# PERFORMANCE COMPARISON OF DIFFERENT ULTRALOW **EMITTANCE UNIT CELLS\***

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# Abstract

The available minimum emittance of a storage ring and the ring performance is closely related to the unit cell of the lattice. Up to now, several ultralow-emittance unit cells have been proposed and applied in the lattice design of the diffraction-limited storage ring light sources. In this study we quantitatively compared the performance of three typical unit cells, based on mainly the parameters of the High Energy Photon Source. The results indicate that the modified-TME unit cell with antibend and longitudinal gradient dipole allows the lowest possible emittance, given a long enough cell length.

### **INTRODUCTION**

Emittance is one of the most important parameters of an electron storage ring. Extremely low cultures in ring-based in colliders or high brightness in ring-based  $\frac{1}{2}$  ference is usually pre-determined in the very early stage of 5 the design and construction of the ring. In the past few decades, many efforts have been made to explore unit cells that are efficient in minimizing emittance and dealing with the challenge of the related physics issues in an ultralow-emit-🛱 tance storage ring.

Before the discussion on unit cell, we first introduce the so-called theoretical minimum emittance (TME) [1], which is usually used to measure the degree of emittance minimization of a unit cell or a practical lattice design, by using the ratio of the natural emittance to the TME,  $R_{\varepsilon} = \varepsilon_0 / \varepsilon_{\text{TME}}$ .

In a storage ring with dipoles having uniform bending field, the TME is of the form

> $\varepsilon_{TME} = \frac{C_q \gamma_L^2 \theta^3}{12\sqrt{15}J_x},$ (1)

with the horizontal optical functions at the dipole centre (with subscript 0) being

$$\beta_0 = \frac{L_B}{2\sqrt{15}}, \quad \alpha_0 = 0, \quad D_0 = \frac{L_B^2}{24\rho}, \quad D_0' = 0,$$
 (2)

 $\stackrel{\text{\tiny D}}{=}$  where  $C_q=3.83*10^{-13}$  m;  $\gamma_L$  is the Lorentz factor;  $J_x$  is the uniform-field dipole;  $\rho = L_B/\theta$  is the bending radius of dipole, with  $L_B$  the dipole length or  $J_{ab}(\theta)$  $\widehat{\mathbf{g}}$  horizontal damping partition number,  $J_x \sim 1$  for the case with

from this In this paper, a TME (TME-like) unit cell refers to a standard cell consisting of one uniform-field dipole and two families of separated-function quadrupoles, the horizontally focusing and defocusing quadrupoles (QFs and QDs), where QFs are located closer to the dipole to provide strong enough focusing to exactly (or approximately) reach the TME conditions in Eq. (2).

This type of unit cell is, however, barely used in practical lattice designs, due to mainly the following reasons. To reach a  $R_{\varepsilon}$  of close to or equal to 1, except the strong focusing, a long cell length is required (saying larger than 5 m). Furthermore, very strong sextupoles will be needed to compensate for the large natural chromaticities associated with the large phase advance (284.5 degrees per cell for  $R_{\varepsilon}=1$ ), leading to extremely large nonlinearities and difficulty in nonlinear optimization.

To help in dealing with the challenge to the nonlinear performance, a variation of the TME unit cell, called modified TME (M-TME) unit cell, was proposed and thoroughly studied [2]. The main feature of the M-TME unit cell is to put the QDs rather than QFs closer to the dipole. Although such a layout allows a  $R_{\varepsilon}$  of only about 3, it promises a more compact layout, a lower phase advance per cell (below 180 degrees), smaller natural chromaticities and more relaxed optical functions than in a TME or TME-like unit cell. As a variant of the M-TME (VM-TME) unit cell, by combining the defocusing gradients in the dipole, the VM-TME unit cell can have an even smaller cell length and a smaller  $R_{\varepsilon}$  (with  $J_{x} > 1$ ).

Actually, the VM-TME unit cell has been used in the standard multi-bend achromat (MBA) of many ultralowemittance designs. For instance, the MAX-IV lattice [3] used five VM-TME unit cells in each 7BA to achieve a natural emittance of 326 pm at 3 GeV ( $R_{\epsilon} \sim 8$ ) with a circumference of 528 m. For the High Energy Photon Source (HEPS), a standard 7BA lattice with VM-TME unit cells [4] was designed to reach a natural emittance of 75 pm at 5 GeV ( $R_{\varepsilon}$ ~2.7) and a circumference of about 1.3 km.

Nevertheless, it was noticed [5] that when continuously pushing the emittance of a standard MBA lattice down to even lower values, sextupoles with even larger lengths than quadrupoles (if using conventional magnet technology) will be needed to compensate for the increasing natural chromaticities coupled with the decreasing dispersion function, which imposes a limitation in reducing the cell length and affects the emittance reduction efficiency.

This difficulty can be overcome with the hybrid-MBA lattice that was first proposed for the ESRF-EBS project [6]. For example, a hybrid 7BA consists of two DBA-like cells with longitudinal gradient dipoles (LGBs) and three VM-TME unit cells in the middle. In the two DBA-like cells dispersion bumps are created with chromatic sextupole located therein, such that the sextupole strengths can

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The hybrid MBA lattice does help in minimizing the natural emittance, especially in a high energy DLSR design. Using 32 hybrid 7BAs, the ESRF-EBS design [6] achieved a natural emittance of 150 pm at 6 GeV ( $R_{\varepsilon}$ ~3.8) with a circumference of 844 m. And in APS-U design, a natural emittance of 67 pm at 6 GeV ( $R_{\varepsilon}$ ~3.3) was reached by using 40 hybrid 7BAs with a circumference of 1104 m [7]. For the HEPS design with 48 hybrid 7BAs, the natural emittance can be pushed down to ~45 pm at 6 GeV ( $R_{\varepsilon}$ ~3.8) [8].

Afterwards, a novel unit cell with LGB and antibend was proposed for the SLS-2 project [9]. The layout is basically similar to the M-TME cell, but with the uniform-field dipole replaced by a LGB and the QF replaced by an antibend (with a small shift relative to the magnetic field centre). The antibend allows independent control of the beta and dispersion functions, which makes it feasible to approach the exact TME conditions with weaker focusing than in a TME unit cell; and the LGB (see, e.g., [10]), in principle, can promise an emittance lower than the TME. By using very aggressive antibends ( $\sim 30\%$  of nominal bending angle) and LGBs (with highest peak filed up to 5 T), a natural emittance of 137 pm at 2.4 GeV ( $R_{\epsilon}$ ~1.1) was reached for the SLS-2 with a circumference of 288 m [9]. Note that this design has a negative momentum compaction, which is different from most of the ultralow- emittance designs and may lead to bunch shortening in the presence of impedance.



Figure 1: Layout of type A, B, and C unit cells (from up to low). A yellow box represents a LGB or a dipole combined with defocusing gradient, a blue box represents a QD, and a red box represents a QF or an antibend (a QF with reverse bending angles).

It was then proposed to combine the antibend into the hybrid 7BA lattice in the APS-U lattice design [11]. By adding reverse bending angles to three families of QFs in each 7BA (especially with large reverse bending angles in the QFs of the middle unit cell), the emittance was reduced to 42 pm ( $R_{e}$ ~2.1). The momentum compaction decreases because of antibends, but remains positive.

Based on the above, it would be interesting to investigate which type of the unit cell promises the 'best' ring performance. To this end, we compared the performance of three types of unit cell, such as the VM-TME cell, the VM-TME cell with antibend, and the M-TME cell with LGB and antibend, as shown in Fig. 1. For simplicity, in the following they are referred as type A, B and C unit cells, respectively.

## UNIT CELL COMPARISON

To do a fair comparison among these unit cells, the ultimate performance of a specific type of unit cell was explored with an iteration of particle swarm optimization and multi-objective genetic optimization [12], where the performance was described with two objectives, i.e.,  $R_{\varepsilon}$  and the normalized natural chromaticity,  $\xi_{norm} = \xi_x \xi_y / v_x v_y$ . In addition, based on mainly the HEPS hybrid 7BA design, as many practical considerations as possible were taken into account.

- It was assumed that the unit cells contain only dipoles and quadrupoles, and it is not necessary to reserve space for sextupoles (as in the middle unit cell of a hybrid 7BA).
- The total bending angle was set to the same value, 1 degree (similar to the HEPS case where 336 dipoles are used). And the nominal energy was set to 6 GeV.
- Two different upper limits of the magnet pole face field were considered, 1 and 1.12 T (corresponding to the maximum gradient of 80 and 90 T/m for a pure quadrupole with a pole radius of 12.5 mm).
- For each type of unit cell, five cell lengths were considered, i.e., 1.8, 2.0, 2.2, 2.6 and 3.0 m.
- The minimum distance between adjacent magnets was set to 6 cm.
- The average bending radius of the dipole (with positive bending angle) should not be less than 30 m, and the average bending radius of the antibend should not be less than 90 m, to avoid too large an energy loss per turn due to synchrotron radiation.
- For type C unit cell, the maximum peak filed (at the central slice) of the LGB should not be larger than 1.7 T [13].

In the optimization of each type of unit cell, all tuneable parameters of elements were varied. For the unit cell of type A, B and C, the variable numbers are 5, 6 and 18, respectively. In each case, 200 seeds were generated based on the parameters of the middle unit cell of the HEPS hybrid 7BA lattice [8], and used as the initial population. Due to small number of variables, the solutions can reach good convergence after evolution of several tens of generations.

The optimization results for the unit cell of type A, B, and C are shown in Figs. 2, 3 and 4. For type A unit cell, except the case with a cell length of 1.8 m and upper limit of pole face field of 1.0 T, a  $R_{\varepsilon}$  of about 2 can be reached with a moderate normalized natural chromaticity,  $\xi_{norm} = 3$ , whatever the maximum pole face field is 1.0 T or 1.12 T.

For the type B unit cell, situation slightly changes. A higher quadrupole pole field helps reach the available minimum emittance in a shorter unit cell. For example, in the

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and case with upper limit of pole face field of 1.12 T. the  $R_{\varepsilon}$  of b a 2.0-m unit cell is already very close to that available with a much larger cell length (e.g., 3 m). While in the case with gupper limit of pole face field of 1.0 T, it needs to use a long enough unit cell (saying, not less than 2.5 m) to push the work, emittance down to close to the lowest possible value at a



Figure 2: Optimization results for the type A unit cell, with the upper limit of the quadrupole pole face field of 1.0 T (left) and 1.12 T (right).



Figure 3: Optimization results for the type B unit cell, with distribution the upper limit of the quadrupole pole face field of 1.0 T (left) and 1.12 T (right).

In addition, the function of the antibend is evident, which E allows a  $R_{\varepsilon}$  of ~1 at  $\xi_{norm}$  = 3, or slightly lower with a price  $\dot{\infty}$  of larger normalized natural chromaticity.

For the type C unit cell, a long cell length is essentially 20]  $\odot$  required to reach a small  $R_{\varepsilon}$ . In Fig. 4, the solutions for the 1.8-m unit cell are out of the range of interest, i.e.,  $R_{\varepsilon}$  below 5. And it is not possible to exert the power of the LGB and antibend unless the cell length is increased to 2.6 m. If usantibend unless the cell length is increased to 2.6 m. If us ing a cell length of 3.0 m and a maximum pole face fiel of 1.12 T, a  $R_{\varepsilon}$  of close to 0.5 can be achieved at  $\xi_{norm} = 3$ . ing a cell length of 3.0 m and a maximum pole face field



Figure 4: Optimization results for the type C unit cell, with used the upper limit of the quadrupole pole face field of 1.0 T (left) and 1.12 T (right). è

Note that in the above results, it was not yet considered may to accommodate a three-pole wiggler (3PW, needs a drift space not less than 0.35 m) in type A and B unit cells (as in the middle unit cell of a hybrid 7BA in ESRF-EBS), and this one or two BPMs (needs a drift space of about 0.1 m per rom BPM) in type C unit cell.

Thus, we did the optimization again for the cases with Content cell lengths of 2.2 and 2.6 m, considering the constraints on the drift space. The results are shown in Fig. 5. It turns out that for a 2.2-m unit cell, the available  $R_{es}$  of type A, B, C unit cells are 2.52 (2.46), 1.41 (1.30) and 1.96 (1.27) respectively, with the upper limit of the pole face field of 1.0 T (1.12 T); and for a 2.6-m unit cell, the available  $R_{\epsilon}$ s of three types of unit cells are 2.26 (2.22), 1.06 (1.03) and 0.88 (0.74), respectively.



Figure 5: Optimization results for the unit cells, with practical constraints on the drift space.

It appears that compared to the VM-TME (type A) unit cell, it is feasible to reach a higher emittance reduction efficiency in both VM-TME unit cell with antibend (type B) and M-TME unit cell with antibend and LGB (type C), with  $R_{\varepsilon}$  reduced by a factor of 50% to 60%, respectively.

From the above comparison, one can know that if using 3PWs for bending magnet beamlines, it seems a good choice to replace the middle unit cell of a hybrid-7BA with a type B unit cell with a  $R_{\varepsilon}$  of close to 1.

On the other hand, this study suggests an alternative way. One can replace the middle unit cell of a hybrid-7BA with a type C unit cell, and use the central slice of the LGB (with a high peak field) instead of a 3PW as the radiator for bending magnet beamline, whose radiation performance appears good enough for users (results not shown here due to limited space). If it is possible to slightly increase the arc length either by increasing the circumference or decreasing the straight section length or both, a long enough type C unit cell with a  $R_{\varepsilon}$  of 0.5 to 0.8 can be used to improve the efficiency in emittance reduction. This is actually what was done in the latest HEPS lattice design as reported in [14, 15].

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