# DETERMINATION OF RF RESONATOR AXIS INCLINATION TO BEAM AXIS IN ELECTRON-POSITRON STORAGE RING 

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## Abstract

We proposed and tested a method of experimental determination of the upper limit of RF resonator axis inclination to the beam axis. The electric field transverse component by such a perturbation deflects the particle trajectory. In the horizontal plane, a distortion of the closed orbit leads to a difference in the energies and spin precession frequencies of electrons and positrons in a storage collider with a single ring.

## EFFECT OF PERTURBATION OF RF RESONATOR AXIS INCLINATION

Since the deflection is caused by the transverse electric field on the beam axis, the closed orbits of electrons and positrons differ from each other. Therefore, in the case of radial deflection angle in the RF resonator, there are differences in the energies (and the spin frequencies) of electrons and positrons. These differences are defined as [1]:

$$
\begin{equation*}
\frac{\delta E}{E}=-2 \alpha_{z} \frac{U_{0}}{E} \frac{\eta_{X}}{\alpha \Pi} . \tag{1}
\end{equation*}
$$

Here $\eta_{x}$ is the dispersion function at the resonator location, $\alpha$ is the momentum compaction factor, and $\Pi$ is the machine circumference. With a $10^{-3} \mathrm{rad}$ inclination angle in the equivalent resonator, which is in the center of the VEPP-4M technical section, the systematic error in the CPT invariance test [2] based on comparison of the spin frequencies of electrons and positrons [1] will be $7.7 \cdot 10^{-9}$ against the planned $5 \cdot 10^{-9}$ accuracy. With the same angular perturbation for each of the five resonators in the technical section [3], the contribution to the energy shift does not exceed $2.5 \cdot 10^{-9}$. Due to the random distribution of the perturbations, the total contribution can be even smaller with the given spread of the angles. The energy recovery of an equivalent particle in each resonator is described by the following equation:

$$
\begin{equation*}
\Delta U_{i}=\sin \varphi_{0} U_{R F, i}, \tag{2}
\end{equation*}
$$

where $\mathrm{U}_{\mathrm{RF} ;}$ is the voltage amplitude in the $i$ th RF resonator, and $\varphi_{0}$ is the equilibrium phase. The total energy losses per revolution are $\sum \Delta U_{i}=U_{0}$. Then the angle of deflection equals to

$$
\begin{equation*}
\chi_{i}=\frac{\alpha_{R F}^{i} \Delta U_{i}}{E}, \tag{3}
\end{equation*}
$$

where $\alpha_{R F}^{i}$ is the corresponding angle of inclination.

## DESCRIPTION OF METHOD

The proposed method consists in resonant excitation of betatron oscillations by modulating the frequency of the RF system master oscillator (see Fig. 1). There are five RF resonators (R2, R3, R4, R5, and R6) operating in the main
mode $\mathrm{E}_{010}$ in the technical section of VEPP-4M. Parameters of the resonators are given in Table 1.

Table 1: Parameters of Resonators

| Resonator | $\begin{aligned} & \boldsymbol{\eta}_{\mathrm{x}} \\ & \text { (m) } \end{aligned}$ | Q | $\begin{gathered} \mathrm{U} \\ (\mathrm{kV}) \end{gathered}$ | $\begin{gathered} \mathbf{f}_{\mathrm{RF}} \\ (\mathrm{MHz}) \end{gathered}$ | Км |
| :---: | :---: | :---: | :---: | :---: | :---: |
| R2 | $0.28$ | 48500 | 11 | 181.83 | 0.0052 |
| R3 | $0.49$ | 46100 | 246 | 181.76 | 0.0055 |
| R4 | $0.35$ | 47600 | 266 | 181.73 | 0.0053 |
| R5 | $0.17$ | 48500 | 7 | 181.76 | 0.0052 |
| R6 | 0.01 | 43500 | 8 | 181.76 | 0.0058 |

So, the total loss per revolution is

$$
\begin{equation*}
\sin \varphi_{0} \cdot \sum_{i=1}^{5} U_{R F, i}=U_{0} \tag{4}
\end{equation*}
$$

where $U_{R F, i}$ is the voltage amplitude on the ith resonator. The RF frequency is modulated as follows: $\omega=$ $\omega_{R F}+\Delta \omega_{\mathrm{M}} \cdot \cos \left(\omega_{\mathrm{M}} t\right)$.


Figure 1: Scheme of the experiment.
The resonator oscillation spectrum has three main frequencies: $\omega_{R F}$ and $\omega_{R F} \pm \omega_{\mathrm{M}}$. A combination of two side band frequencies produces oscillations of the form $\sin \left(\omega_{R F} t\right) \cdot \sin \left(\omega_{\mathrm{m}} t\right)$. All the resonators act in-phase on an equilibrium particle, and therefore $\sin \left(\omega_{R F} t\right)$ can be considered as a constant. Thus, it is possible to use phase modulation (PM) to build up transverse oscillations at resonant frequencies $\omega_{\mathrm{m}}=f_{0}\left(1-\left\{\nu_{x, y}\right\}\right)$, where $\}$ is the noninteger part of the betatron frequency in the X and Z directions, and $f_{0}$ is the revolution frequency.

Monitoring of the betatron resonance was carried out with the help of two scintillation counters of the Touschek polarimeter of the VEPP-4M collider [3]. The intensity of scattering of Touschek particles is inversely proportional to the beam volume. Therefore, at resonance the load of the counters falls down due to increase respective in the transverse size of the beam.

## Excitation of Betatron Oscillations by Phase Modulation of RF System

Three series of experiments were carried out, the first two involving the RF resonators. The frequency of the external generator, that sets the phase modulation, was scanned in the range of $372.5-379.5 \mathrm{kHz}$ for the X resonance (radial oscillations) and of $350.5-357.5 \mathrm{kHz}$ for the Z resonance (vertical oscillations). Most of the experiments were conducted with resonators R3 and R4 tuned and the rest resonators untuned. It is obvious that the main perturbation is produced by the tuned resonators with a large field amplitude (4).


Figure 2: Typical behavior of the Touschek counters over time during scanning the PM frequency near the vertical betatron resonance.

Figure 2 shows the typical behavior of the counters load near the Z resonance in scanning the RF generator modulation frequency. The resonance FWHM $\Delta v_{x, z} / v_{x, z}$ is about $10^{-3}$ in relative units.

In the second series of the experiments, the Z oscillations were excited at a frequency of 342 kHz . We studied the influence of the driving transverse force (the magnetic component in the $\mathrm{E}_{010}$ mode) on the beam varying the vertical displacement of the closed orbit in the resonators.

The first three experiments were performed with the electrostatic separation plates turned on. In this case, in addition to the effect of the resonator axis inclination, the particles are under the action of a magnetic force, which is proportional to the orbit displacement. Experiments with the plates turned off showed the resonance peak to decrease more than two-fold (Figs. 3 and 4).
In addition, the influence of the vertical TZ "bump" [3] of the closed orbit in the technical section was investigated with the electrostatic separation plates turned off. This bump is to compensate for the effect of the electrostatic separation plates in beam injection. It was found that variation of the TZ correction within $\pm 1 \mathrm{~mm}$ did not lead
to a significant change in the resonance particle scattering rate, which was approximately $5 \%$. This observation may indicate presence of a stronger factor, which we associate with the resonator axis inclination to the beam axis in the vertical plane.


Figure 3: Scan with the electrostatic separation plates turned on.


Figure 4: Scan of the PM frequency with the electrostatic separating plates turned off and a vertical displacement of the closed orbit down of 0.82 mm due to the TZ bump.

All the experiments were carried out on the same PM control signal span. So, we can assume that the value of the effect under study is within $5 \%$.

## Calibration Experiments with Kicker

The third series of experiments is related to calibration. The external generator was connected to the depolarizing kicker vertical plates. It was necessary to determine the threshold for the transverse electric force in the resonators. This threshold was found from the agreement of the resonance effect in the calibration series with the data of the PM experiments with minimal influence of the orbit transverse displacement. To do this, the vertical electrostatic separation of orbit in the technical section was turned off. The kicker excited the Z resonance at a frequency of 342 kHz . Thus, the calibration is related to evaluation of the vertical angle of the resonator axis inclination. Figures 5 and 6 show typical dependences of the behavior of the load of the counters for large and small voltages applied to the kicker.


Figure 5: Scan of the PM frequency at a doubled signal amplitude on the depolarizer plates of 3.1 V .


Figure 6: Scan of the PM frequency at a doubled signal amplitude on the depolarizer plates of 0.52 V .

Figure 7 shows the resonance effect vs. the voltage across the kicker plates. At low voltages, the effect is linear, whereas at higher voltages there appears saturation.


Figure 7: Resonance effect vs. voltage across the kicker plates.

A $5 \%$ effect is achieved with a voltage $2 \mathrm{U}_{\mathrm{K}}=0.4 \mathrm{~V}$ at the kicker plates. The upper limit of the RF resonator axis inclination influence in the vertical plane is determined by a value of $5 \%$ too.

## Determination of Vertical Axis Perturbation of Resonators

The deflection angle acquired by a particle in the kicker with a counter TEM wave is equal to

$$
\begin{equation*}
\chi_{K}=2 \frac{H_{e f f} l}{H R} \tag{5}
\end{equation*}
$$

where $H_{e f f}$ is defined through the amplitude of the voltage across the plates, and $H R$ is the magnetic rigidity at the energy of the experiments of 1.85 GeV . From the calculations we have $\chi_{K}=3.2 \cdot 10^{-9} \mathrm{rad}$.

The voltage across the resonator for an equilibrium particle with a phase $\varphi_{0}$ is written as

$$
\begin{equation*}
U_{R F} \sin \varphi_{0}+2 U_{R F} \cos \varphi_{0} \cdot J_{1}\left(\Delta \varphi_{\mathrm{M}}\right) \cos \omega_{\mathrm{m}} t \tag{6}
\end{equation*}
$$

$J_{l}\left(\Delta \varphi_{m}\right)$ is the first-order Bessel function. The second term of this sum corresponds to the driving force. Then the angle of deflection in the resonator is determined from the following expression:

$$
\begin{equation*}
\chi_{R F}=2 \alpha_{Z} \frac{e U_{R F} \cos \varphi_{0} \cdot j_{1}\left(\Delta \varphi_{\mathrm{M}}\right) K_{\mathrm{M}}}{E} \tag{7}
\end{equation*}
$$

where $\alpha_{z}$ is the angle of the resonator axis inclination, $K_{\mathrm{M}}=$ $1 / \sqrt{1+(2 Q \Delta f / f)^{2}}$ is the modulation factor, with $Q$-factor, and $\Delta f=\omega_{\mathrm{M}}$, and $\Delta \varphi_{M}$ is the phase modulation amplitude. This amplitude is found from the following relationship: a PM circuit voltage amplitude of 1 V corresponds to $\Delta \varphi_{\mu}=$ $10^{\circ}$ [4]. The experiments were carried out at modulation amplitude of 1.5 V . Approximately,

$$
\begin{equation*}
\chi_{R F} \approx \alpha_{Z} \frac{e U_{R F} \cos \varphi_{0} \cdot \Delta \varphi_{\mathrm{m}} K_{\mathrm{M}}}{E} \tag{8}
\end{equation*}
$$

We find the average vertical angle of axis inclination for the effective RF resonator ( $\mathrm{R} 3+\mathrm{R} 4$ ) with $\mathrm{U}_{\mathrm{RF}}=440 \mathrm{kV}$ and $K_{M}=0.0054$. The relationship between the deflection angles in the kicker $\chi_{\kappa}$ and in the resonator $\chi_{\text {RF }}$ is given by the following expression:

$$
\begin{equation*}
\chi_{K}=\chi_{R F} \sqrt{\frac{\overline{\beta_{Z(R F)}}}{\beta_{Z(K)}}} \tag{9}
\end{equation*}
$$

where $\bar{\beta}_{Z(R F)}=\sqrt{\beta_{R 3}^{2}+\beta_{R 4}^{2}+2 \beta_{R 3} \beta_{R 4} \cos \Delta \Psi_{Z}}$ and
$\Delta \Psi_{\mathrm{Z}}=2.348 \mathrm{rad}$ is the betatron phase incursion between resonators R3 and R4. From this equality we obtain that $\chi_{\mathrm{RF}}=3.7 \cdot 10^{-9}$. Then the vertical angle of inclination of the resonator axis found from formula (7) is $\alpha_{\mathrm{z}} \leq 0.01 \mathrm{rad}$.

## CONCLUSION

One can suppose that similar angles of inclination may occur in the median plane. Hence, the estimated contribution of the RF resonator axis angle perturbation to the error of the CPT test experiment will not exceed $10^{-8}$ for resonator R5. At the same time, this value is $<10^{-9}$ for resonator R6, due to the small dispersion function value at its location. Thus, it is desirable to perform the CPT test using resonator R6.
More precise determination of the contribution of the resonators to the CPT test experiment error requires conduction of a calibration experiment on radial oscillations using a kicker with the radial plates. Such variant, in principle, is possible but needs special technical preparation.

## REFERENCES

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