ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPML083

ITERATIVE LEARNING CONTROL TO CANCEL BEAM LOADING EFFECT ON AMPLITUDE AND PHASE OF THE ACCELERATING FIELD*

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Abstract

Iterative learning control (ILC) is an open loop control strategy that improves the performance of a repetitive system through learning from previous iterations. ILC can be used to compensate for a repetitive disturbance like the beam loading effect in resonators. Assuming that the beam loading disturbance is identical for all iterations, the learning law can be non-causal; it can anticipate the disturbance and preemptively counteract its effect. In this work, we aim to use ILC to cancel beam loading effect on amplitude and phase. Feedback controllers are not fast enough for this purpose. A normal feed forward controller may not be sufficient as well if there is a difference between the feed forward signal and the beam loading current. Therefore, the goal is to use ILC to adaptively cancel the beam loading effect.

INTRODUCTION

In a linear accelerator, such as a cavity resonator, the goal is to establish and maintain a standing wave electromagnetic field with constant amplitude and phase. A feedback control loop is responsible for maintaining constant amplitude and phase despite various disturbances. The electromagnetic field within the cavity can be assumed as stored energy. When a bunch of particles passes through the cavity, a portion of the energy is transferred from the field to the beam, resulting in a drop in the accelerating field. This effect is referred to as beam loading. In superconducting cavity resonators, this effect can be significant as the ratio of the beam loading energy loss to the total energy stored in the cavity could be up to 4% [1].

Feedback controllers are not fast enough to compensate the beam loading effect. It is preferred to use feedforward controllers to preemptively counteract with the energy drop by increasing the cavity voltage just before the beam arrival. At Japan Proton Accelerator Research complex (J-PARC), a multi-harmonic RF feedforward system is used to compensate beam loading in 3 GeV rapid cycling synchrotron (RCS) [2]. The feedforward controller uses the wall current monitor (WCM) to pick up the beam signal I_{beam}. The controller then generates an additional signal equal to $-I_{beam}$ on top of the driving RF current. This control system compensates the beam loading of the three main harmonics ($h = 2, 4, 6$). In TESLA linear accelerator, adaptive feedforward control is used to compensate the beam loading and dynamic Lorentz force detuning [3].

In this work, we aim to use iterative learning control (ILC) to cancel the beam loading effect. The idea of iterative learning control (ILC) is to improve the performance of a repetitive system through learning from previous iterations [4]. It can be used on systems that perform the same task multiple times like a robot arm tracking the same path repeatedly, or to compensate for a repetitive error like the beam loading effect in resonators.

ILC is basically an open loop control strategy that improves as a result of repetition and learning. For a repetitive system, a nonlearning feedforward controller leads to the same tracking error on each iteration. For such controllers, the error signals obtained from previous repetitions remain unused, although they can provide valuable information on the controller performance. The goal of ILC is to incorporate the error signals from previous iterations toward improving the performance of the controller [4]. Since ILC is essentially an open loop feedforward controller, it has to be used in conjunction with a feedback loop. The feedback loop is responsible to compensate non-repeating disturbances, noise and model uncertainties. The ILC, on the other hand, rejects repeating disturbances faster than the feedback loop can.

SYSTEM MODELLING

Amplitude and Phase in Self-Excited Loop

A block diagram of the self-excited loop (SEL) is shown in Figure 1 [5]. The filter represents the cavity resonator. In time domain, the relationship between the input v_g and the output voltage v of the resonator is given by

$$\ddot{v}(t) + 2(1+\beta)\zeta_u\omega_0\dot{v}(t) + \omega_0^2 v(t) = 2\beta\zeta_u\omega_0\dot{v}_g(t), \qquad (1)$$

where β is the coupling coefficient, ζ_u is the unloaded damping ratio, and ω_0 is the natural frequency. Since SEL is a positive feedback loop, it oscillates at a frequency for which the total phase shift in the loop is an integer multiple of 2π radians.

To keep the loop frequency or phase of the oscillation constant despite variations in the filter transfer function, it is desired to compensate for the changes in the phase of the oscillation by adding a controllable amount of loop phase shift. This is done by adding a signal in quadrature as shown in Figure 1 [5].

* Work supported by TRIUMF. TRIUMF receives federal funding via a contribution agreement with the National Research Council of Canada
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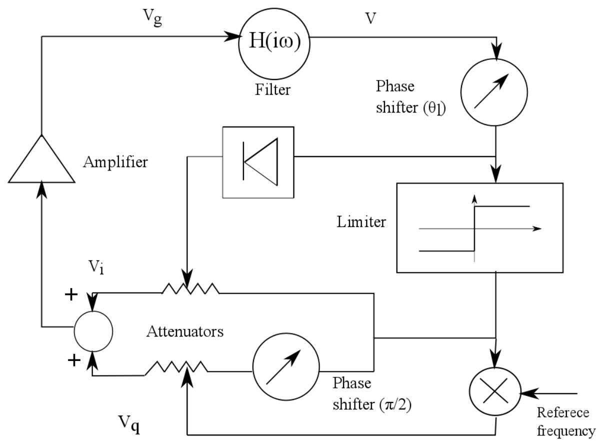

Figure 1: Block diagram of a self-excited loop, with in phase and in quadrature signals for phase stabilization.

Assuming that θ_l is chosen such that the loop oscillates at cavity's resonance frequency ω_0, we will form *in phase* and *in quadrature* signals as shown in Figure 1, referred to as v_i and v_q respectively. The addition of in quadrature signal introduces an additional phase shift given by $\chi = \tan^{-1}\frac{v_q}{v_i}$, and the amplitude of the signal before going through the amplifier is given by $A = \sqrt{v_i^2 + v_q^2}$.

To analyze the effect of adding in phase and in quadrature signals, one can use the input-output differential equation of a resonator given by Eq. (1), using the following expressions for input and output of the resonator in Figure 1

$$\begin{aligned} v &= Ve^{i\phi}, \\ v_g &= e^{i(\theta+\phi)}(v_i + iv_q). \end{aligned} \tag{2}$$

Calculating the time derivative of the input and output and substituting them into Eq. (1) leads to

$$\begin{aligned} &\ddot{V}e^{i\phi} + i2\omega\dot{V}e^{i\phi} - \omega^2 Ve^{i\phi} + 2\zeta\omega_0\dot{V}e^{i\phi} \\ &+ i2\zeta\omega_0\omega Ve^{i\phi} + \omega_0^2 Ve^{i\phi} = \\ &2\gamma\zeta\omega_0(i\omega e^{i(\theta+\phi)}(v_i + iv_q) + e^{i(\theta+\phi)}(\dot{v_i} + i\dot{v_q})). \end{aligned} \tag{3}$$

Removing $e^{i\phi}$ and using $\zeta\omega_0 = \frac{1}{\tau}$ gives

$$\begin{aligned} &\ddot{V} + i2\omega\dot{V} + V(\omega_0^2 - \omega^2) + \frac{2}{\tau}\dot{V} + \frac{i2}{\tau}\omega V \\ &= \frac{2\gamma e^{i\theta}}{\tau}\left(i\omega(v_i + iv_q) + \dot{v}_i + i\dot{v}_q\right). \end{aligned} \tag{4}$$

Assuming that voltage variations are much slower than an RF cycle ($\dot{V} \ll \omega V$) and using the approximation $\omega_0^2 - \omega^2 \approx -2\omega(\omega - \omega_0)$ leads to

$$V(\frac{i2\omega}{\tau} - 2\omega\Delta\omega) + i2\omega\dot{V} = \frac{2\gamma e^{i\theta}}{\tau}\left(i\omega(v_i + iv_q)\right) \tag{5}$$

Multiplying both sides by $\frac{-i\tau}{2\omega}$,

$$\tau\dot{V} + V(1 + i\tau\Delta\omega) = \gamma e^{i\theta}(v_i + iv_q). \tag{6}$$

We will now assume that δv_i and δv_q are independent variables in the control loop, and δV and $\delta\omega$ are dependent variables as follows

$$\begin{aligned} &\tau(\dot{V} + \delta\dot{V}) + (V + \delta V)(1 + i\tau(\omega + \delta\omega) - i\tau\omega_0) \\ &= \gamma e^{i\theta}(v_i + \delta v_i + iv_q + i\delta v_q). \end{aligned} \tag{7}$$

Subtracting Eq. (6) from Eq. (7), ignoring $\delta V\delta\omega$ and applying Laplace transform

$$\begin{aligned} &s\tau\delta V + \delta V + i\tau\Delta\omega\delta V + iV\tau\delta\omega \\ &= \gamma(\cos\theta\delta v_i - \sin\theta\delta v_q + i(\cos\theta\delta v_q + \sin\theta\delta v_i)). \end{aligned} \tag{8}$$

The real and imaginary parts of Eq. (8) can be separated and written in matrix form as follows

$$\begin{bmatrix} s\tau + 1 & 0 \\ \tan\phi & \tau V \end{bmatrix} \begin{bmatrix} \delta V \\ \delta\omega \end{bmatrix} = \gamma \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \delta v_i \\ \delta v_q \end{bmatrix}, \tag{9}$$

where $\tan\phi = \tau\Delta\omega$. Assuming $V \neq 0$ we have

$$\begin{bmatrix} \delta V \\ \delta\omega \end{bmatrix} = \begin{bmatrix} \gamma\frac{\cos\theta}{s\tau+1} & -\gamma\frac{\sin\theta}{s\tau+1} \\ \frac{\gamma}{\tau V}\left(\sin\theta - \frac{\tan\phi\cos\theta}{s\tau+1}\right) & \frac{\gamma}{\tau V}\left(\cos\theta + \frac{\tan\phi\sin\theta}{s\tau+1}\right) \end{bmatrix} \begin{bmatrix} \delta v_i \\ \delta v_q \end{bmatrix}. \tag{10}$$

Assume that we have chosen $\theta = \phi = 0$ so that the loop frequency ω tracks the resonator frequency ω_0, in order to have maximum voltage at the cavity. Therefore, Eq. (10) will be simplified as

$$\begin{bmatrix} \delta V \\ \delta\omega \end{bmatrix} = \begin{bmatrix} \frac{\gamma}{s\tau+1} & 0 \\ 0 & \frac{\gamma}{\tau V} \end{bmatrix} \begin{bmatrix} \delta v_i \\ \delta v_q \end{bmatrix}. \tag{11}$$

Since the instantaneous frequency of the loop ω is derivative of the phase of the loop signal, or $\delta\omega = s\delta\phi$, Eq. (11) can be written in terms of amplitude and phase of the resonator as follows

$$\begin{bmatrix} \delta V \\ \delta\phi \end{bmatrix} = \begin{bmatrix} \frac{\gamma}{s\tau+1} & 0 \\ 0 & \frac{\gamma}{sV\tau} \end{bmatrix} \begin{bmatrix} \delta v_i \\ \delta v_q \end{bmatrix}. \tag{12}$$

Therefore, two approximately decoupled control loops can be assumed for amplitude and phase given by $\delta V = \frac{\gamma}{s\tau+1}\delta v_i$ (a first order low pass filter) and $\delta\phi = \frac{\gamma}{sV\tau}\delta v_q$ (an integrator), respectively.

Iterative Learning Control

A block diagram of the ILC controller is shown in Figure 2. The output of the ILC at the $j+1$ iteration is u_{j+1} and its inputs are the error from the previous iteration e_j and the ILC output from the previous iteration u_j. The data from the previous iteration is stored in memory blocks. A common ILC updating law is [4]

$$u_{j+1}(k) = Q(q)[u_j(k) + L(q)e_j(k+1)], \tag{13}$$

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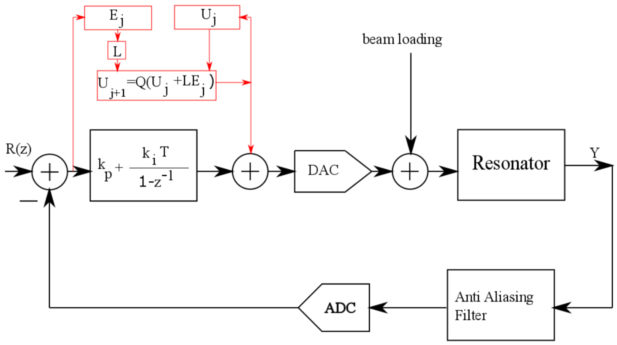

Figure 2: iterative learning control block diagram in the control loop.

where k is the time index, q is the forward time-shift operator, $Q(q)$ is the Q-filter and $L(q)$ is defined as the learning function. The control law in Eq. (13) uses the control output at the previous iteration at the same time ($u_j(k)$), and error at the previous iteration corresponding to *future* time ($e_j(k+1)$). Since it is assumed that the error is repeating, and the disturbance is not varying from iteration to iteration, the learning function $L(q)$ can be either causal or non-causal. A non-causal learning function utilizes the "future" values of error to proactively cancel its effect.

SIMULATION RESULTS

The block diagram in Figure 2 was implemented on Simulink. Two decoupled control loops were assumed for controlling amplitude and phase. In the amplitude control loop, the transfer function of the plant (the resonator is Figure 2) is $\frac{a}{s+a}$; whereas in the phase control loop, it is $\frac{1}{as}$. The cutoff frequency of the anti-aliasing filter is assumed to be 10 times the resonator bandwidth to minimize its effect on the loop signal. The plant and the anti-aliasing filter are simulated in continuous time, and the PI and ILC controller are simulated in discrete time domain. The sampling time is assumed to be $T = 1$. Beam loading is modelled as an input disturbance, a square wave with duty factor of $\frac{1}{2}$ and period of 200 seconds. Iteration length is equal to the period of beam loading. Assuming $a = 0.1$, the gain of the PI controller for the amplitude and phase control loops is shown in Table 1.

Table 1: PI Parameters for Amplitude and Phase Control Loops

	Amplitude loop	Phase loop
P	2.82	16.32
I	0.37	0.30

The simulation results for the amplitude and phase control loops ares shown in Figures 3 and 4, respectively, for

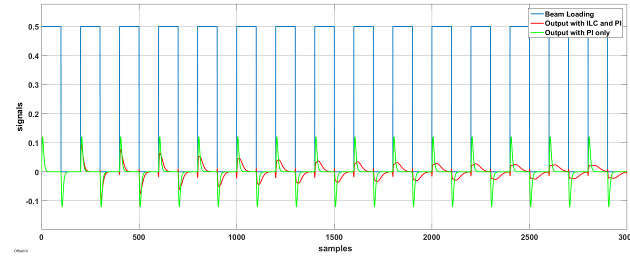

Figure 3: Simulation results for the amplitude control loop.

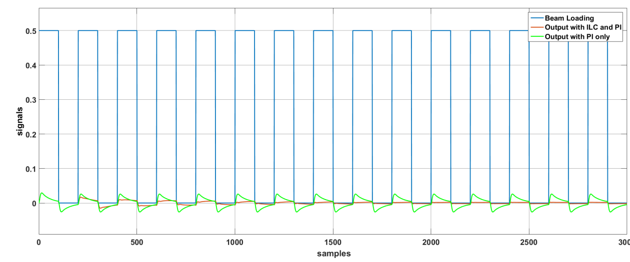

Figure 4: Simulation results for the phase control loop.

15 iterations. The signal shown in blue is the beam loading. The green signal is the output of the anti-aliasing filter when the ILC controller is not connected. It shows how the system would respond with only the PI controller in the loop. The red curve shows the system response with the ILC in the loop. The figure shows that the ILC controller can reduce the error at the edges of beam loading to 0.17 for amplitude and 0.08 for phase, compared to the PI. The learning filter used in this simulation is an average of the next three "future" error samples, and $Q = 0.95$.

These simulation results show that the controller improves its performance at each iteration by using the error information from the previous trial; whereas a non-learning controller, like the PI, results in the same error every iteration.

CONCLUSION

Beam loading effect is a repetitive disturbance and a feed forward controller can be used to deal with it faster than a feedback loop. Iterative learning control is an open loop control strategy that uses the error information from the previous iteration to improve the control output at the current iteration. In this work, ILC was implemented on amplitude and phase control loops in SEL, and the simulation results showed that the system performance was improved significantly within the first 15 iterations.

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