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DISPERSIVE ELECTRON COOLING FOR JLEIC*

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Abstract

JLEIC is the electron ion collider under design at Jefferson Lab, which will provide a luminosity up to 10^{34} cm^{-2}s^{-1}. To reach the high luminosity, short ion and electron bunches with high charge density colliding in high frequency are proposed. The high charge density of the ion beam leads to strong intrabeam scattering effect, which enlarges the ion beam emittance and ruins the luminosity if not mitigated. Magnetized electron cooling is implemented to overcome the intrabeam scattering effect and to reduce or maintain the ion beam emittance. In this paper, we discuss the redistribution of the cooling effects in the longitudinal and the transverse directions by introducing the dispersion of the ion beam in the cooling section. When the charge density of the cooling electron beam varies transversely, the dispersion of the ion beam leads to an increase of the transverse cooling rate and a reduction of the longitudinal cooling rate. This allows us to use the extra longitudinal cooling to help the horizontal cooling in JLEIC. Both theoretical analysis and numerical calculation are presented.

ELECTRON COOLING

Electron cooling is a method to reduce the emittance of an ion beam by introducing an accompanying electron beam of a lower temperature into one of the straight sections of the ion orbit. Due to Coulomb scattering of the particles, the ion gas is cooled in the electron one. After multiple passes of the interaction region, the size and the energy spread of the ion beam decrease to some equilibrium values.

When an ion particle moves in the accompanying electron beam, it experiences the friction force resulted from the superposition of the Coulomb interaction forces with individual electrons. The friction force can be calculated as [1]

$$\boldsymbol{F} \approx -\frac{4\pi Z^2 e^4 n_e'}{m} \int L(u) \frac{\boldsymbol{u}}{u^3} f(\boldsymbol{r}, \boldsymbol{v}_e) d^3 v_e$$

in the co-moving reference frame, where Z is the charge number of the ion, e the electron charge, n_e' the electron density, m the electron mass, $f(\boldsymbol{r}, \boldsymbol{v}_e)$ the electron phase space distribution, $\boldsymbol{v}_e$ electron velocity, $\boldsymbol{u} = \boldsymbol{v} - \boldsymbol{v}_e$ the relative particle velocity, and $L(u)$ the Coulomb logarithm.

Introducing a longitudinal magnetic field along the electron beam moving direction helps to compensate the repulsion of electrons by the space charge field, as well as an angular beam divergence. In the magnetic field, the electron velocity component transverse to the field $\boldsymbol{v}_\perp$ rotates with the Larmor frequency $\boldsymbol{\Omega} = -e\boldsymbol{B}/mc$, with c the speed of light. In the co-moving frame, electrons move in circles with radii $r_L = v_\perp/\Omega$. In the lab frame, they move in spirals with radii r_L and steps $l_L = 2\pi\gamma\beta c/\Omega$ where β and γ are Lorentz factors. In a Coulomb interaction, the momentum and energy exchange of colliding particles diverges logarithmically in the region of large impact parameters and must be cut off at some macroscopic parameter $\rho_{\max}$. In the region of impact distance ρ satisfying the condition $r_L < \rho < \rho_{\max}$, duration of collisions and intensity of the exchange do no depend on the electron Larmor velocity $v_{e\perp}$ and are determined only by the proton velocity with respect the Larmor "circle" :

$$\boldsymbol{u}_A = \boldsymbol{v} - \boldsymbol{v}_{e\parallel} = v_\perp + u_\parallel$$

Thus in the "logarithmic" approximation, all the collisions in magnetic field are divided into two types:

1. Fast collisions with effective interaction time small compared to the electron Larmor period ($\tau = \rho/u_A < 1/\Omega$) - a region of impact distance $\rho < u_A/\Omega$;
2. Adiabatic collision with Larmor circles at distances $\rho > \max\{r_L, u_A/\Omega\}$.

The contribution of the fast oscillations is the same with non-magnetized cooling. The friction force due to the adiabatic collision can be calculated as [1]

$$\boldsymbol{F}^A = \frac{2\pi Z^2 e^4 n_e'}{m} \frac{\partial}{\partial \boldsymbol{v}} \langle \frac{1}{u_A^2} \left(\frac{v_\perp^2}{u_A^2} L^A + 1 \right) \rangle \tag{1}$$

$$L^A = \ln(\rho_{\max}^A / \rho_{\min}^A),\ u_A = v - v_{e\parallel} = v_\perp + u_\parallel$$

$$\rho_{\max}^A = \min\{u_A t, \frac{u_A}{\omega_e}\},\ \rho_{\min}^A = \max\{r_L, \frac{u_A}{\Omega}, \frac{ze^2}{mu_A^2}\}$$

where ω_e is the Langmuir frequency of the electron beam. When $v_\perp$ is comparable with $u_\parallel$, the accuracy of L^A is unit. For practice we can ignore the term 1 in Eq. (1) and we have

$$F_\parallel^A \approx -\frac{6\pi Z^2 e^4 n_e'}{m} \langle u_\parallel \frac{v_\perp^2}{u_A^5} L^A \rangle$$

$$\boldsymbol{F}_\perp^A \approx -\frac{2\pi Z^2 e^4 n_e'}{m} \langle \frac{v_\perp^2 - 2u_\parallel^2}{u_A^5} L^A \rangle \boldsymbol{v}_\perp$$

A complete description of the cooling process of a focused beam in a ring requires computation of cooling rates of the 3-dimensions particle oscillators. The betatron oscillation of an ion can be described as follows.

$$x_b = \sqrt{2I_x\beta} \cos\psi_x\ ,\ \ x_b' = -\sqrt{\frac{2I_x}{\beta}} (\sin\psi_x - \frac{\beta'}{2} \cos\psi_x)$$

with $\psi' = 1/\beta$, $\beta = \beta_0 + z^2/\beta_0$, and $\beta' = 2z/\beta_0$. The Courant-Snyder invariants are

$$I_x = \frac{1}{2} \left[\frac{1}{\beta} x_b^2 + \beta \left(x_b' - \frac{\beta'}{2\beta} x_b \right)^2 \right]$$

$$I_y = \frac{1}{2} \left[\frac{1}{\beta} y_b^2 + \beta \left(y_b' - \frac{\beta'}{2\beta} y_b \right)^2 \right]$$

With dispersion, the transverse directions and the longitudinal direction are coupled together. The difference of energy leads to a difference of transverse position: $x_b = x - D_x q$, $y_b = y - D_y q$, $x_b' = x' - D_x' q$, $y_b' = y' - D_y' q$,

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where $D_{x,y}$ is the dispersion in x or y direction and q is the energy spread to the reference particle.

The dynamic invariant $I_{x,y}$ changes due to the dissipative force. Assuming $D'_{x,y}=0$ at the cooling section, we obtain:

$$p\dot{I}_x = \langle\frac{1}{\gamma}\left(\beta_x x' - \frac{\beta'_x}{2}x_b\right)F_x - \frac{D_x}{\beta_x}F_\parallel x_b\rangle$$

$$p\dot{I}_y = \langle\frac{1}{\gamma}\left(\beta_y y' - \frac{\beta'_y}{2}y_b\right)F_y - \frac{D_y}{\beta_y}F_\parallel y_b\rangle$$

$$\dot{p}_\parallel = p\dot{q} = \langle F_\parallel\rangle$$

where the dot represents the derivative with respect to time, and the brackets represents averaging over time, which can be replaced by averaging over phase ψ. Now we can calculate the cooling rates for the friction force of adiabatic collisions of ions with magnetized electrons in solenoid. For simplicity we neglect terms connected to change of β function in cooling section, influence of solenoid field and rest forces on ion motion, and assume constant dispersion functions across the cooling section. The resulting initial expressions for the cooling rates are as follows:

$$\frac{\langle\dot{I}_\parallel\rangle}{I_\parallel} = -\frac{6g}{v_{\parallel a}}\langle n_e L^A \frac{v_\perp^2}{u_A^5}u_\parallel \sin\psi_\parallel\rangle,\quad g=\frac{2\pi Z^2 e^4}{\gamma^2 mM}$$

$$\frac{\dot{I}_x}{I_x} = -2g\left[\langle n_e L^A\frac{v_\perp^2-2u_\parallel^2}{u_A^5}\sin^2\psi_x\rangle + \frac{3D_x}{Va_x}\langle n_e L^A\frac{v_\perp^2}{u_A^5}u_\parallel\cos\psi_x\rangle\right]$$

$$\frac{\dot{I}_y}{I_y} = -2g\left[\langle n_e L^A\frac{v_\perp^2-2u_\parallel^2}{u_A^5}\sin^2\psi_y\rangle + \frac{3D_y}{Va_y}\langle n_e L^A\frac{v_\perp^2}{u_A^5}u_\parallel\cos\psi_y\rangle\right]$$

where M is the ion mass, V is the velocity of the reference particle, n_e is the electron density in the lab frame, $v_{\parallel a}$ is the amplitude of the longitudinal ion velocity, *i.e.* $v_\parallel = v_{\parallel a}\sin\psi$, and $a_{x,y}$ is the amplitude of betatron oscillation in the respective direction.

Transverse temperature of high energy relativistic beams is usually large with respect to the longitudinal one, *i.e.* $\gamma\theta \gg \delta\gamma/\gamma$, with θ the angular spread. This is due to the following factors: (1) Adiabatic acceleration leads to increase of transverse temperature while the longitudinal one usually is maintained (could be even decreased, in principle). In result, at initial cooling at top energy we have: $\Delta_{v\perp}/\Delta v_\parallel \geq \sqrt{\gamma_{\text{top}}/\gamma_{\text{Injection}}}$, (2) Intrabeam scattering (IBS) around the ring at high energies excites betatron horizontal transverse emittance through the dispersion. (3) Coupling around the orbit leads to proportional excitation of vertical emittance. Due to the non-linear dependence of cooling rates on emittances, this asymmetry in the initial or steady state beam leads to a big difference in the partial cooling rates.

DISPERSIVE ELECTRON COOLING OF HIGH TRANSVERSE TEMPERATURE

The asymmetry in cooling rates can be corrected by introduction of the following measurements. (1) split in average longitudinal velocity of ion beam relative the electron beam: $f(v_{e\parallel}) = \frac{1}{\sqrt{2\pi}\Delta_\parallel}\exp\left[-\frac{(v_{e\parallel}-v_0)^2}{2\Delta_\parallel^2}\right]$ with $\Delta_\parallel$ the longitudinal velocity spread of the electron beam, (2) ion dispersion function in cooling section, plus (3) transverse space-gradient of the longitudinal friction force. This gradient can be arranged in two possible ways: *A*) introduce a correspondent e-dispersion across the drift mode space, or *B*) positioning e-beam (of the sizes twice smaller than that of the ion beam) asymmetrically relative the ion beam center.

Let us consider option *B* for realization of transverse gradient of the longitudinal friction force as more simple and straightforward, representing it via gradient of electron density as follows:

$$n_e(x,y) = n_0 + \frac{\partial n_e}{\partial x}x + \frac{\partial n_e}{\partial y}y;\ \frac{\partial n_e}{\partial x}\approx\frac{n_0}{\sigma_{ex}};\ \frac{\partial n_e}{\partial y}\approx\frac{n_0}{\sigma_{ey}}.$$

where $\sigma_{ex,y}$ is the transverse size of the electron beam. The dispersive contribution to cooling is derived as:

$$\langle\dot{I}_x\rangle_D = \frac{6g}{V}\langle L^A u_\parallel\frac{v_\perp^2}{u_A^5}x_b^2\rangle\frac{D_x}{\beta_x}\frac{\partial n_e}{\partial x}.$$

Here we limit our consideration by conditions: $v_{xa}, v_{ya} \gg |u_\parallel|$ and $v_0, v_{\parallel a} \ll \Delta_\parallel$, where $v_{xa,ya}$ is the amplitude of the ion velocity in the respective direction.. Taking into account that integration over betatron phases is concentrated near $\psi_x=\psi_y=0$, we can produce this integration. Then the integration is reduced to averaging over the electron velocities distribution and synchrotron oscillations of ions:

$$\langle\dot{I}_\parallel\rangle \approx -I_\parallel\frac{8gn_0}{\pi v_{xa}v_{ya}v_{\parallel a}}\langle L^A\frac{u_\parallel}{|u_\parallel|}\sin\psi_\parallel\rangle + \langle\dot{I}_\parallel\rangle_D$$

$$\langle\dot{I}_\parallel\rangle_D \approx -I_\parallel\frac{8g}{\pi v_{xa}v_{ya}V}\langle L^A(u_\parallel)\frac{u_\parallel}{|u_\parallel|}\sin^2\psi_\parallel\rangle\left(\frac{\partial n_e}{\partial x}D_x + \frac{\partial n_e}{\partial y}D_y\right)$$

$$\langle\dot{I}_x\rangle_D \approx I_x\frac{8g}{\pi v_{xa}v_{ya}V}\langle L^A(u_\parallel)\frac{u_\parallel}{|u_\parallel|}\rangle\frac{\partial n_e}{\partial x}D_x$$

$$\langle\dot{I}_y\rangle_D \approx I_y\frac{8g}{\pi v_{xa}v_{ya}V}\langle L^A(u_\parallel)\frac{u_\parallel}{|u_\parallel|}\rangle\frac{\partial n_e}{\partial y}D_y$$

Performing the final averaging in asymptotic situations $v_0, v_{\parallel a} \ll \Delta_\parallel$, we obtain the following results:

$$\langle\dot{I}_\parallel\rangle \approx -2I_\parallel\left(\frac{2}{\pi}\right)^{3/2}\frac{gn_0L^A(\Delta_\parallel)}{v_{xa}v_{ya}\Delta_\parallel}\left[1 - \frac{v_0}{V}\frac{1}{n_0}\left(\frac{\partial n_e}{\partial x}D_x + \frac{\partial n_e}{\partial y}D_y\right)\right]$$

$$\dot{I}_{xD} = -4I_x\left(\frac{2}{\pi}\right)^{3/2}\frac{gL^A(\Delta_\parallel)}{v_{xa}v_{ya}\Delta_\parallel}\frac{v_0}{V}\frac{\partial n_e}{\partial x}D_x$$

$$\dot{I}_{yD} = -4I_x\left(\frac{2}{\pi}\right)^{3/2}\frac{gL^A(\Delta_\parallel)}{v_{xa}v_{ya}\Delta_\parallel}\frac{v_0}{V}\frac{\partial n_e}{\partial y}D_y$$

with $D_{x,y}q < \sigma_{ex,y}$.

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DISPERSIVE COOLING FOR JLEIC

The JLab Electron Ion Collider (JLEIC) will provide an electron beam with energy up to 10 GeV, a proton beam with energy up to 100 GeV, and heavy ion beams with corresponding energy per nucleon with the same magnetic rigidity. To achieve the ultrahigh luminosity close to 10^{34} $cm^{-2}s^{-1}$ per detector with large acceptance, the traditional electron cooling will be implemented strategically. [2]

The JLEIC ion complex consists of ion sources, an SRF linac, a booster ring and a collider ring, as shown in Fig 1. In the booster ring, a low energy DC cooler will be installed only for heavy ion beam injection. In the collider ring, both a DC cooler and an ERL based bunched beam cooler will be installed. The DC cooling is applied to reduce the ion beam emittance at low energy (7 GeV/u) and the ERL-based bunched beam cooling is applied during collision to mitigate the IBS effect and to maintain the emittance.

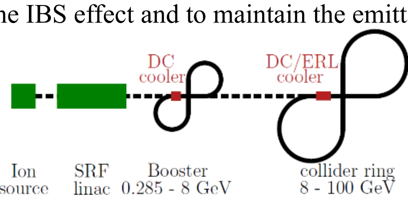

Figure 1: Components of JLEIC ion complex.

In the following we only consider the bunched beam cooling for the proton beam at the collision energy (100 GeV). The property of the proton beam for the center-of-mass (CM) energy of 44.7 GeV is listed in Table 1. The property of the bunched beam cooler is listed in Table 2. The initial cooling rate and the IBS rate in all the three directions are listed in Table 3. As it shows, the IBS expansion rate in the horizontal direction is more than 10 times larger than those in the other two directions. Although the electron beam provides enough cooling in the vertical direction and the longitudinal directions, in the horizontal direction the IBS effect is stronger than the cooling effect and the proton beam will expand in this direction. Redistribution of the IBS effect and the cooling effect for the equilibrium in all the directions is the problem we face.

Table 1: JLEIC Proton Beam for 44.7 CM Energy

Energy	GeV	100
Particle number	10^{10}/bunch	0.98
Norm. emit. (rms)	mm·mrad	0.5/0.1
Bunch size (rms)	mm	0.528
Bunch length (rms)	cm	1
Momentum spread		8×10^{-4}

Table 2: JLEIC ERL Bunched Cooler Parameters

Length	m	30×2
Magnetic field	T	1
Electron charge	nC/bunch	3.2
Electron bunch shape	beer can	
Electron bunch size	mm	0.528
Electron bunch length	cm	2
Electron temperature	eV	0.246/0.184

Table 3: Initial Expansion Rate Without Dispersion

		Rx	**Ry**	**Rs**
Cooling	10^{-3}/s	-4.708	-11.787	-0.554
IBS	10^{-3}/s	12.894	0.669	0.992
Total	10^{-3}/s	8.186	-11.118	-0.455

Transverse coupling redistributes the the IBS effect between the horizontal and the vertical directions, which allows us to use the extra cooling in the vertical direction. Introducing dispersions at the cooling sections has two benefits. First, it transfers the extra cooling in the longitudinal direction to the transverse directions. Second, it helps to prevent overcooling in the longitudinal direction. If the momentum spread and the bunch length reduces too fast, the abrupt increase of proton charge density will lead to a sudden increase of IBS effect, which will make the transverse cooling more difficult. Figure 2 shows an example simulation [3, 4] of the cooling process for the proton beam of slightly reduced current (82%), increased emittance (0.5/0.15 mm·mrad) and increased rms bunch length (1.5 cm) with dispersion (1.8/0.3 m) at the cooling section and 40% transverse coupling. The emittances are maintained. But the large dispersion (1.8 m) may cause dynamic problems. We are investigating other solutions with smaller dispersion.

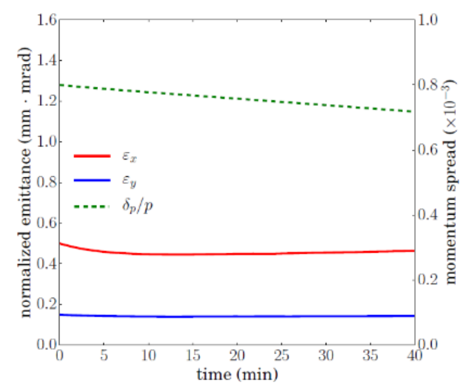

Figure 2: Cooling for proton beam with dispersion.

SUMMARY

We have presented that ion beam dispersion at the cooling section redistributes the cooling between the transverse and the longitudinal directions. For JLEIC, the proton beam at the collision energy has much stronger IBS effect in the horizontal direction than the other directions. Dispersive cooling, together with other means, *e.g.* transverse coupling, helps to find a preferred equilibrium, which maintains the emittance during the collision.

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[4] `https://github.com/zhanghe9704/electroncooling`

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