# ALIGNMENT OF CURRENT STRIPS AT THE CANADIAN LIGHT SOURCE 

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## Abstract

The Quantum Materials Spectroscopy Centre beamline at the Canadian Light Source will employ a 180 mm period elliptically polarizing undulator (EPU180), which will have significant impacts on beam dynamics with large tune shifts and reductions in dynamic aperture. Current strips mounted to the vacuum chamber are intended to mitigate the effects of EPU180 with each strip powered by an independent power supply. It is important to accurately model the current strips in order to calculate the required compensation. We model the current strips as straight wires, parallel to the electron beam, with small horizontal and vertical displacements from their nominal positions. As the real current strips are not completely straight, this is an effective model, but justified as we are mostly interested in the magnetic field integrated along the strips. By activating two strips and measuring the ratio of the two currents needed to minimize closed orbit distortion in the horizontal and vertical planes, we can find the effective horizontal and vertical displacements of the straight wires in the model. Our goal is to create an effective model of the strips from beam-based measurements.

## INTRODUCTION

The Quantum Materials Spectroscopy Centre (QMSC) beamline at the Canadian Light Source (CLS) uses a double elliptically polarizing undulator (EPU) as its photon source. One of the EPUs, referred to as EPU180, is a quasiperiodic device with a period of 180 mm . This EPU has a significant effect on beam dynamics [1] and we plan to mitigate these effects using current strips adhered to the vacuum chamber [1-6].

The QMSC beamline is currently commissioning using horizontal polarized light. However, the beamline ultimately wishes to operate in universal mode [1,2] where the EPU will produce light with an arbitrary polarization. This is an additional complication as instead of having two degrees of freedom, gap and phase, the EPU will operate with three degrees of freedom: gap, linear phase and elliptical phase. We have calculated tolerances for current strip alignment with the relative alignment of the strips required to be $100 \mu \mathrm{~m}$ and absolute alignment of the strips to the EPU center required to be $400 \mu \mathrm{~m}$ [1]. As a result of these alignment requirements and universal mode operation, we require a model of the current strips built from beam-based measurements.
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## THE MODEL

We begin with the Biot-Savart law $[7,8]$, which allows us to calculate the magnetic field for a line current, $I$,

$$
\begin{equation*}
\vec{B}(\vec{r})=\frac{\mu_{0} I}{4 \pi} \oint \frac{d \overrightarrow{r^{\prime}} \times \vec{R}}{R^{3}} \tag{1}
\end{equation*}
$$

where $\mu_{0}$ is the permeability of free space, $\vec{r}$ is the location where we observe the magnetic field, $\vec{r}^{\prime}$ is the location of the current carrying wire and $\vec{R}=\vec{r}-\overrightarrow{r^{\prime}}$. Ultimately, it will not be sufficient to use a infinitesimal wire, but such a simplified model can be used to give justification to the procedure. We can parameterise a wire, primarily laid in the $\hat{y}$ direction, as it snakes through 3-dimensional space as

$$
\begin{equation*}
\overrightarrow{r^{\prime}}\left(s^{\prime}\right)=x^{\prime}\left(s^{\prime}\right) \hat{x}+z^{\prime}\left(s^{\prime}\right) \hat{z}+s^{\prime} \hat{y} \tag{2}
\end{equation*}
$$

and we observe the magnetic field on the $x=z=0$ line, which gives the parameterisation $\vec{r}(s)=s \hat{y}$.

For an ideal, straight wire with no distortions, we have $x^{\prime}\left(s^{\prime}\right)=x_{0}^{\prime}$ and $z^{\prime}\left(s^{\prime}\right)=z_{0}^{\prime}$, which are constants. We can write the magnetic field as

$$
\begin{equation*}
\vec{B}=\frac{\mu_{0} I\left(x_{0}^{\prime} \hat{z}-z_{0}^{\prime} \hat{x}\right)}{2 \pi\left(x_{0}^{\prime 2}+z_{0}^{\prime 2}\right)} \tag{3}
\end{equation*}
$$

Unfortunately, the vacuum chamber that the current strips are adhered to is not perfectly straight. As such, the parameterisation of the wire now becomes

$$
\begin{equation*}
x^{\prime}\left(s^{\prime}\right)=x_{0}^{\prime}+\delta x^{\prime}\left(s^{\prime}\right) \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
z^{\prime}\left(s^{\prime}\right)=z_{0}^{\prime}+\delta z^{\prime}\left(s^{\prime}\right) \tag{5}
\end{equation*}
$$

where $\delta x^{\prime}\left(s^{\prime}\right)$ and $\delta z^{\prime}\left(s^{\prime}\right)$ are intended to be small perturbations on the position of the wire. We calculate the resulting magnetic fields from the Biot-Savart law and obtain

$$
\begin{align*}
& B_{z}(s)= \\
& \frac{\mu_{0} I}{4 \pi} \int_{-\infty}^{\infty} \frac{\left(x_{0}^{\prime}+\delta x^{\prime}\left(s^{\prime}\right)+\frac{d \delta x^{\prime}}{d s^{\prime}}\left(s-s^{\prime}\right)\right)}{\left[\left(x_{0}^{\prime}+\delta x^{\prime}\left(s^{\prime}\right)\right)^{2}+\left(z_{0}^{\prime}+\delta z^{\prime}\left(s^{\prime}\right)\right)^{2}+\left(s-s^{\prime}\right)^{2}\right]^{\frac{3}{2}}} d s^{\prime} \tag{6}
\end{align*}
$$

for the vertical field which is significantly more complex than Equation 3. The horizontal field has a similar form. Instead of calculating the magnetic field, which will have complicated behavior in the $\hat{y}$-direction, we find it useful

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to calculate the field integral, which will integrate out the $s$ dependence,

$$
\begin{align*}
I_{z} & \equiv \int_{-\infty}^{\infty} B_{z}(s) d s  \tag{7}\\
& =\frac{\mu_{0} I}{2 \pi} \int_{-\infty}^{\infty} \frac{x_{0}^{\prime}+\delta x^{\prime}\left(s^{\prime}\right)}{\left(x_{0}^{\prime}+\delta x^{\prime}\left(s^{\prime}\right)\right)^{2}+\left(z_{0}^{\prime}+\delta z^{\prime}\left(s^{\prime}\right)\right)^{2}} d s^{\prime}
\end{align*}
$$

We now define

$$
\begin{equation*}
\Delta x^{\prime} \equiv \frac{1}{L} \int_{-\infty}^{\infty} \delta x^{\prime}\left(s^{\prime}\right) d s^{\prime} \text { and } \Delta z^{\prime} \equiv \frac{1}{L} \int_{-\infty}^{\infty} \delta z^{\prime}\left(s^{\prime}\right) d s^{\prime} \tag{8}
\end{equation*}
$$

If the terms $\delta x^{\prime}\left(s^{\prime}\right)$ and $\delta z^{\prime}\left(s^{\prime}\right)$ are sufficiently small, we can use the series expansion $(1+x)^{-1}=1-x+O\left(x^{2}\right)$ to simplify the expression for the field integral to

$$
\begin{equation*}
I_{z} \simeq \frac{\mu_{0} I L}{2 \pi} \frac{x_{0}^{\prime}+\Delta x^{\prime}}{\left(x_{0}^{\prime}+\Delta x^{\prime}\right)^{2}+\left(z_{0}^{\prime}+\Delta z^{\prime}\right)^{2}} \tag{9}
\end{equation*}
$$

The horizontal field integral has a similar form. Note that if we take $\Delta x^{\prime} \rightarrow 0$ and $\Delta z^{\prime} \rightarrow 0$, we recover the result for the perfectly aligned wire of Equation 3.

This result implies that we can model the imperfect strip as a straight wire with the position offset by $\Delta z^{\prime}$ and $\Delta x^{\prime}$, when the misalignments are sufficiently small.

## THE RATIO MATRIX METHOD

The QMSC insertion device vacuum chamber has 12 current strips on the top and 12 on the bottom for a total of 24 strips. Using the results of the previous section, we will model these strips as long, infinitesimal wires with position $\left(x_{i}, z_{i}\right)$ relative to the electron beam. For this section, we will drop the prime notation needed to distinguish quantities for the Biot-Savart law in the previous section.

If we balance the currents on two strips such that $B_{x}=0$ or $B_{z}=0$ at the electron beam, the beam will experience no magnetic field in one plane and there will be no closed orbit distortion in the opposite plane. The resulting ratio of strip currents can give us information about the strip positions.

Working with the horizontal magnetic field, let all but two strips have zero current and strips $i$ and $j$ have current $I_{x, i}$ and $I_{x, j}$ respectively. The magnetic fields experienced by the electron beam due to strips $i$ and $j$ are

$$
\begin{equation*}
B_{x, i}=\frac{-\mu_{0} I_{x, i} z_{i}}{2 \pi\left(x_{i}^{2}+z_{i}^{2}\right)} \text { and } B_{x, j}=\frac{-\mu_{0} I_{x, j} z_{j}}{2 \pi\left(x_{j}^{2}+z_{j}^{2}\right)} \tag{10}
\end{equation*}
$$

which come from Equations 3 and 9 recast into this section's notation. We adjust $I_{x, i}$ and $I_{x, j}$ to balance the magnetic fields such that

$$
\begin{equation*}
0=B_{x, i}+B_{x, j}=-\frac{\mu_{0}}{2 \pi}\left(\frac{I_{x, i} z_{i}}{\left(x_{i}^{2}+z_{i}^{2}\right)}+\frac{I_{x, j} z_{j}}{\left(x_{j}^{2}+z_{j}^{2}\right)}\right) \tag{11}
\end{equation*}
$$

which implies

$$
\begin{equation*}
-\frac{z_{i}}{z_{j}}\left(\frac{x_{j}^{2}+z_{j}^{2}}{x_{i}^{2}+z_{i}^{2}}\right)=\frac{I_{x, j}}{I_{x, i}} \equiv R_{x, i j} \tag{12}
\end{equation*}
$$

We have defined the matrix $R_{x}$ to be the ratio of the two strip currents that cancel the horizontal magnetic field. Likewise,

$$
\begin{equation*}
-\frac{x_{i}}{x_{j}}\left(\frac{x_{j}^{2}+z_{j}^{2}}{x_{i}^{2}+z_{i}^{2}}\right)=\frac{I_{z, j}}{I_{z, i}} \equiv R_{z, i j} \tag{13}
\end{equation*}
$$

and we define the matrix $R_{z}$ to be the ratio of the two strip currents, $I_{z, i}$ and $I_{z, j}$, that cancel the vertical magnetic field. Thus, by measuring $R_{x}$ and $R_{z}$ we can calculate $\left(x_{i}, z_{i}\right)$ for each strip by minimizing the objective function

$$
\begin{align*}
& f\left(x_{1}, x_{2}, \ldots x_{24}, z_{1}, z_{2}, \ldots z_{24}\right)= \\
& \sum_{i, j}\left[1+R_{x, i j}\left(\frac{z_{j}}{z_{i}}\right)\left(\frac{x_{i}^{2}+z_{i}^{2}}{x_{j}^{2}+z_{j}^{2}}\right)\right]^{2}+ \\
& \sum_{i, j}\left[1+R_{z, i j}\left(\frac{x_{j}}{x_{i}}\right)\left(\frac{x_{i}^{2}+z_{i}^{2}}{x_{j}^{2}+z_{j}^{2}}\right)\right]^{2} . \tag{14}
\end{align*}
$$

It is important to note that the infinitesimal strip model should only be used to gain understanding of the method. For real calculations, we must use a model of the strips that takes into account the finite transverse dimensions of the strips. This can be done by calculating simulated ratio matrices

$$
\begin{equation*}
R_{x, i j}^{s i m}=-\frac{B_{x, i}^{1 \mathrm{~A}}}{B_{x, j}^{1 \mathrm{~A}}} \text { and } R_{z, i j}^{s i m}=-\frac{B_{z, i}^{1 \mathrm{~A}}}{B_{z, j}^{1 \mathrm{~A}}} \tag{15}
\end{equation*}
$$

where $B_{x, i}^{1 \mathrm{~A}}, B_{x, j}^{1 \mathrm{~A}}, B_{z, i}^{1 \mathrm{~A}}$ and $B_{z, j}^{1 \mathrm{~A}}$ are the calculated magnetic fields for strips $i$ and $j$ with finite dimensions and 1 A current. Note that $B_{x, i}^{1 \mathrm{~A}}$ and $B_{z, i}^{1 \mathrm{~A}}$ depend on $\left(x_{i}, z_{i}\right)$. We can again find ( $x_{i}, z_{i}$ ) by constructing a new objective function comparing $R_{x}^{\text {sim }}$ directly with $R_{x}$ and $R_{z}^{\text {sim }}$ directly with $R_{z}$.

## MEASUREMENTS AND RESULTS

In our implementation, we use particle swarm optimization [9] to minimize the objective function as it is simple to implement, robust and we are familiar with the algorithm [10]. We tested the algorithm by creating a model of misaligned current strips with finite length and width that were straight and with centroid position $\left(x_{i}^{\text {test }}, z_{i}^{\text {test }}\right)$. Using this model, we calculated ratio matrices and then calculated the strip positions $\left(x_{i}, z_{i}\right)$ from the ratio matrices. The algorithm converged and returned the correct positions.

We have tested our algorithm on real data, and the results are curious. Figure 1 shows ratio matrices measured at CLS using the QMSC current strips. Note that we only measure the bottom-right portion of the matrices as the top-right portion is determined by $R_{x, i j}=R_{x, j i}^{-1}$ and $R_{z, i j}=R_{z, j i}^{-1}$. Also note that $R_{x, i i}=R_{z, i i}=-1$ by definition.

We apply the algorithm to the measured data of Fig. 1 to determine the strip positions $\left(x_{i}, z_{i}\right)$ in order to build an


Figure 1: Measured current strip ratio matrices $R_{x}$ (top) and $\stackrel{\rightharpoonup}{\infty} R_{z}$ (bottom).
effective model. The calculated strip positions are shown
in Fig. 2. The positions shown in Fig. 2 have some strange
behavior, especially on the bottom strips where there is an
apparent discontinuity in the middle and the strips furthest to the right appear to sag significantly. The features of this plot are not yet understood and study is ongoing.
First, we recognize that we are attempting to build an effective model based on a simplifying approximation. This has a reduced our model parameters to a number that we can hope to measure using the ratio matrix technique. It is possible that, even with the odd features observed in Fig. 2, the effective model may work sufficiently well for our needs. The next test, once sufficient machine time becomes available, is to offset the beam in the straight and measure several ratio matrices with different horizontal and vertical beam offsets. If the algorithm returns the same result as in Figure 2 plus the applied offset, it will build confidence in the method.
It may also be the case that the algorithm needs to be altered. We may be converging to a false minimum of the objective function or we may need to perform weighting of different ratio matrix elements.
It may also be the case that our approximation of the strips as long, straight strips with only a single horizontal and a

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Figure 2: Current strip positions calculated from the measured $R_{x}$ and $R_{z}$ ratio matrices of Fig. 1 plotted against their ideal positions
single vertical misalignment parameter is unrealistic. In this case, we must re-evaluate our base assumptions. One item of note is that while measuring $R_{x}$ and $R_{z}$, the closed orbit distortion rarely went to zero, but to a non-zero minimum, as shown in Fig. 3. The model may need to take into account


Figure 3: Measurement of $R_{x, 69}$ performed by holding $I_{9}=$ -15 A and performing a search using $I_{6}$ to minimize the vertical RMS closed orbit distortion as measured by the beam position monitors (BPMs). The minimum occurs at $R_{x, 69}=-3.29$, but the closed orbit distortion never goes entirely to zero.
the second field integral as well as the first. We may also consider realigning the vacuum chamber to reduce distortions of the strips.

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## REFERENCES

[1] W. A. Wurtz, D. Bertwistle, L. O. Dallin and M. J. Sigrist, "Simulation of a long-period EPU operating in universal mode at the Canadian Light Source", IPAC14, 1995 (2014).
[2] J. Bahrdt and G. Wüstefeld, Phys. Rev. ST AB 14, 040703 (2011).
[3] J. Bahrdt, W. Frentrup, A. Gaupp, M. Scheer and G. Wuestefeld, "Active shimming of the dynamic multipoles of the BESSY UE112 APPLE undulator", EPAC08, 2222 (2008).
[4] B. Singh, R. Bartolini, R. Fielder, E. C. Longhi, I. P. Martin, S. P. Mhaskar and R. P. Walker, "Active shimming of dynamic multipoles of an APPLE II undulator in the Diamond storage ring", IPAC13, 1997 (2013).
[5] Q.L. Zhang, B.C. Jiang, S.Q. Tian, Q.G. Zhou, Z.T. Zhao, "Study on beam dynamics of a knot-APPLE undulator proposed
for SSRF", IPAC15, 1669 (2015).
[6] T. Tanabe, Y. Hidaka, C. Kitegi, D. Hidas, M. Musardo, D. A. Harder, J. Rank, P. Cappadoro, H. Fernandes, and T. Corwin, "Latest experiences and future plans on NSLS-II insertion devices", AIP Conference Proceedings 1741, 020004 (2016).
[7] D. Griffiths, Introduction to electrodynamics, Prentice Hall, (1999).
[8] J. D. Jackson, Classical electrodynamics, 3rd ed, Wiley (1999).
[9] J. Kennedy and R. Eberhart, "Particle swarm optimization," Proceedings of the IEEE International Conference on Neural Networks 4, 1942 (1995).
[10] W.A. Wurtz, "Coupling control and optimization at the Canadian Light Source", Nucl. Instr. and Meth. A 892, 1 (2018).

