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STUDY OF THE IMPACT OF LINEAR COUPLING ON OFF-AXIS INJECTION*

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Abstract

The next generation of storage ring light sources will likely operate with high linear coupling, which could potentially prevent the use of off-axis injection as large horizontal oscillation of the injected beam is coupled to the vertical plane. We did experiments on the SPEAR3 storage ring to study how linear coupling impact the dynamic aperture and the off-axis injection efficiency. The results show that the dynamic aperture is significantly reduced and injection efficiency can drop to zero when operated on the coupling resonance. However, with large nonlinear detuning, the dynamic aperture and high injection efficiency can survive with the stored beam at full coupling because the injected beam is shifted away from the coupling resonance.

INTRODUCTION

In existing 3rd generation light sources, the vertical emittance is typically much smaller than the horizontal emittance, with an emittance ratio below 1%. However, for future diffraction limited storage rings (DLSRs), it is not advisable to maintain the same level of vertical-to-horizontal emittance ratio. This is because the horizontal emittance will already be diffraction-limited and hence there is no need to make vertical emittance any smaller, and small vertical emittance will cause significant intra-beam scattering (IBS) and Touschek beam loss. A round beam with equal horizontal and vertical emittances is usually assumed in DLSR designs, and most designs plan to generate round beams by operating on the linear coupling resonance, with fully coupled horizontal and vertical motion. This would be acceptable if on-axis, swap-out injection [1, 2] will be used. However, for the DLSRs that plan to use the traditional off-axis injection scheme, such as PEP-X [3] and PETRA-IV, large coupling between the two transverse planes could cause injection difficulties. With off-axis injection, the injected beam will have large vertical oscillation amplitude through coupling and the initial horizontal offset and may get lost to the small vertical apertures such as the small-gap insertion devices.

Off-axis injection with high coupling has been studied in Refs. [4, 5]. In Ref. [5] it was showed that if the coupling resonance strength is corrected to a low level the injection efficiency does not suffer a loss as the coupling resonance is crossed. To understand the dynamics of beam motion near the coupling resonance and to investigate its potential impact to off-axis injection, we conducted similar experiments on the SPEAR3 storage ring.

In the experiments we changed the strength of the linear difference coupling resonance. At each level we moved the horizontal and vertical tunes toward each other to cross the coupling resonance, which changed the coupling ratio. Dynamic aperture and injection efficiency were measured as the betatron tunes were crossed. In the measurements we also observed safe crossing of the linear coupling resonance ($\nu_x = \nu_y$ for the stored beam) when the coupling strength was corrected. However, the dynamic aperture and injection efficiency suffered significant loss at a point after the crossing, which then recovered as the betatron tunes moved further away. An explanation based on the nonlinear detuning of the injected beam is given for the observation, which may serve as a guide for the design of DLSR lattices with off-axis injection.

In the following we will first briefly discuss the theory of beam motion with linear coupling, which is followed by a detailed description of the experiments and a discussion of the results.

LINEAR COUPLING MOTION

The Hamiltonian for coupled beam motion between the horizontal and vertical planes due to skew quadrupole component, $a_1 = \frac{1}{B\rho}\frac{dB_x}{dx}$, is given by

$$H = \frac{1}{2}x'^2 + \frac{1}{2}y'^2 + \frac{1}{2}K_x(s)x^2 + \frac{1}{2}K_y(s)y^2 - a_1(s)xy, \quad (1)$$

where the last term, $H_1 = -a_1(s)xy$, can be considered a perturbation to the otherwise uncoupled linear motion. By taking a few canonical transformations toward the action-angle coordinates, (J_x, ϕ_x), expanding the perturbation term in Fourier series, and keeping only the term with slow varying phase factor near the linear difference resonance $\nu_x - \nu_y = l$, we arrive at a new Hamiltonian [6]

$$\bar{H} = \nu_x J_x + \nu_y J_y + G\sqrt{J_x J_y}\cos(\phi_x - \phi_y + \chi - l\theta), \quad (2)$$

where $\theta = s/R$, R is the ring radius and

$$Ge^{i\chi} = -\frac{1}{2\pi}\oint a_1(s)\sqrt{\beta_x\beta_y}e^{i\Psi_-}e^{-il\theta}ds, \quad (3)$$

with $\Psi_- = \psi_x - \psi_y - (\nu_x - \nu_y)\theta$, $\psi_{x,y} = \int_0^s \frac{1}{\beta_{x,y}(s')}ds'$, and $\beta_{x,y}$ are beta functions.

Taking another canonical transformation to the resonant processing frame with the generating function [6]

$$F_2(\phi_x, \phi_y, J_1, J_2) = (\phi_x - \phi_y + \chi - l\theta)J_1 + \phi_y J_2 \quad (4)$$

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and new coordinates

$$\phi_1 = \phi_x - \phi_y + \chi - l\theta, \qquad J_1 = J_x, \tag{5}$$

$$\phi_2 = \phi_y, \qquad J_2 = J_x + J_y, \tag{6}$$

the Hamiltonian becomes

$$\tilde{H} = H_1(J_1, \phi_1, J_2) + H_2(J_2), \quad \text{with} \tag{7}$$

$$H_1 = \Delta J_1 + G\sqrt{J_1(J_2 - J_1)}\cos\phi_1, \tag{8}$$

and $H_2 = \nu_y J_2$, with $\Delta = \nu_x - \nu_y - l$.

If a particle is launched initially from a horizontal plane with action variable $J_{1,max} = J_2$, from Eq. (8) we obtain

$$G^2 J_1 \cos\phi_1^2 = \Delta^2 (J_2 - J_1), \tag{9}$$

which leads to the maximum action value of the vertical action variable

$$J_{y,\max} = J_2 \frac{G^2}{\Delta^2 + G^2}. \tag{10}$$

Therefore, the ratio of maximum action variables for the two planes, which is the same as the emittance ratio in a storage ring, is

$$\frac{\epsilon_y}{\epsilon_x} = \frac{J_{y,max}}{J_{x,max}} = \frac{G^2}{\Delta^2 + G^2} \tag{11}$$

The coupling ratio increases with the coupling strength G and becomes 100% on the difference coupling resonance.

THE EXPERIMENTS

We did experiments to study the impact of high linear coupling to injection efficiency on SPEAR3, which is a 3rd generation light source. The nominal working point is at [14.106, 6.177]. The injected beam comes into SPEAR3 at the end of a Lambertson septum magnet with a horizontal separation from the stored beam. During normal operation the coupling ratio is corrected to about 0.1%, using 13 skew quadrupole magnets.

In the experiments we changed the strength of the difference coupling resonance by the skew quadrupole magnets. We first used LOCO [7], the orbit response matrix based method, to correct linear coupling to a low level. The coupling resonance strength was changed by scaling the skew quadrupole setpoints to 70% of the correction values, and fully turned off. A lattice model was obtained with LOCO fitting for each case. Additional skew quadrupole variables in the model were used to more accurately represent the coupling information in the orbit response matrices. At the nominal working point, the coupling ratio was 0.07% with full correction, 0.13% with skew quadrupoles at 70%, and 0.95% with skew quadrupoles off, respectively.

At each of the three levels of coupling resonance strengths, the horizontal and vertical tunes were moved toward each other in equal steps. In this process the tune separation, $\Delta = \nu_x - \nu_y$, moved from negative territory toward the resonance condition, $\nu_x - \nu_y = 0$, and over to the positive side. As the working point moves toward the difference resonance line, the coupling ratio increases. The injection efficiency and the dynamic aperture were measured at each stop.

Figure 1 shows the injection efficiency as a function of $\nu_x - \nu_y$ for the three cases. With skew quadrupoles off (“no corr”), there is a loss of injection efficiency over a wide range of tune separation about the resonance condition. With skew quadrupoles powered at 70% of the correction values (“70% corr”), the width of the region with injection efficiency loss is much smaller (between $\Delta = -0.006$ and 0.014). The center of this region is shifted toward the positive direction. When the skew quadrupoles are at the full correction strength (“full corr”), the region with loss of injection efficiency is even smaller. On the resonance condition, the injection efficiency is still at about 80%. However, as the tune separation moves away from the resonance condition, at $\Delta = 0.008$, the injection efficiency drops to nearly zero.

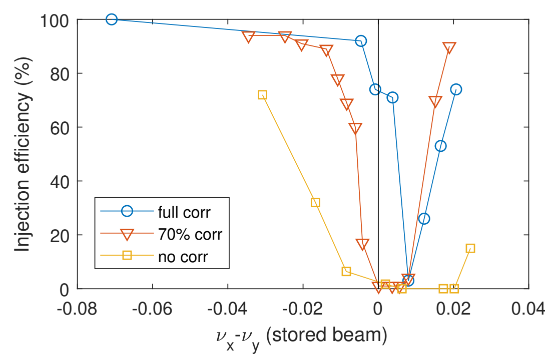

Figure 1: Injection efficiency vs tune separation at three levels of linear difference resonance strengths.

We measured the dynamic aperture by kicking a short train of stored beam bunches until the beam was lost with one injection kicker. Figure 2 shows the dynamic aperture measurements for the case with full skew quadrupole correction, where the fraction of surviving beam is plotted against the kicker voltage. We may define dynamic aperture arbitrarily as the kicker voltage when 80% of the initial beam current is lost. Figure 3 shows the measured dynamic aperture at various tune separations for the three cases. Roughly speaking, a voltage of 1 kV corresponds to a dynamic aperture of 11 mm for this lattice. The dynamic aperture measurements generally agree with the injection efficiency measurements. For the case with minimum coupling strength (“full corr”), the dynamic aperture is on an acceptable level when the working point is nearly on resonance ($\Delta = -0.0008$ and 0.0038), while suffering a significant reduction at $\Delta = 0.008$.

Using the lattice models obtained with LOCO, we can calculate the coupling ratios, which are shown in Figure 4. The coupling strength parameter is $G = 0.56 \times 10^{-3}$, 1.65×10^{-3}, and 3.51×10^{-3} for the three cases, respectively, with the lowest value for the full correction case.

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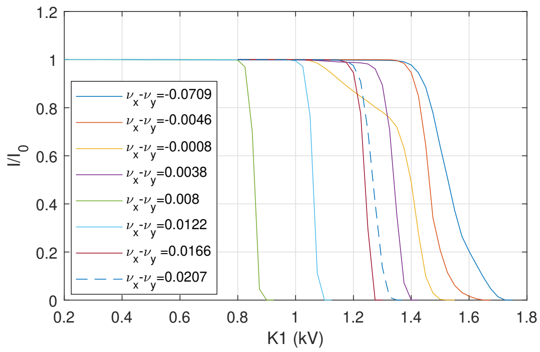

Figure 2: Dynamic aperture measurements with full coupling correction.

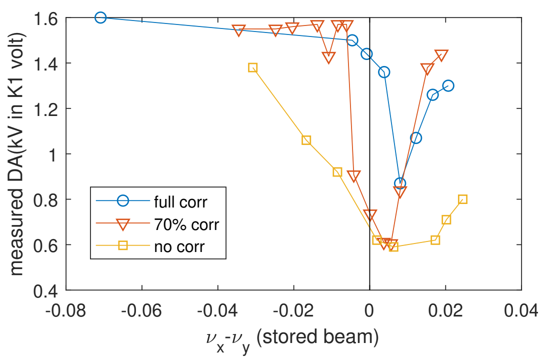

Figure 3: Dynamic aperture vs tune separation for the three coupling levels.

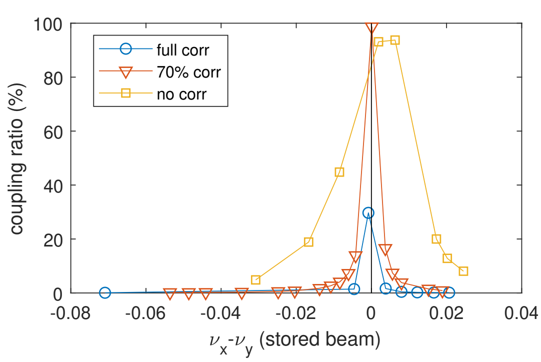

Figure 4: Coupling ratio (stored beam) for the three cases.

EXPLANATION AND DISCUSSION

The observation we made in the experiments appears to be puzzling at first sight: why is the injection efficiency significantly reduced when the working point is not on the difference resonance while not much reduction occurs when the beam is right on resonance? A close examination indicates that this can be explained by the differences between the betatron tunes of the stored beam and the injected beam due to nonlinear detuning by the horizontal oscillation of the latter.

Nonlinear detuning coefficients for SPEAR3 have been measured previously [8]. The tune shifts due to horizontal oscillation are $\frac{d\nu_x}{d(2J_x)} = 1590\ \mathrm{m}^{-1}$ and $\frac{d\nu_y}{d(2J_x)} = 2200\ \mathrm{m}^{-1}$, respectively, where $2J_x = x^2/\beta_x$. The initial offset of the injected beam is $x = 11$ mm and the horizontal beta function at the septum is $\beta_x = 9.0$ m, from which the tune shifts of the injected beam are calculated to be [0.0214, 0.0296]. Therefore, when the stored beam is on resonance, the injected beam has a tune separation of $\Delta = 0.0082$, as illustrated in Figure 5 for the full correction case, where the red dashed lines represent the stopband of the linear difference resonance, defined here at $\pm 2G$ from the resonance line.

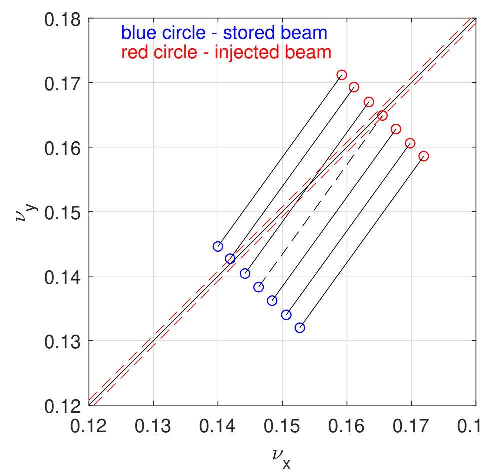

Figure 5: Tune diagram for the full correction case.

Clearly, it is the betatron tunes of the injected beam that determine its coupled motion, not that of the stored beam. If the stopband of the linear difference resonance is corrected to a low level, and the nonlinear detuning of the lattice is properly designed, it is possible to achieve full coupling for the stored beam and not to suffer a loss of dynamic aperture. DLSR designers could take advantage of this principle to generate round beam for the stored beam without losing injection efficiency for off-axis injection. What is important is for the tune footprint of the injected beam to avoid the linear difference resonance.

CONCLUSION

Our experiments on the SPEAR3 storage ring showed that when the injected beam is in the stopband of the linear difference resonance, there will be a reduction of dynamic aperture. Because the injected beam has different betatron tunes from the stored beam due to nonlinear detuning, one can put the stored beam on the linear difference resonance without losing injection efficiency. This could be achieved by correcting coupling resonance strength and designing the nonlinear detuning for the injected beam footprint to move away from the resonance.

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