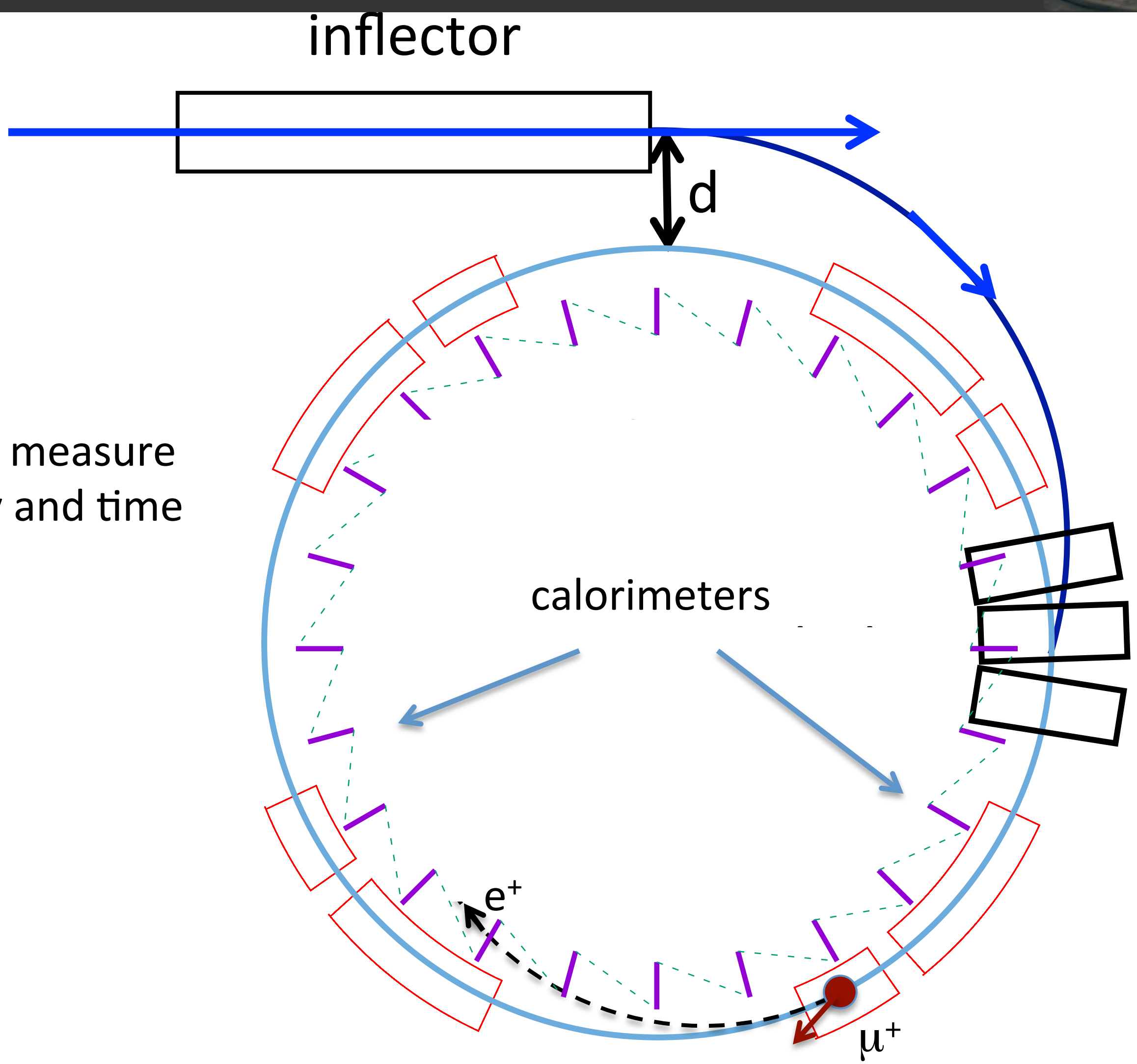
=> Highest energy positron

Correlation of positron energy with muon spin is measure of muon polarization

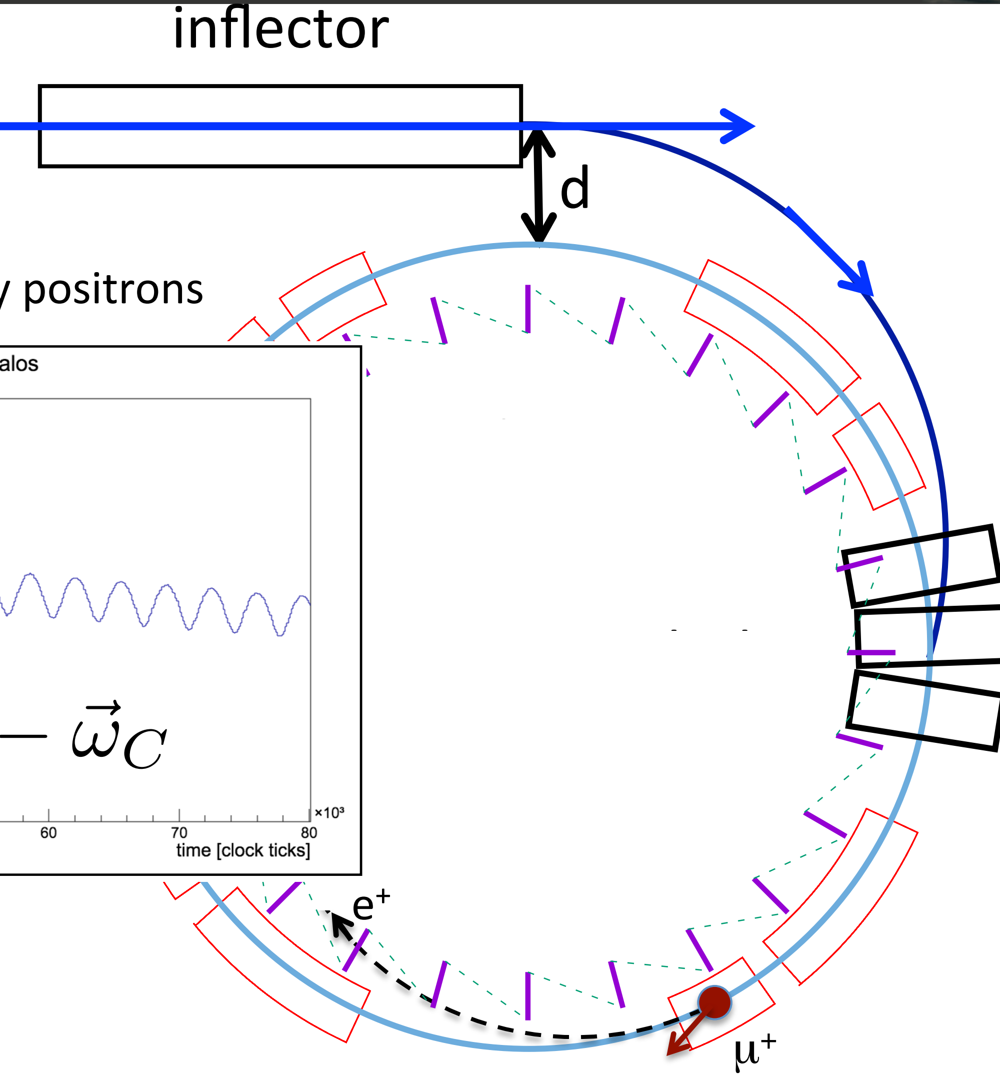


Calorimeters

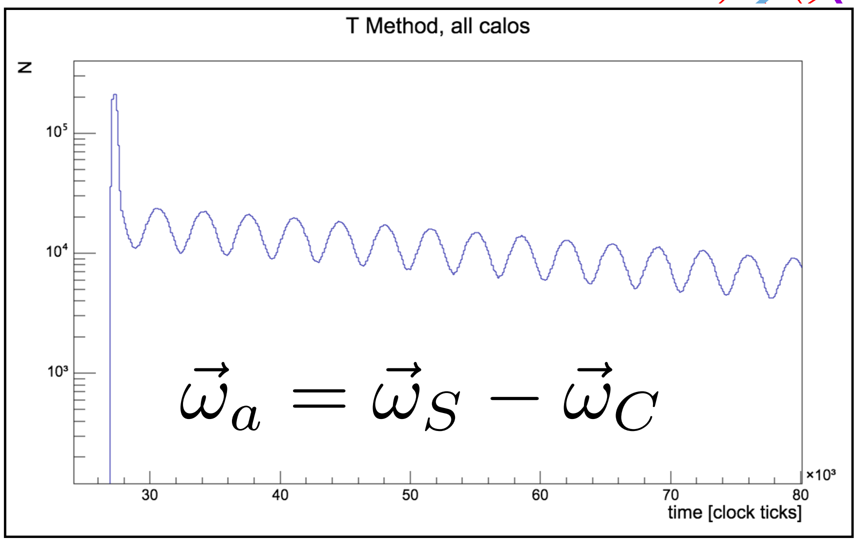
24 calorimeters measure positron energy and time



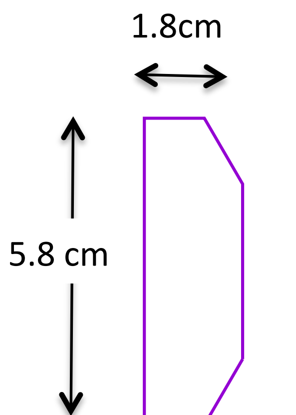
Calorimeters



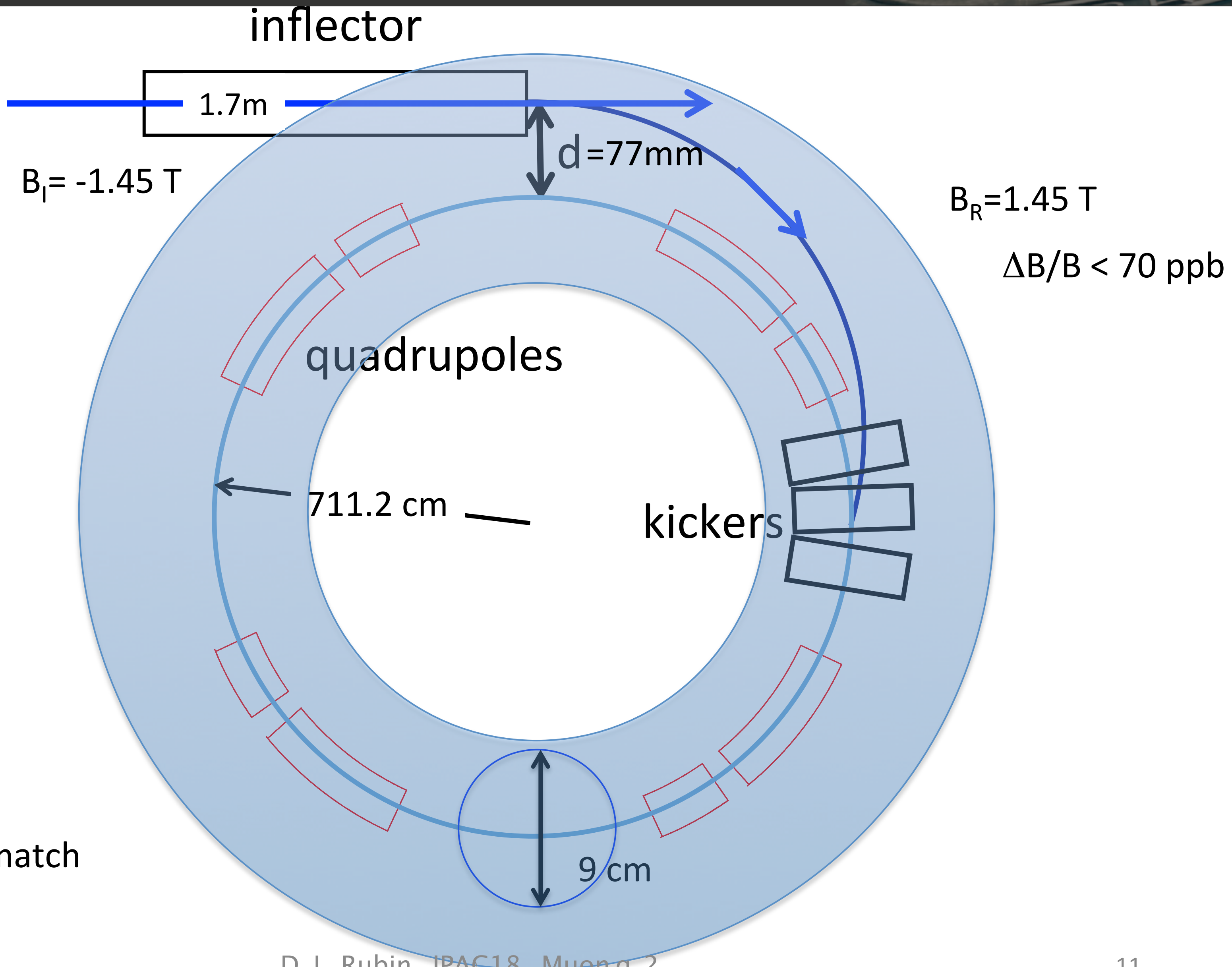
Histogram of high energy positrons



Injection



Inflector aperture



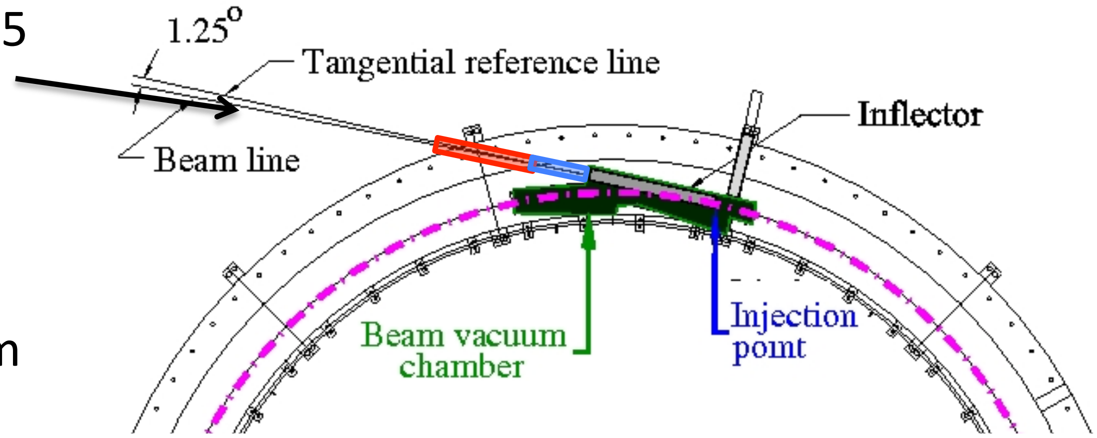
Injection mismatch

Injection

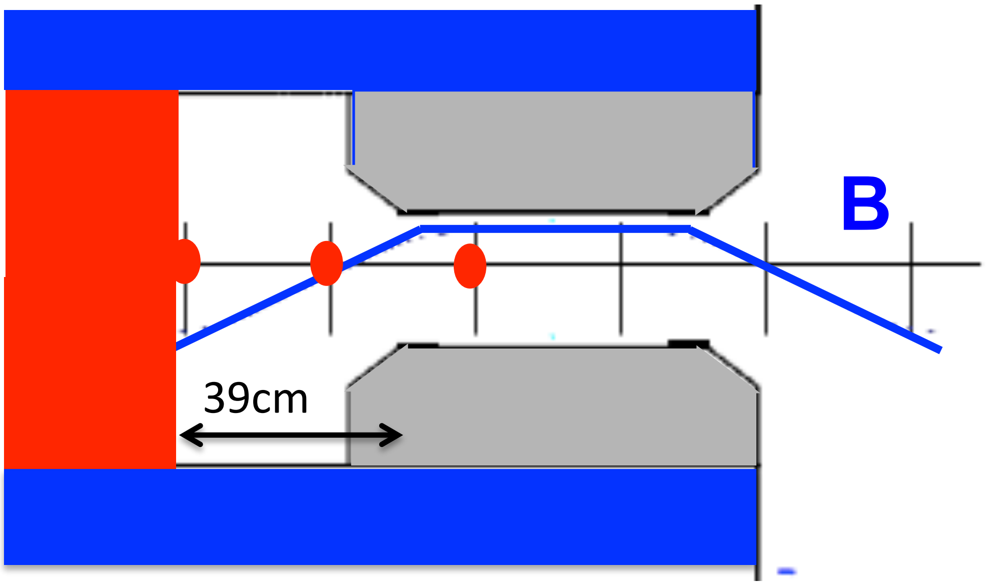
Muons come to the end of the M5 line and then propagate through:

- Hole in magnet yoke
- Dipole fringe field
- Inflector

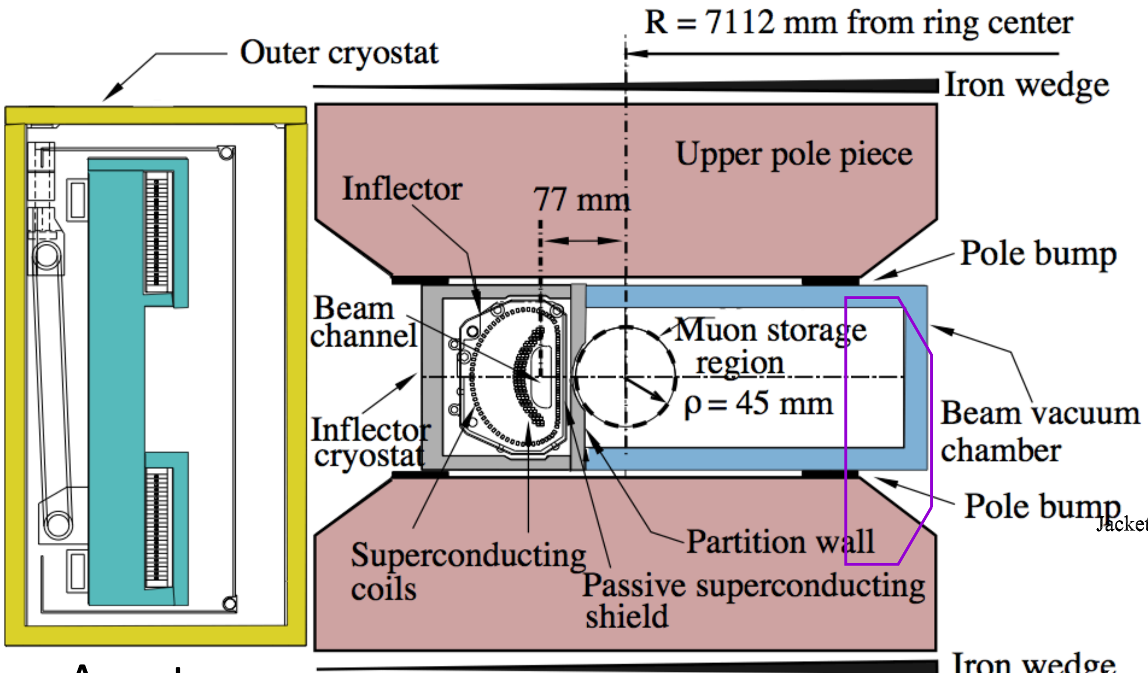
And exit the inflector 77 mm from the center of the dipole aperture



The magnetic field is near zero at the inner surface of the yoke, and rises to 1.45T between the magnet poles, over a distance of ~39cm



Inflector



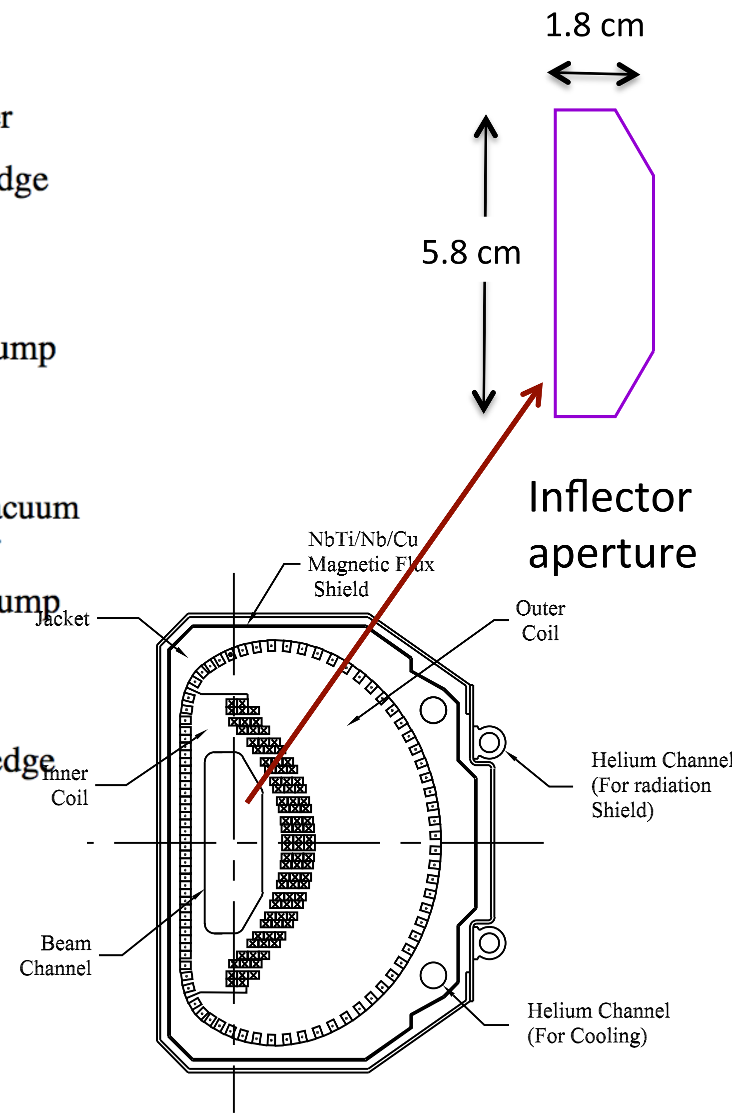


Fig. 7. Inflector cross-section.

Apertures

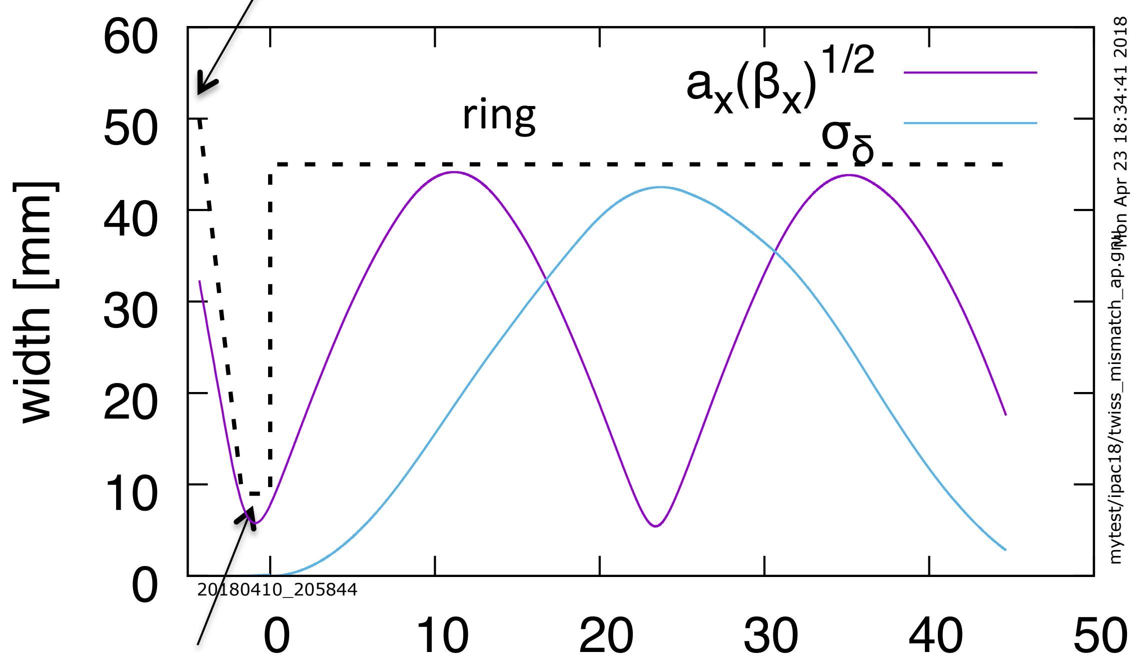
- Inflector half-width = 9mm
- Inflector half-height = 28mm
- Storage volume radius = 45mm

Injection phase space

Injected beam
ε = 40 mm-mrad (95%)
$\Delta p/p$ = 2%

- Beam is focused to a waist to clear inflector aperture
- (Muons are scattered in coil overlapping inflector aperture)
- Dispersion is zero in inflector

Hole in backleg iron

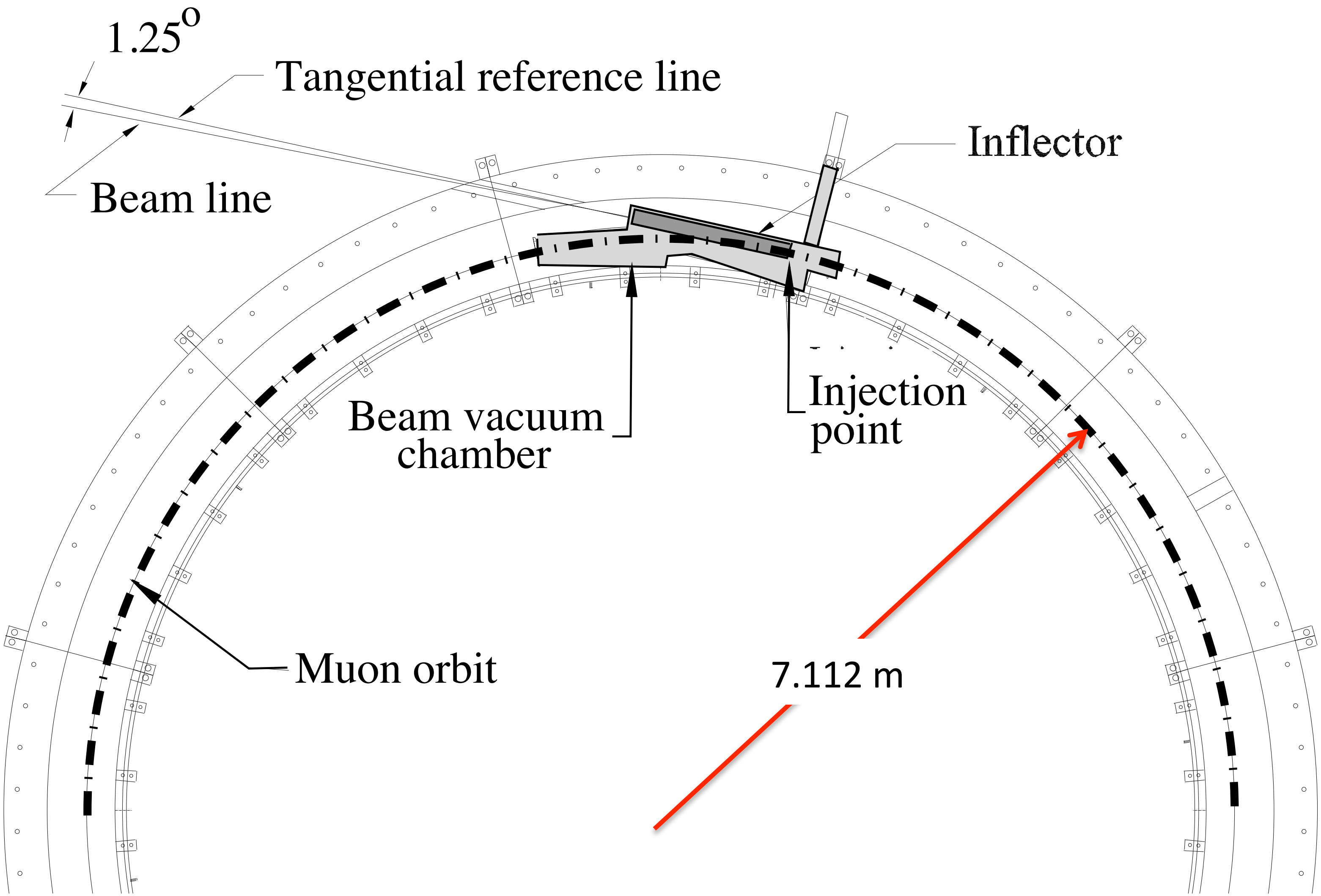


inflector

=> Dispersion and phase space mismatch

Ring
β_x = 7.8 m
η = 8.2 m

Injection channel

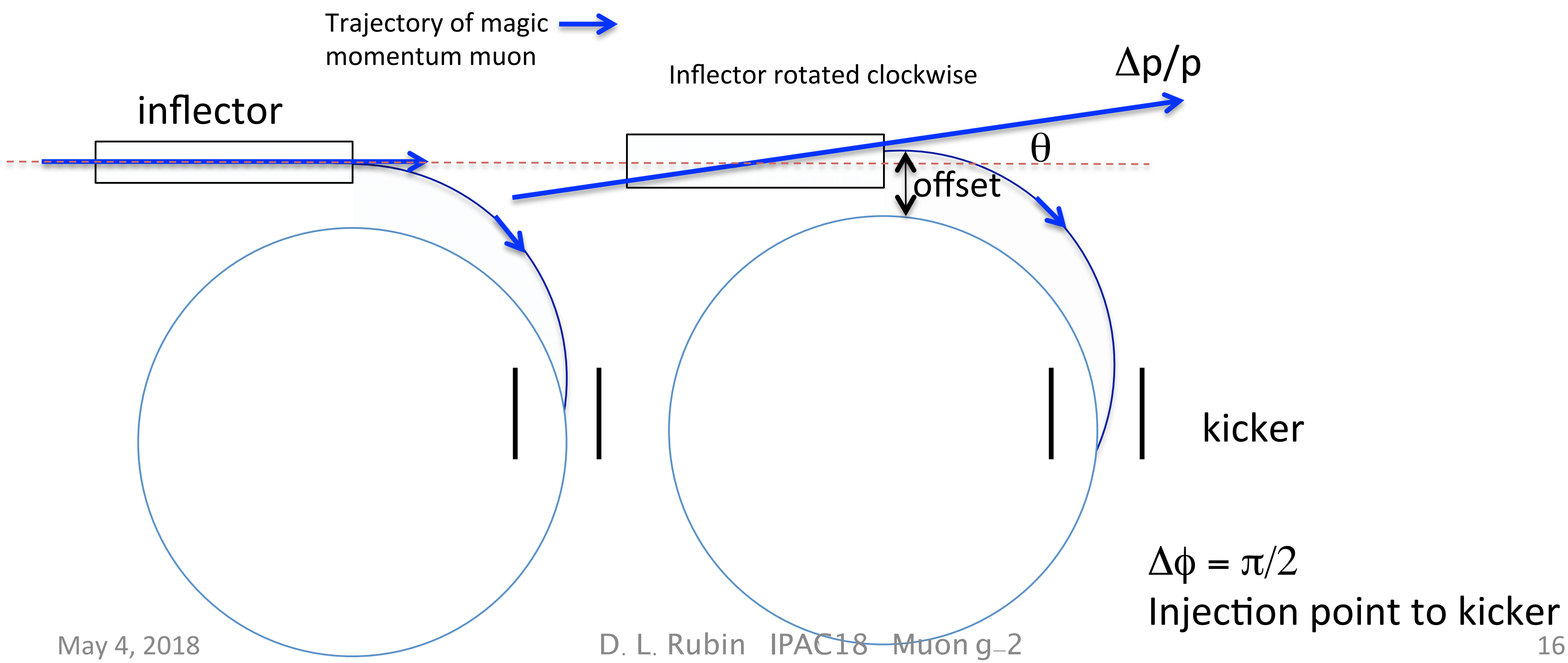


Injection Dependencies

Injected particles oscillate about the closed orbit with amplitude that depends on; momentum, injection angle, offset, and kick angle

Amplitude and envelope of betatron oscillation of stored beam depends on

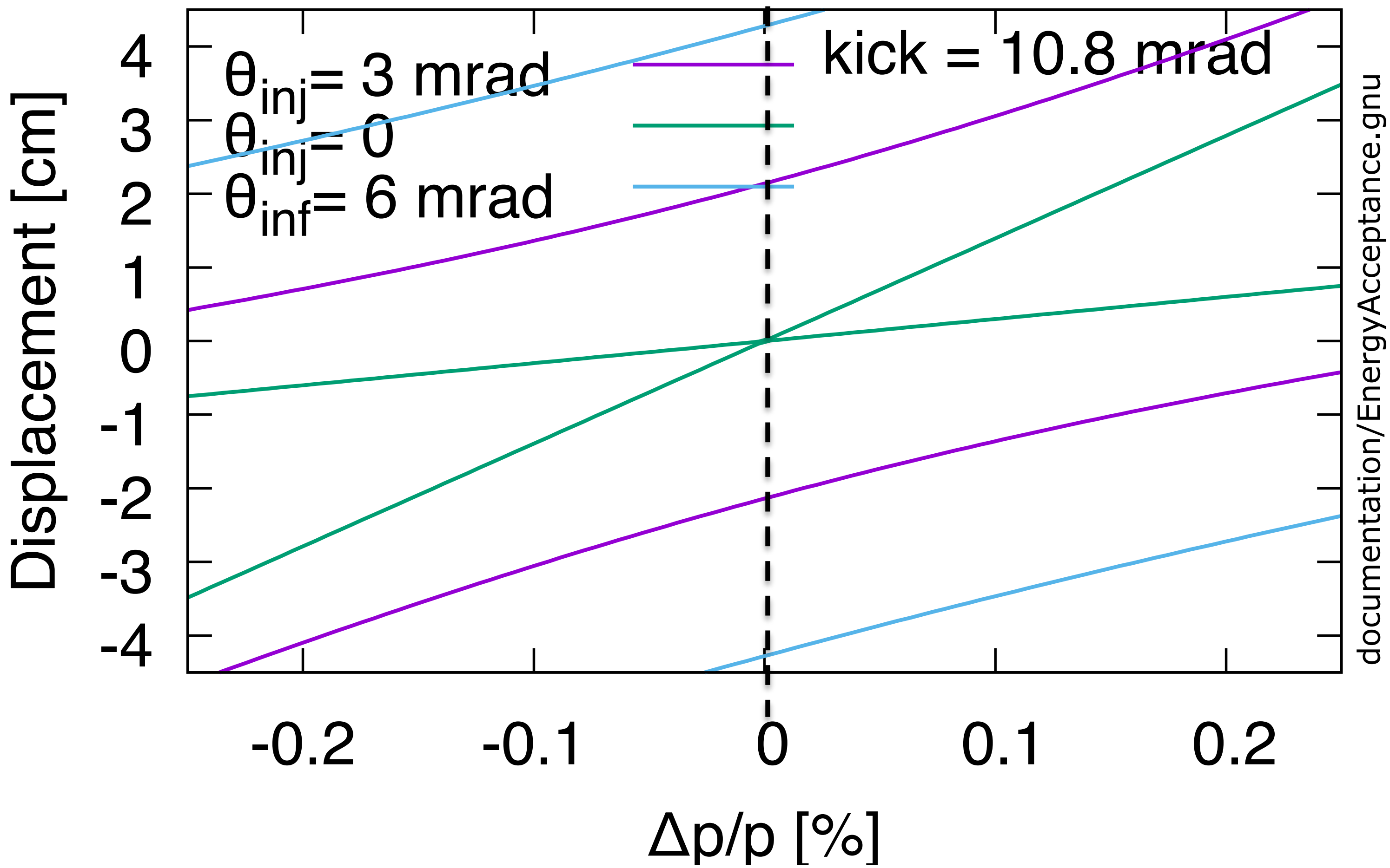
- injection angle θ and
- Kicker angle
- Momentum



Muon capture

Envelope of betatron motion of stored muon

(10.8 mrad kick angle puts on momentum muon on axis)



Muon capture

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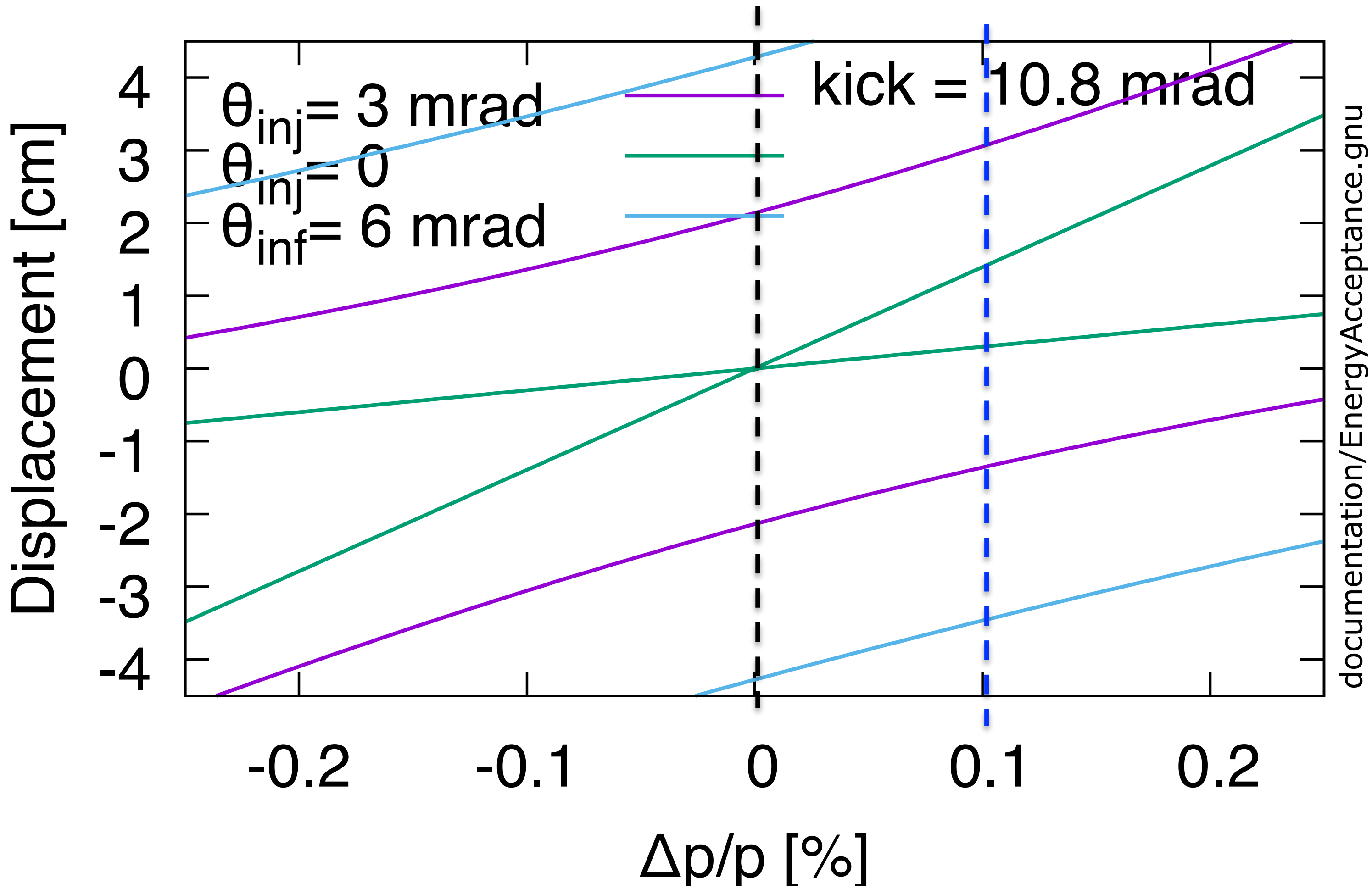
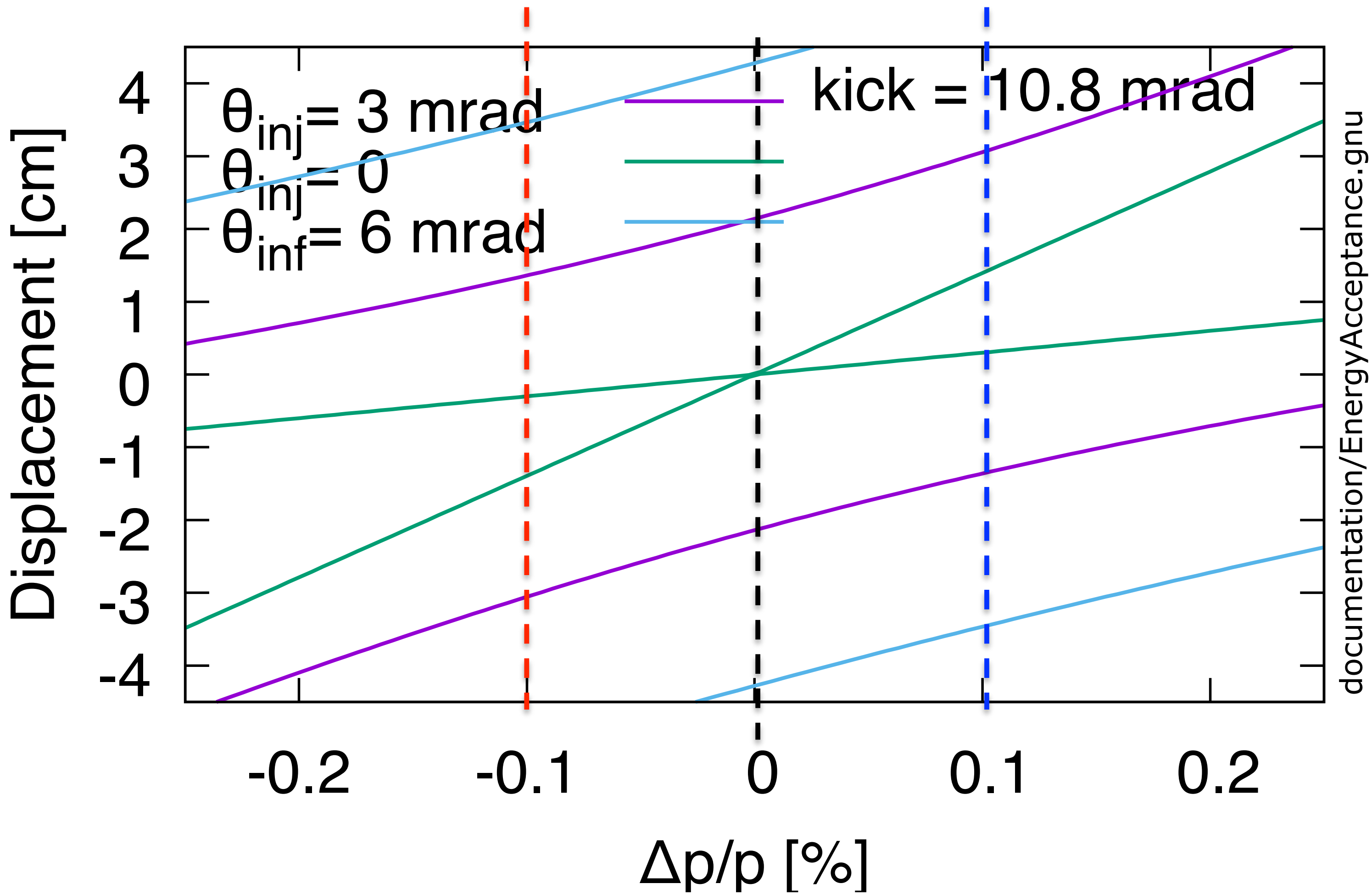


Muon capture

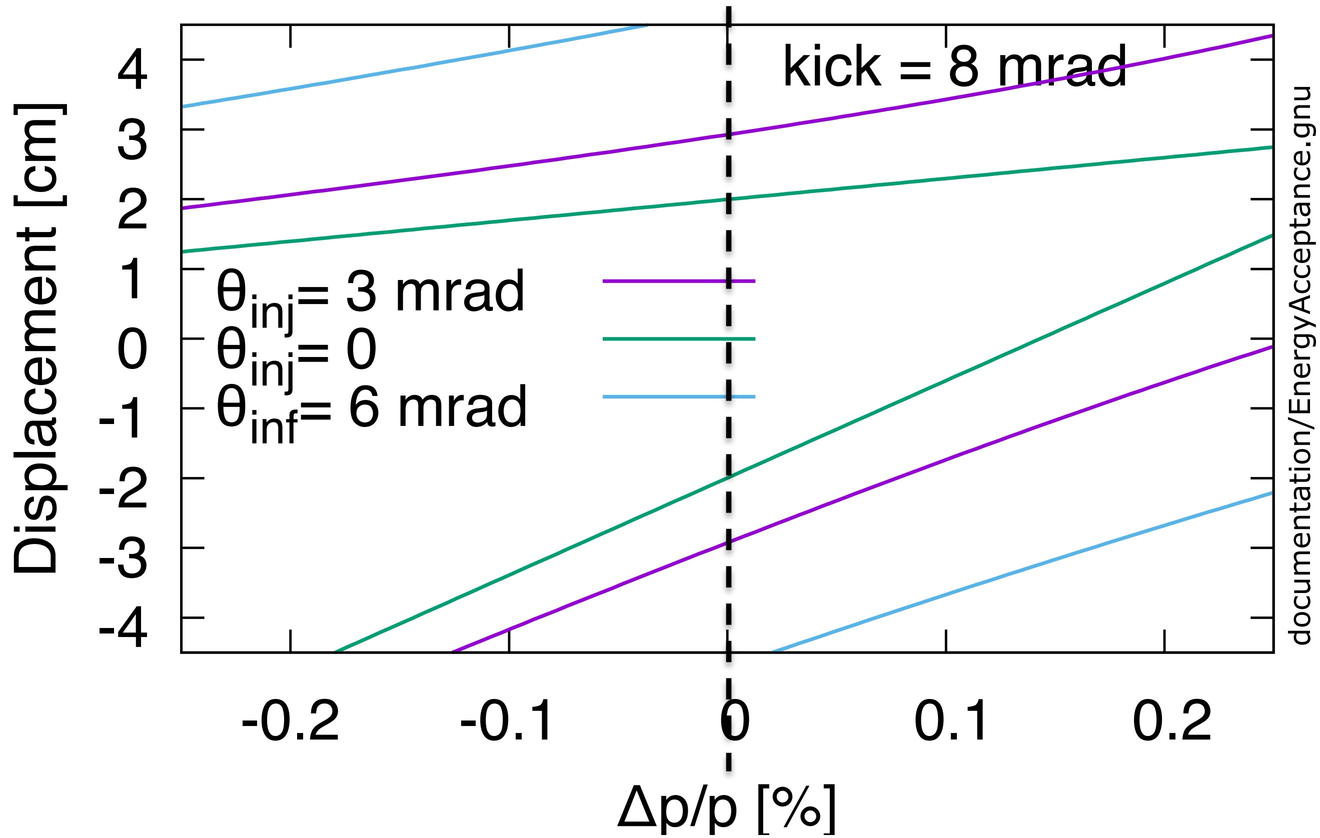
Envelope of betatron motion of stored muon



Muon capture

Envelope of betatron motion of stored muon

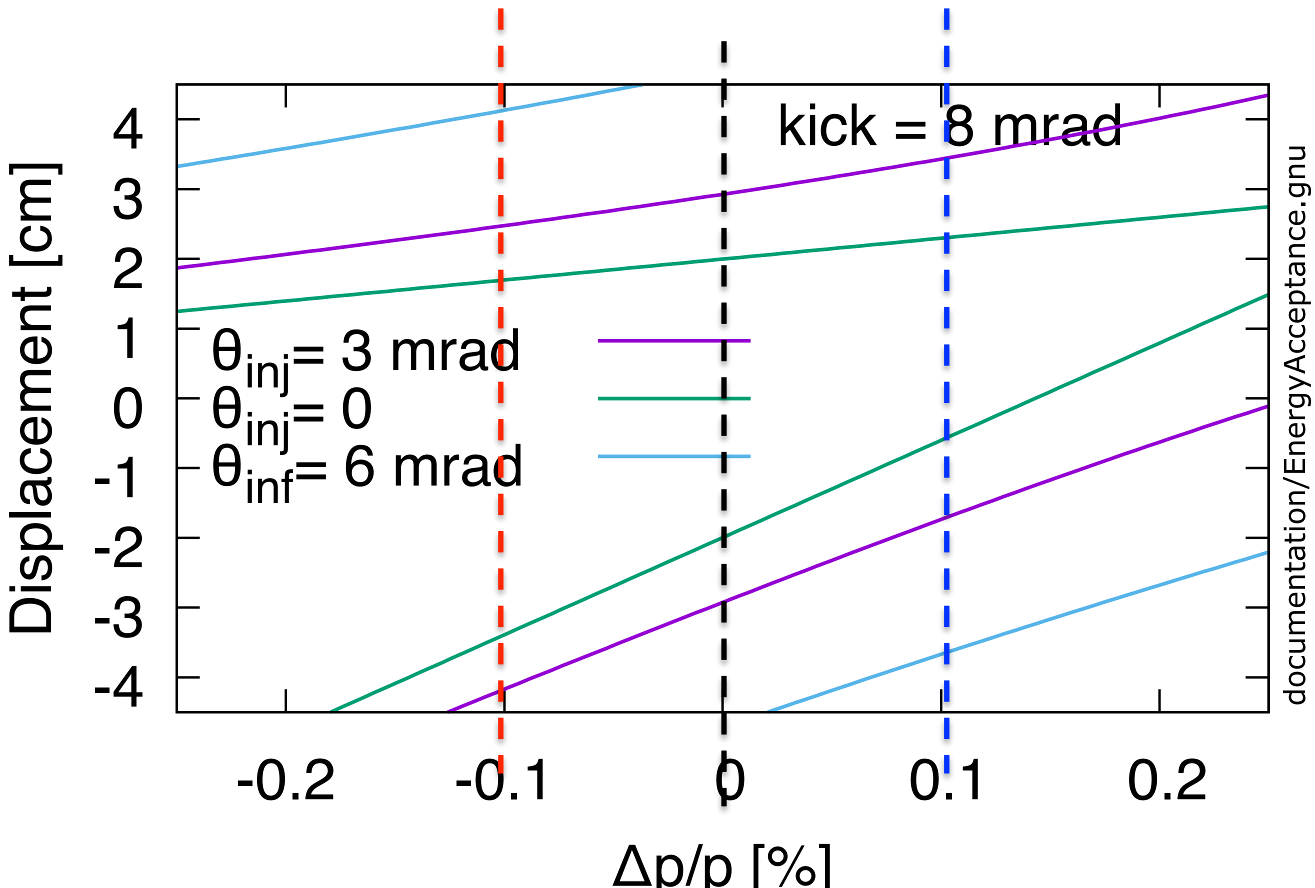
(8 mrad kick = 75% nominal – under kicking)



Muon capture

In the event of *under kicking*

- Acceptance is skewed to higher momenta



Measurement

Systematics

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B}) - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

In general

- Finite momentum distribution
- Transverse motion
- Non zero E-field

Pitch correction

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B}) - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Pitch correction:

- Contribution depends on vertical phase space distribution

$$\beta_y = p_y/(\gamma m c)$$

- Correction requires measurement of

vertical phase space

E-field correction

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B}) - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

E-field correction:

- Choose momentum.

At $\vec{p}_{\mathrm{magic}} = 29.3\ \mathrm{GeV/c} \rightarrow a_\mu = \dfrac{1}{\gamma^2 - 1}$

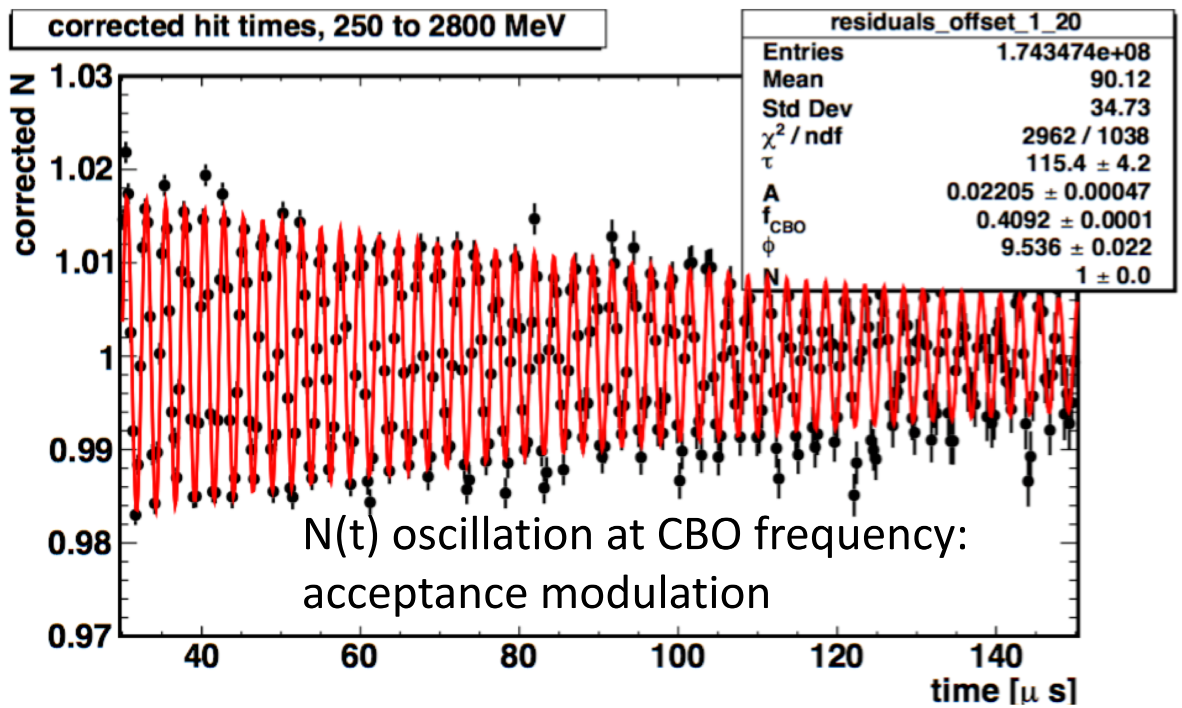
Correcting for contribution from non-magic momentum muons requires

- Measurement of *momentum distribution*
- Measurement of *radial distribution* $\vec{E} = k(x\hat{i} - y\hat{j})$

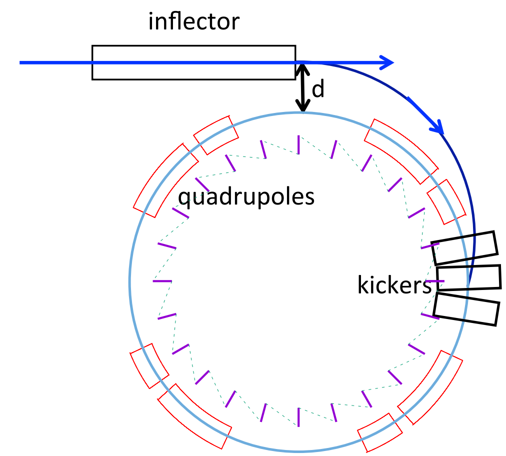
Acceptance correction

The acceptance of the calorimeters depends on the radial offset of the decay muon

- Radial offset varies with modulation of beam width
- And with oscillation of the centroid



N(t) oscillation at CBO frequency:
acceptance modulation



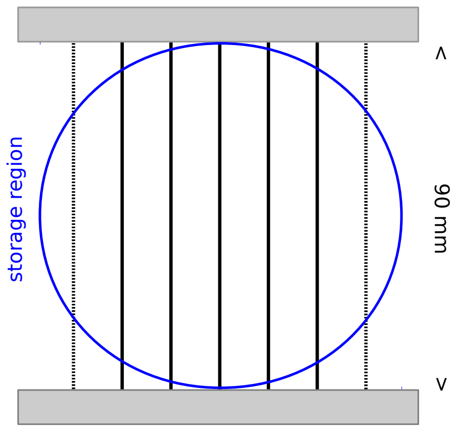
Correction requires measurement of
evolution of radial distribution

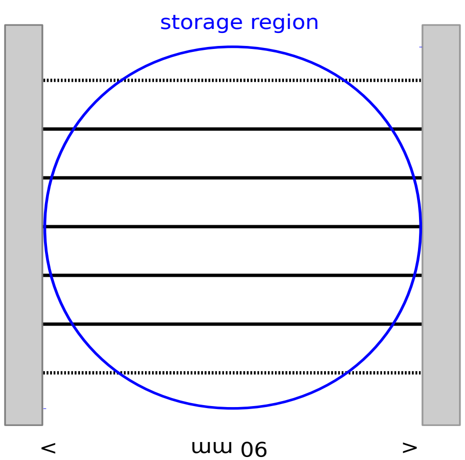
Detectors

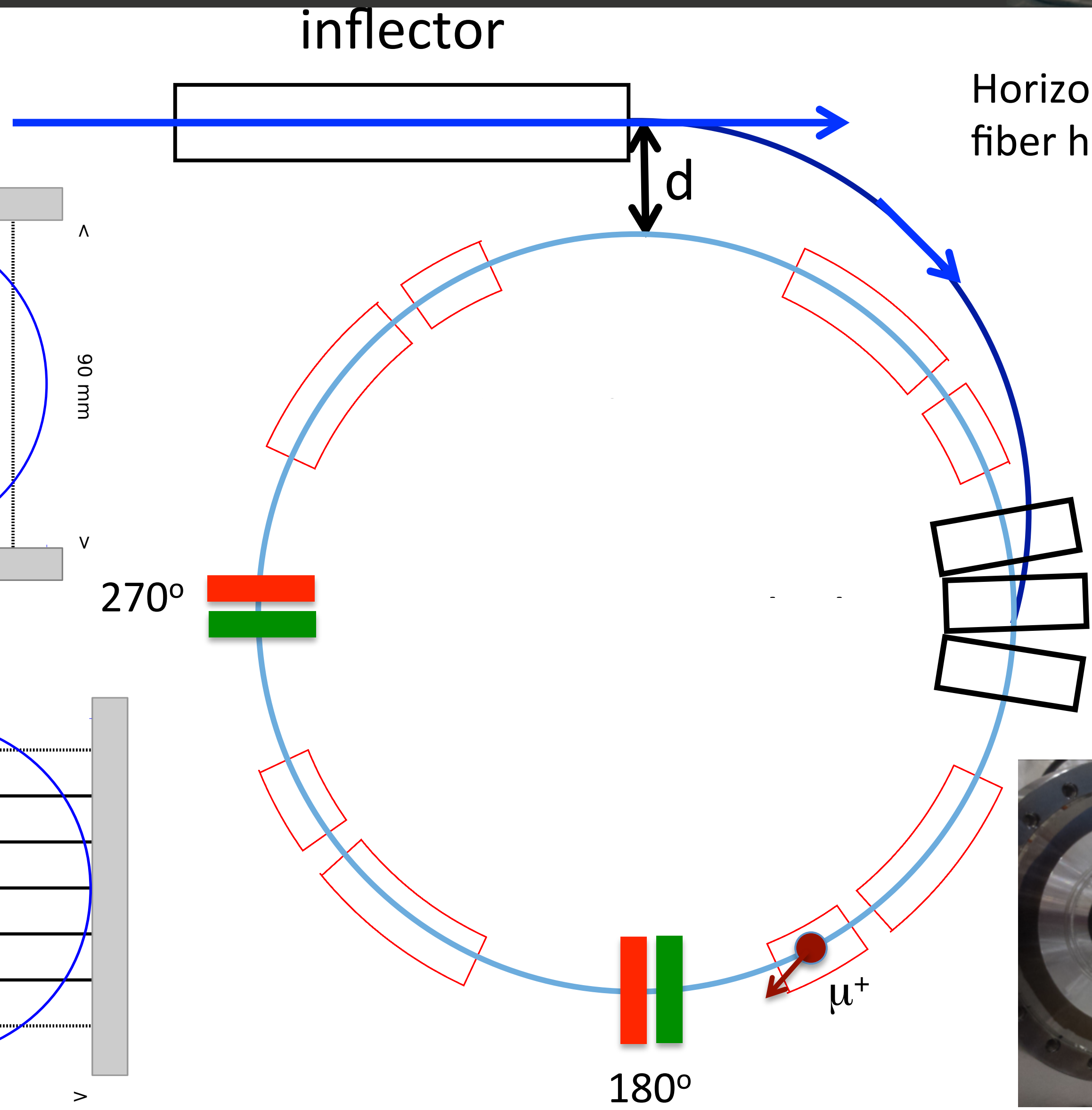
Fiber Harps
Traceback trackers
Calorimeters

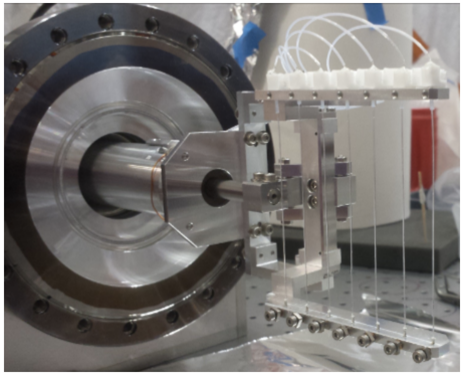
Fiber Harps

Horizontal and vertical fiber harps at 180 and 270



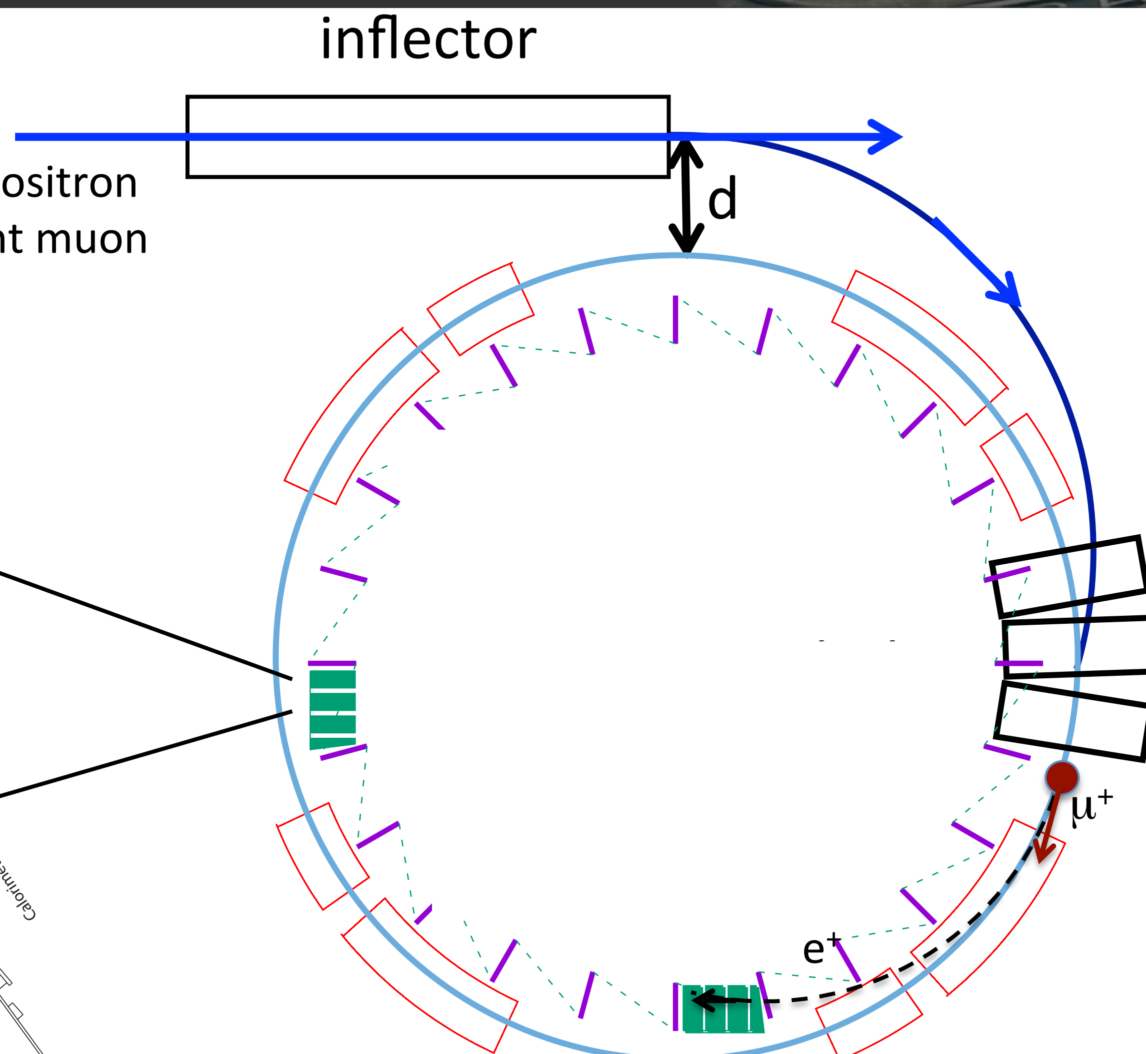


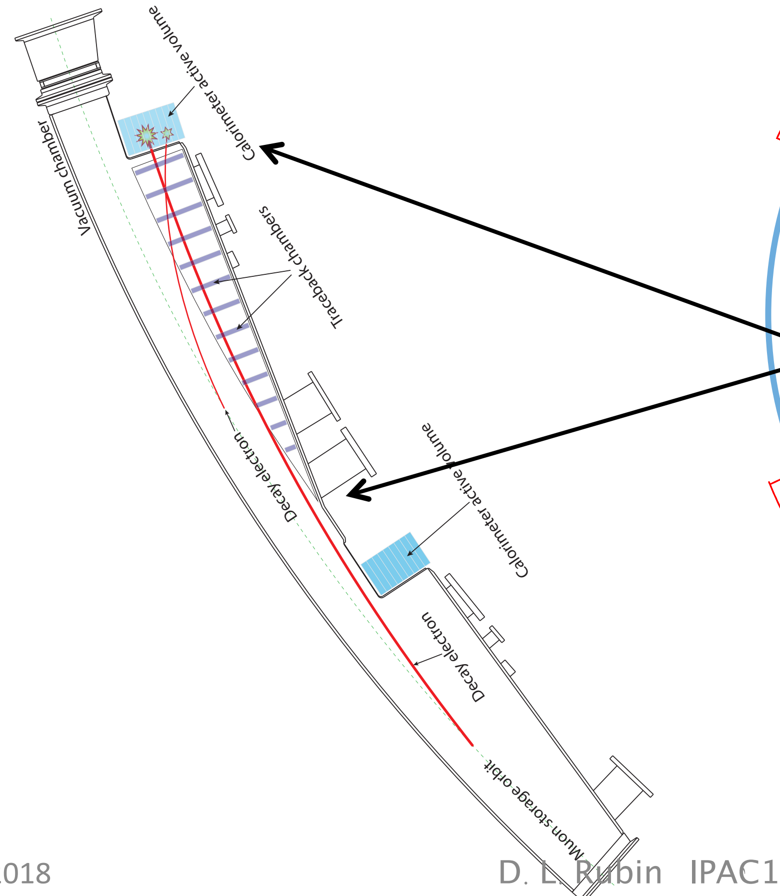




Traceback trackers

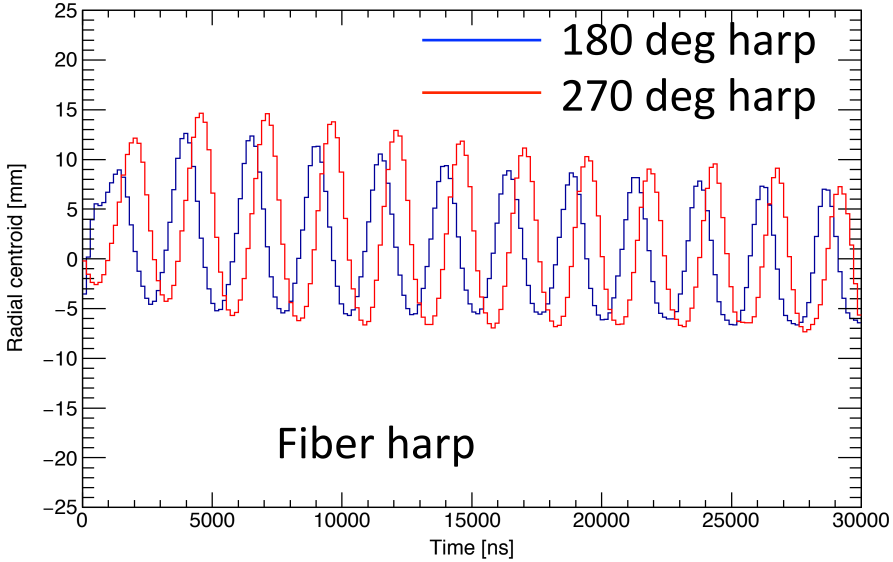
- Measure trajectory of decay positron
- Traceback to position of parent muon

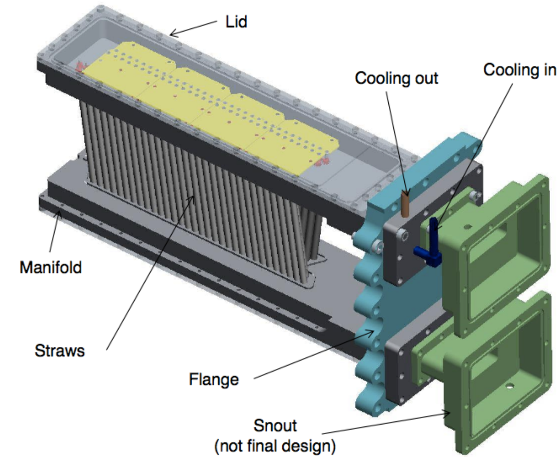


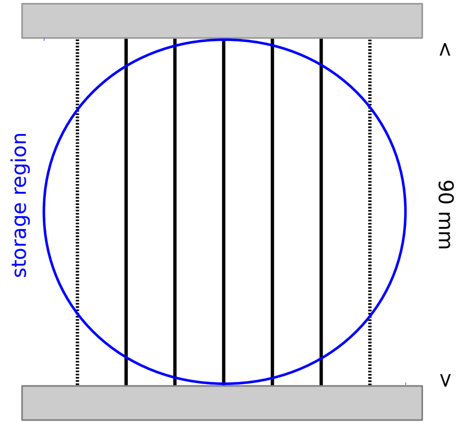


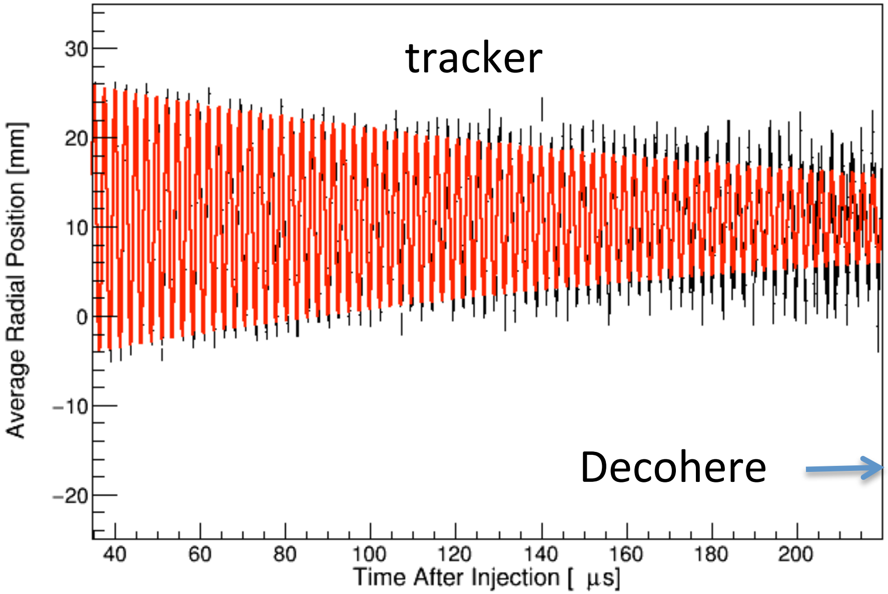
Beam centroid

Measurement of evolution of centroid



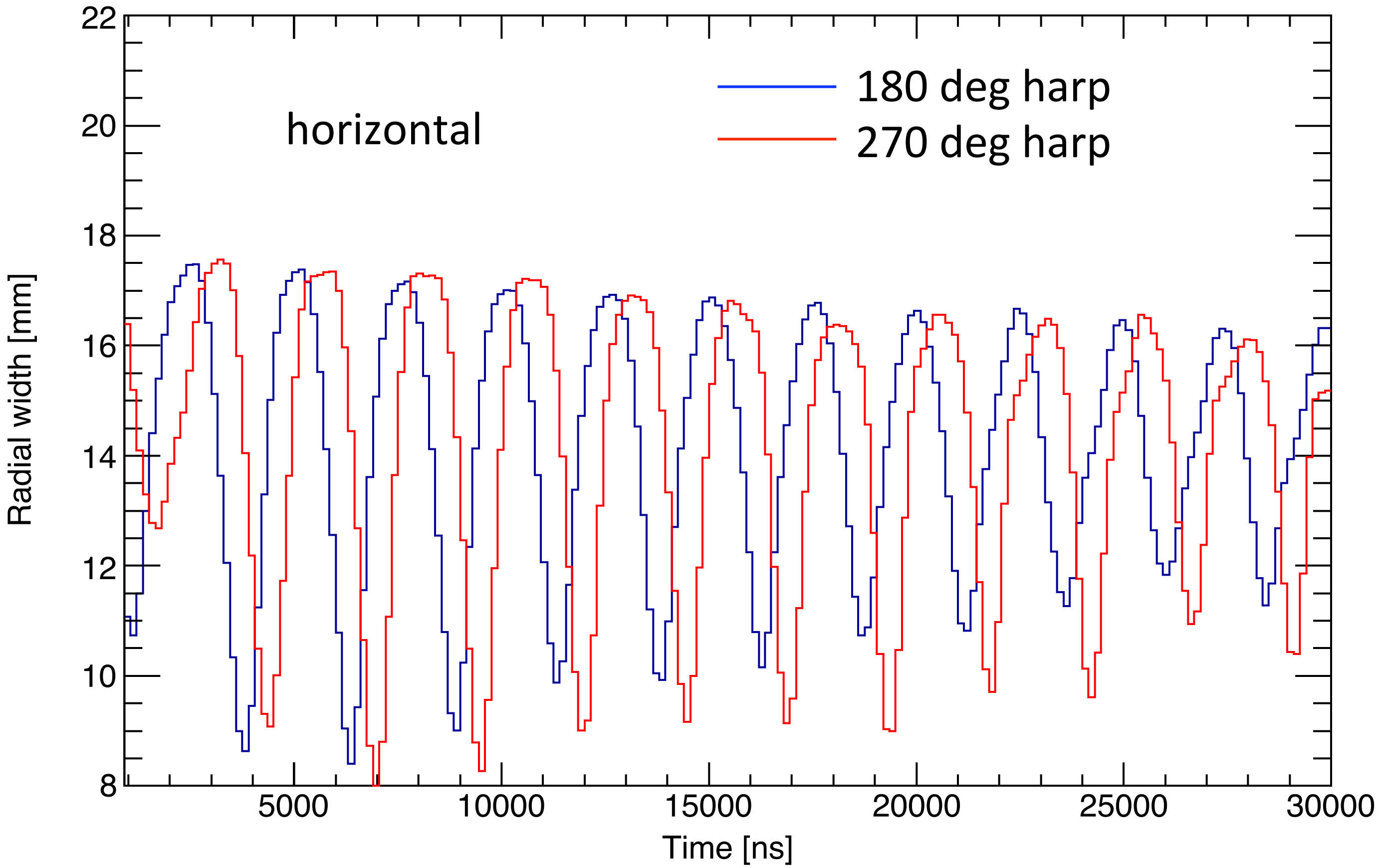






Fiber harp

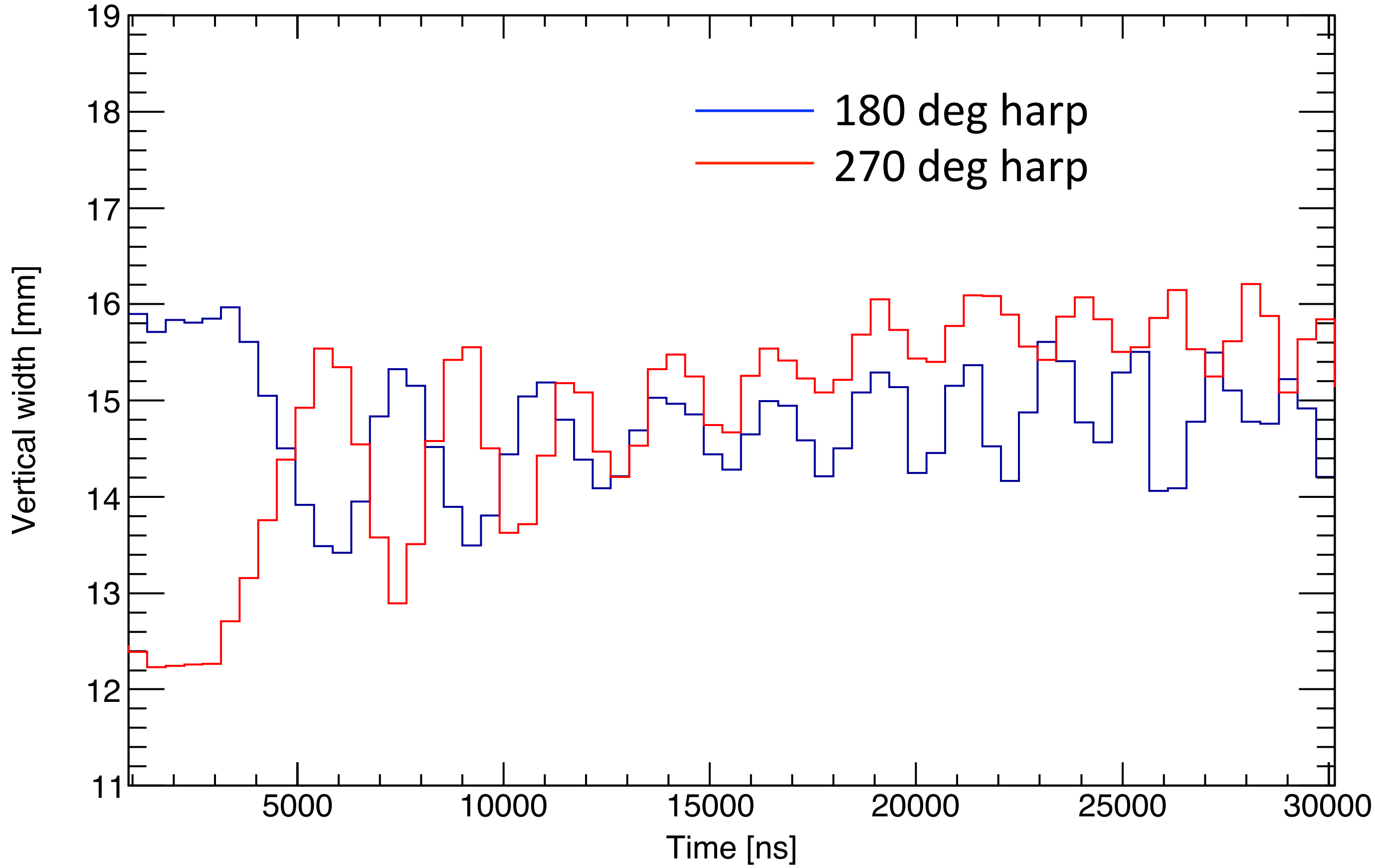
Radial width / 150 ns
- Phase space and dispersion mismatch at injection
- Width modulated at Q_x and $2Q_x$ => dispersive and betatron components
-



Fiber Harp

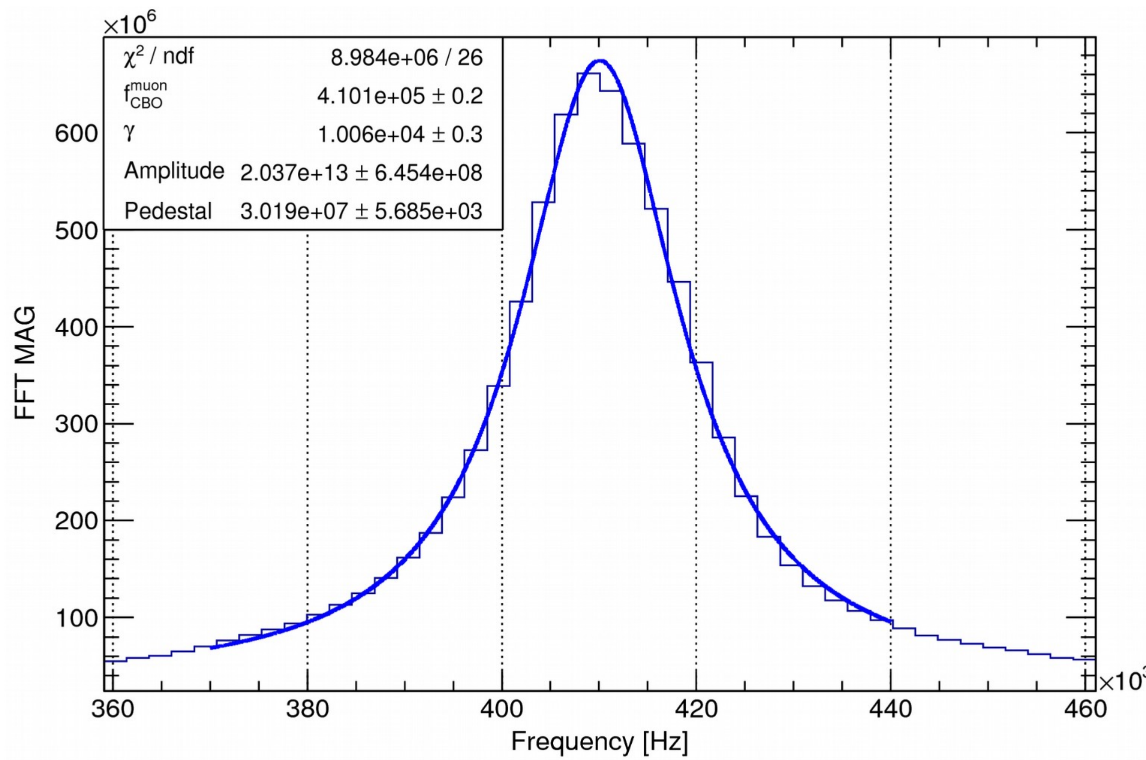
Vertical height / 150 ns

- Modulation at $2Q_y$ => phase space mismatch at injection
- Informs pitch correction



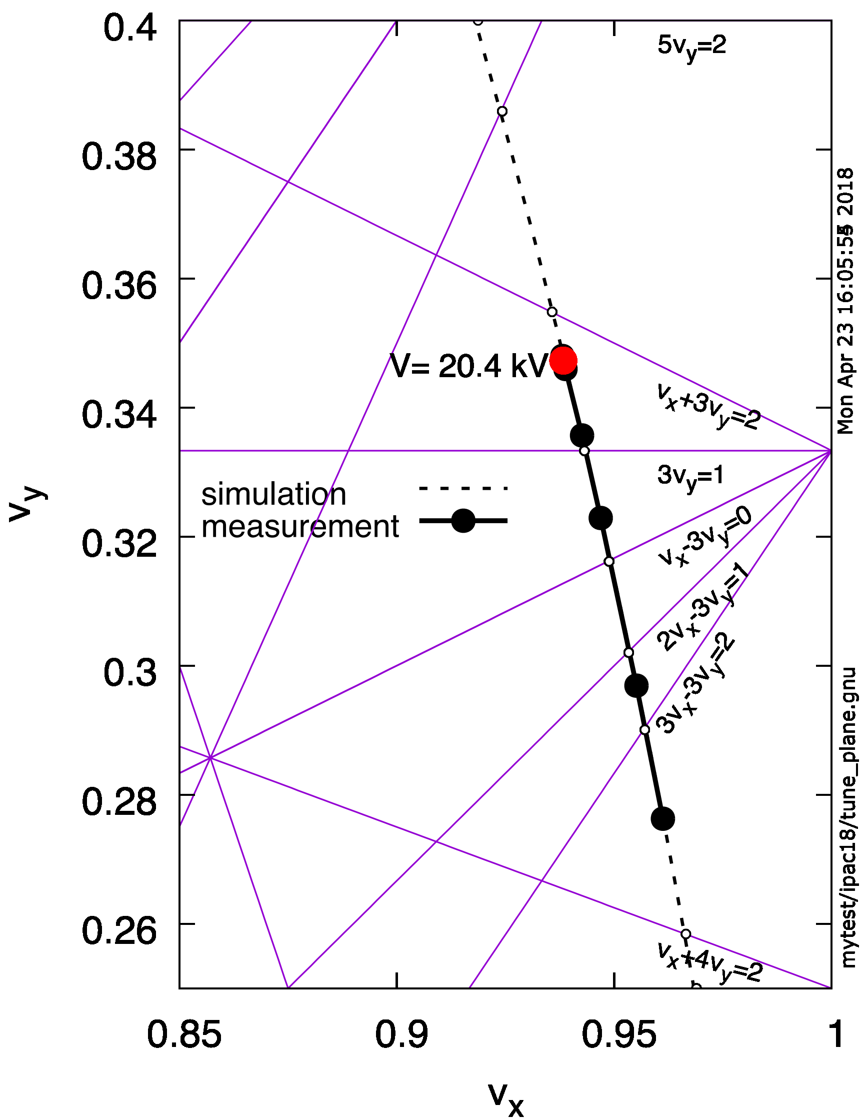
Tune

FFT of fiber harp centroid

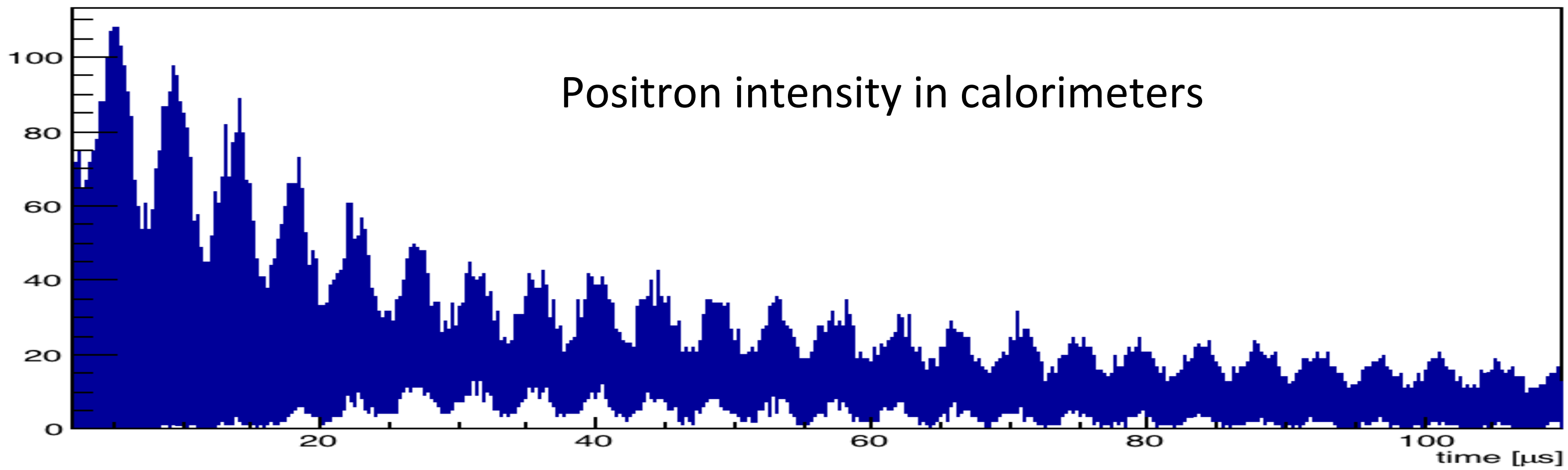


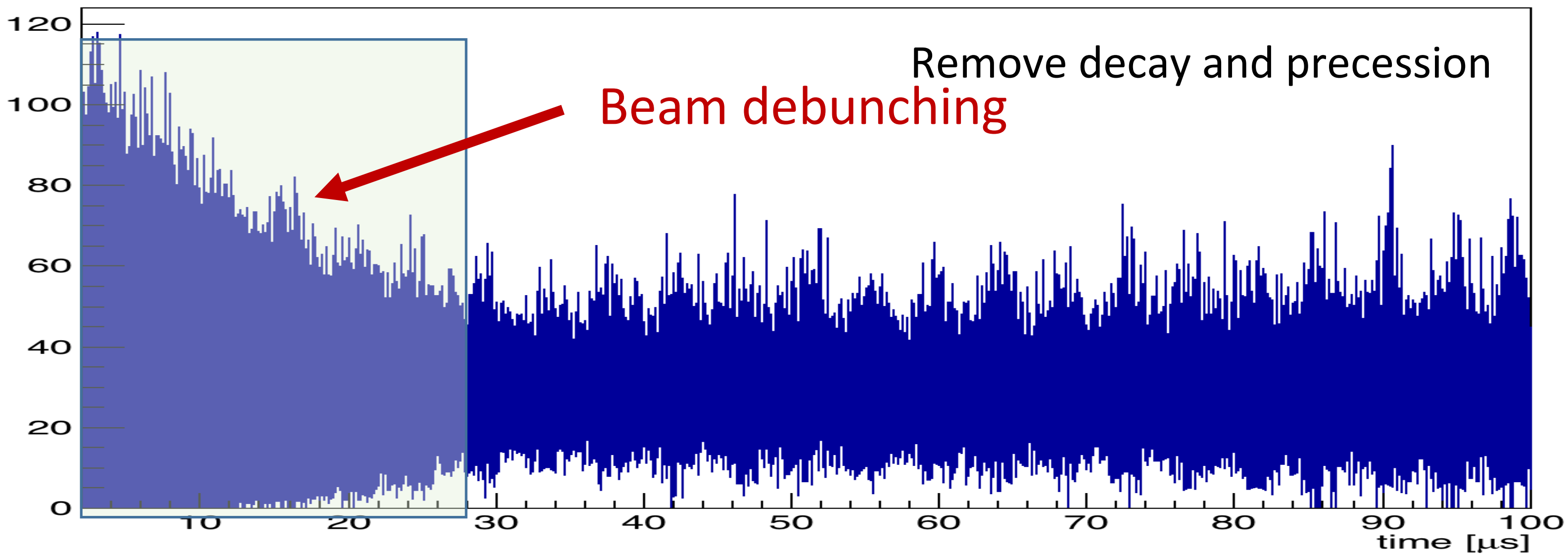
Tune scan

Tune along quad voltage contour



Momentum distribution





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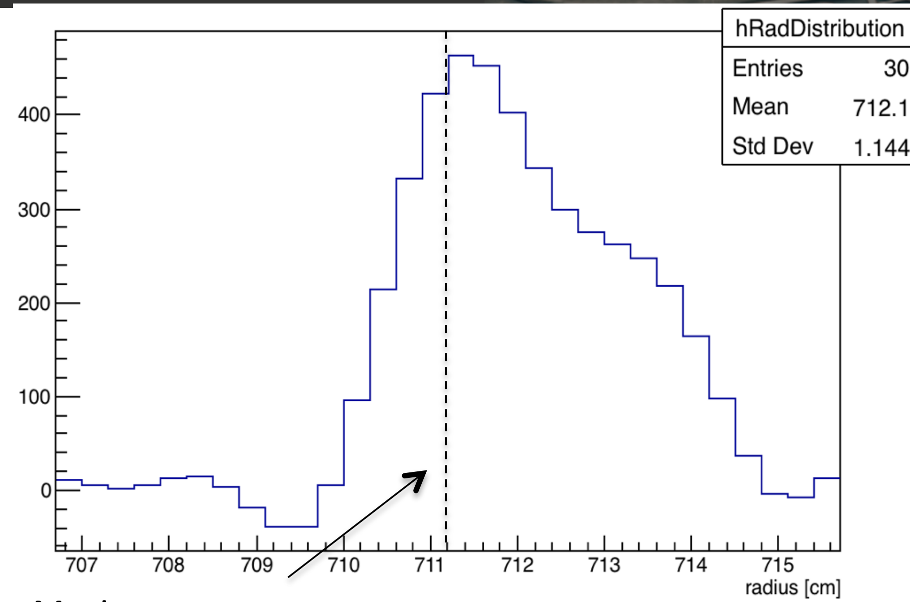
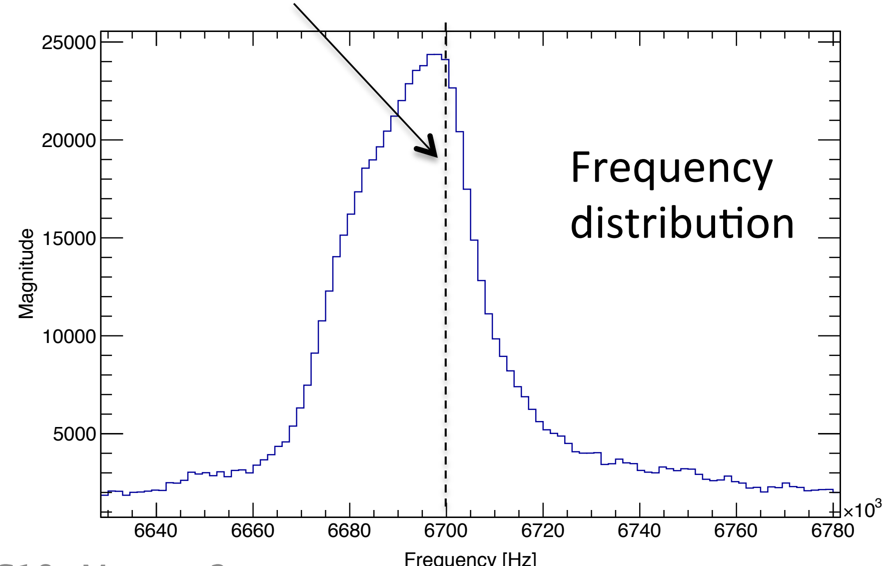
Momentum Distribution

Fit momentum distribution to debunching signal



Magic momentum

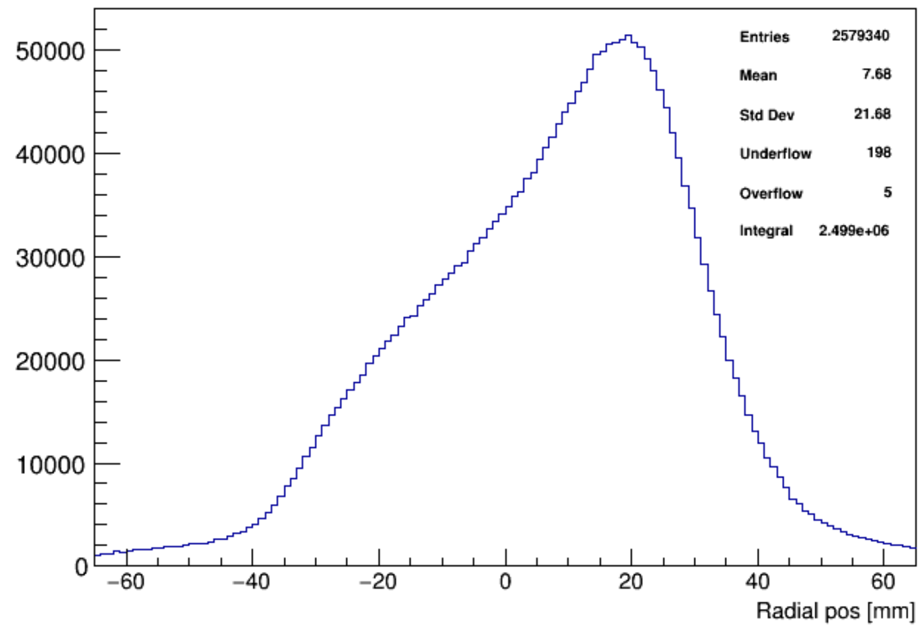


Fourier analysis of fast rotation signal

Distribution of revolution frequencies **is** the distribution of momenta. Fourier transform of the fast rotation signal => frequency distribution

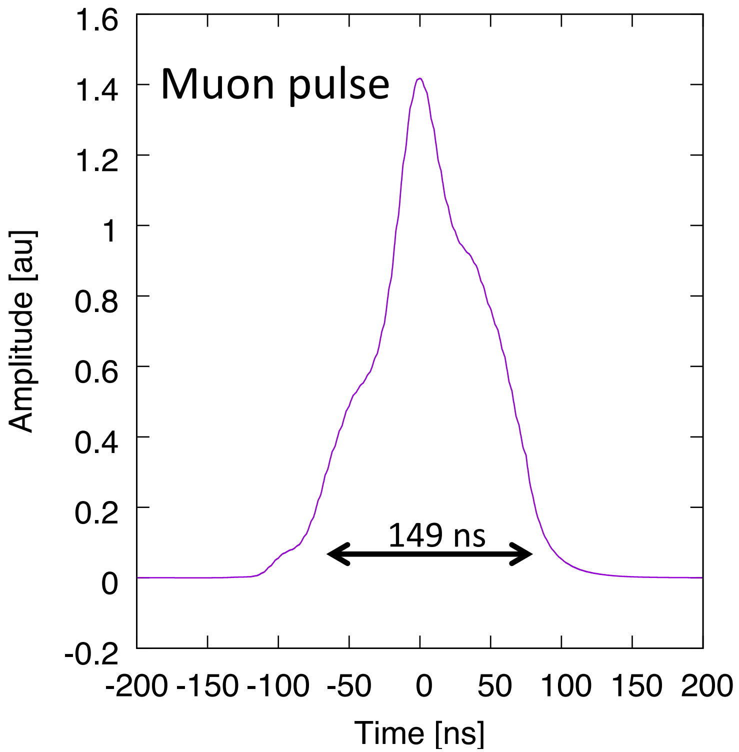
Radial distribution

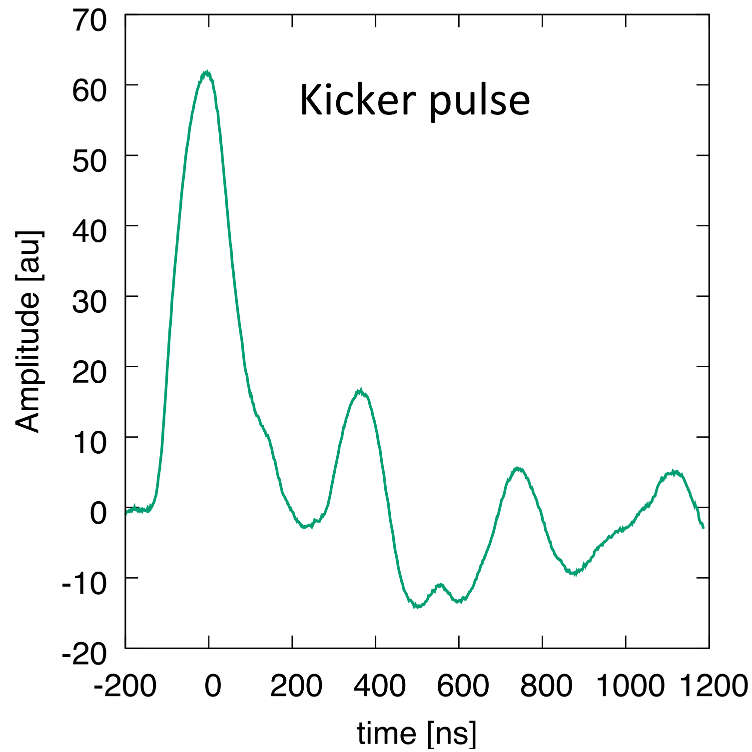
Average radial distribution measured with traceback tracker
(Radial distribution convoluted with detector acceptance)



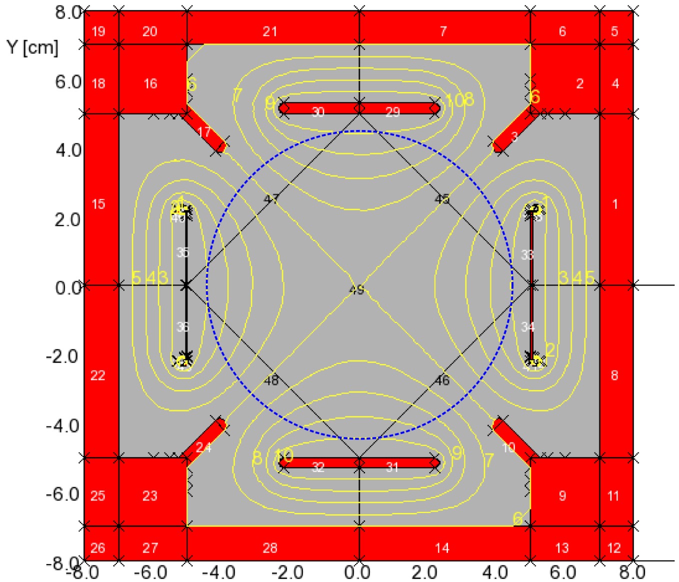
Momentum distribution

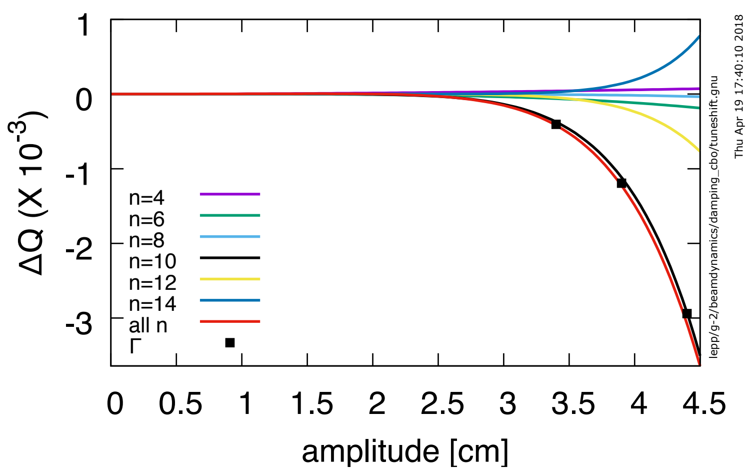
Momentum distribution is ‘high’
⇒ Injection kicker is under-kicking and/or tails of the muon pulse are in the rising and falling tails of the kick



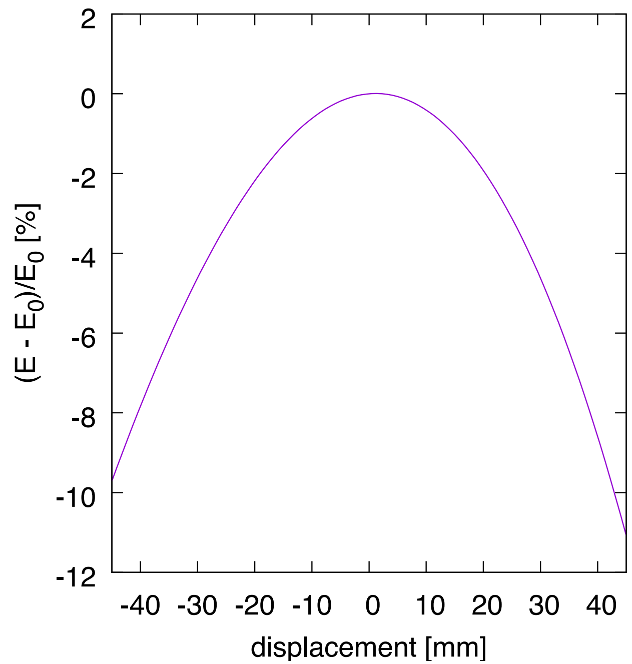


Nonlinearity





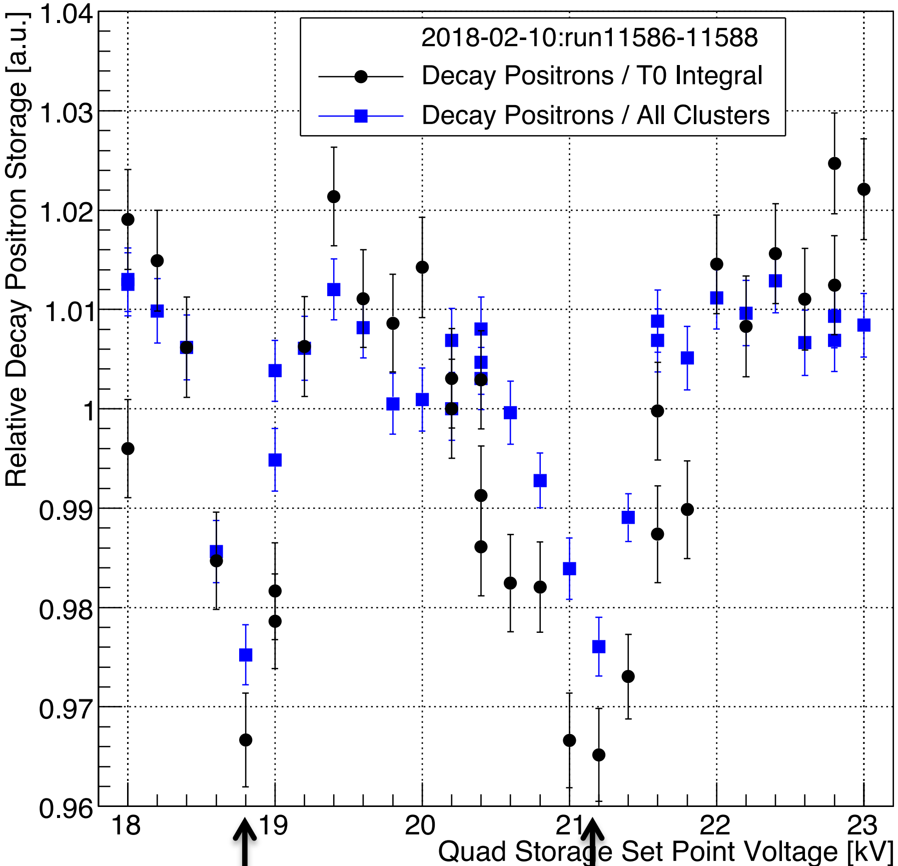
Quad field error



Resonances

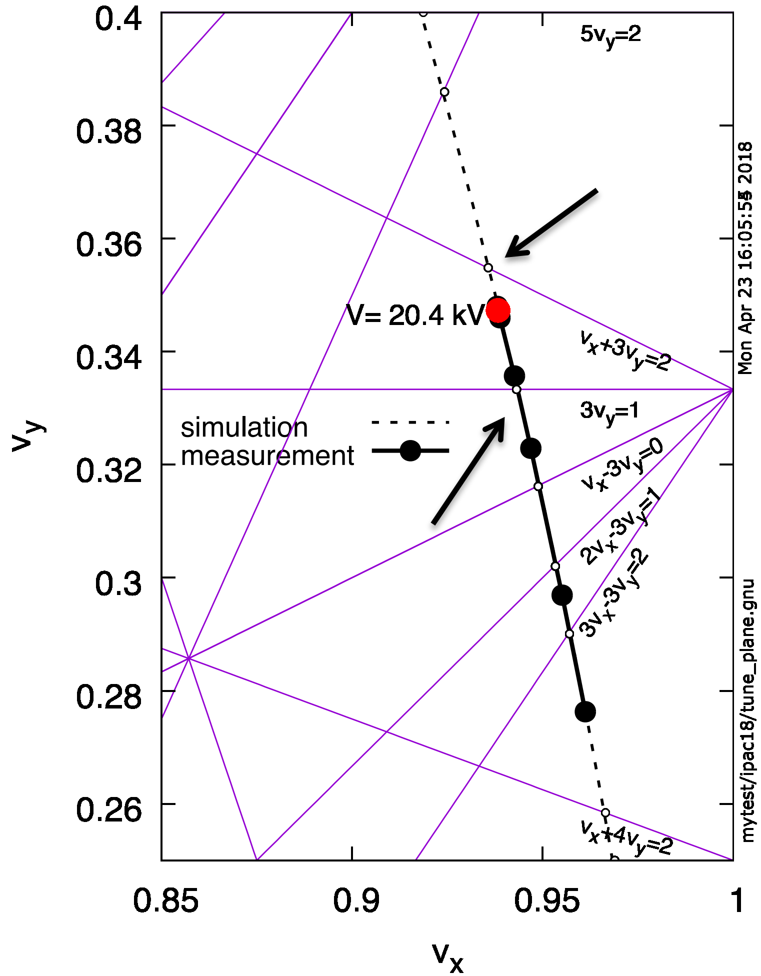
Tune scan

Stored muons



$3\nu_y = 1$

$\nu_x + 3\nu_y = 2$

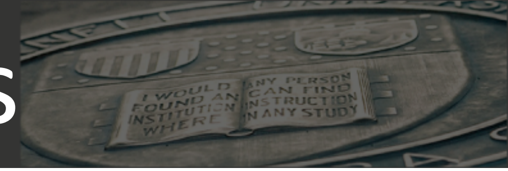


Quad voltage

Summary

- A quantitative understanding of the evolution of the muon distribution over the course of the fill is essential to limiting the systematic uncertainty in the measurement of the anomalous magnetic moment to the 70 ppb (systematic) target.
- The Muon g-2 experiment is equipped with detectors that can measure phase space and momentum distribution in some detail.
- Beam dynamics simulations informed by the measurements complete the description.
- The preliminary data presented was collected during the commissioning phase of the experiment.
- Based on that data it is clear that the fiber harp and tracker systems are an effective window on the behavior of the circulating distribution.

Thanks



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J.D.Crnkovic, W. Morse, V. Tishchenko, Brookhaven National Laboratory
W. Wu, University of Mississippi
K. S. Khaw, A.T. Feinberg, University of Washington

All of the Muon g-2 Collaboration (E989)

- Commissioning the Superconducting Magnetic Inflector System for the Muon g-2 Experiment − N.S.Froemming *et al*. WEPAF014
- Commissioning the Electrostatic Quadrupole System for Muon g-2 - J.D. Crnkovic *et al*. WEPAF015
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