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EMITTANCE PRESERVATION IN PLASMA-BASED ACCELERATORS WITH ION MOTION*

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Abstract

In a plasma-accelerator-based linear collider, the density of matched, low-emittance, high-energy particle bunches required for collider applications can be orders of magnitude above the background ion density, leading to ion motion, perturbation of the focusing fields, and, hence, to beam emittance growth. By analyzing the response of the background ions to an ultrahigh density beam, analytical expressions, valid for non-relativistic ion motion, are obtained for the perturbed focusing wakefield. Initial beam distributions are derived that are equilibrium solutions, which require head-to-tail bunch shaping, enabling emittance preservation with ion motion.

INTRODUCTION

Plasma accelerators (PAs) have received substantial interest because of their ability to produce large accelerating gradients, orders of magnitude larger than that in conventional accelerators, enabling compact accelerating structures [1–3]. The rapid development and properties of PAs make them interesting candidates for applications to future high-energy linear colliders (LCs) [4–6]. It has been anticipated that future LCs will require a center-of-mass energy $\gtrsim 1$ TeV and a luminosity $\gtrsim 10^{34}\,\mathrm{cm^{-2}s^{-1}}$ [7]. This implies using bunches with $N_b \sim 10^{10}$ particles and normalized horizontal and vertical emittances such that $(\varepsilon_{n,x}\varepsilon_{n,y})^{1/2} \ll 1\,\mu\mathrm{m}$ in order to guarantee a sufficiently small bunch size at the interaction point [5, 6].

Preservation of the bunch emittance during the acceleration process, which usually requires cascading several PA stages, is of fundamental importance to the viability of a PA-based LC. Emittance preservation is ensured by matching the bunch in the plasma. This is possible because the transverse wakefield in a PA operating in the blowout regime [1], or in the quasi-linear regime using a near-hollow plasma channel [8], varies linearly with the transverse position. Denoting by $\mathbf{W}_\perp$ the amplitude of the transverse wakefield (the force experienced by a relativistic bunch electron is $\mathbf{F}_\perp \simeq -e\mathbf{W}_\perp$, where $W_x = E_x - B_y$, $W_y = E_y + B_x$, and where $E_{x,y}$ and $B_{x,y}$ are the transverse electric and magnetic fields in the wake), we have

$$\frac{\mathbf{W}_\perp}{E_0} = \frac{k_p \mathbf{r}}{2}, \tag{1}$$

here $\mathbf{r} = (x, y)$ represents the transverse coordinates, $k_p = \omega_p/c$, and $E_0 = mc\omega_p/e$, where $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency, n_0 the plasma density, c the speed of light, and e (m) the electron charge (mass). Using Eq. (1), the matched rms bunch sizes are $\sigma_{x[y]} = k_p^{-1/2}(2\varepsilon^2_{n,x[y]}/\gamma_b)^{1/4}$, where $\gamma_b \gg 1$ is the beam energy normalized to mc^2.

The linear dependence of the transverse wakefield on the transverse coordinates relies on the assumptions that the background ion distribution is uniform and stationary. However, as the bunch accelerates and the matched beam sizes adiabatically decrease, the bunch density, $n_{b,0}$, increases, and so does the amplitude of the bunch space-charge fields. When such fields become large enough so that the ions move significantly during the bunch transit, the transverse wake can be strongly perturbed (e.g., the wakefield strength acquires a nonlinear dependence from the transverse coordinates and changes slice-by-slice along the bunch), resulting in a potentially severe emittance degradation [9–11]. This is anticipated to occur for

$$\Gamma \equiv Z_i \frac{m}{M_i}\frac{n_{b,0}}{n_0}(k_p L_b)^2 \sim 1, \tag{2}$$

where Z_i is the ion charge state, M_i the ion mass, and L_b the bunch length [11].

Ion motion and the related emittance growth is potentially a serious issue for future PA-based LCs. For instance, for the multi-TeV beam-driven plasma-wakefield LC design presented in Ref. [6], the bunch parameters are $N_b = 10^{10}$, $L_b \simeq 20\,\mu\mathrm{m}$, $\varepsilon_{n,x} = 10\,\mu\mathrm{m}$, $\varepsilon_{n,y} = 35\,\mathrm{nm}$, and the bunch energy in the first PA stage ($n_0 \simeq 10^{17}\,\mathrm{cm^{-3}}$, blowout regime) is 25 GeV. We obtain $\sigma_x \simeq 1\,\mu\mathrm{m}$, $\sigma_y \simeq 60\,\mathrm{nm}$, yielding $n_{b,0}/n_0 \simeq 12000$, and so $\Gamma \simeq 10$ for a Hydrogen ion background. We expect ion motion to be important in this case. Numerical modeling of beam evolution for these parameters performed with the code INF&RNO [12] shows that the projected rms bunch emittance increases by 20% after a 2 cm propagation in plasma [11]. Furthermore, the slice-dependent nature of the wake perturbation causes the saturated (i.e., after mixing) bunch emittance to be slice-dependent (i.e., the emittance of a bunch slice located towards the tail of the bunch is higher than that of a slice towards the head of the bunch).

Bunch emittance preservation in the presence of ion motion is a challenging task. In Ref. [11] it was proposed to longitudinally tailor the transverse bunch profile and phase-space in such a way that, even though ion motion is enabled, the bunch transverse distribution at each longitudinal location is an equilibrium solution, and so the bunch remains matched at all times. Achieving this requires that the transverse phase-space distribution of the bunch is a stationary solution of the Vlasov equation. Numerical examples on how to construct such equilibrium solutions were discussed in Ref. [11]. Exact equilibrium solutions completely eliminate bunch emittance growth in presence of ion motion.

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In these proceedings we present and discuss analytical results concerning the construction of the equilibrium bunch distribution valid in the non-relativistic ion motion limit.

TRANSVERSE WAKEFIELD IN PRESENCE OF ION MOTION

General analytical expressions for the transverse (confining) wakefield perturbed by ion motion, valid in the non-relativistic ion motion limit, are given in Ref. [11]. In these proceedings we consider a symmetric (round) bunch with a density profile of the form $n_b(\zeta, r) = n_{b,0}\, g_{\|}(\zeta) g_{\perp}(r;\zeta)$, where $\zeta = z - ct$ is the co-moving longitudinal coordinate (z and t being, respectively, the longitudinal coordinate and the time), $r = |\mathbf{r}| = (x^2 + y^2)^{1/2}$, $g_{\|}(\zeta)$ and $g_{\perp}(r;\zeta)$ describe, respectively, the longitudinal and the $\zeta-$dependent transverse profile of the bunch. We assume that the bunch head is located at $\zeta = 0$ and that the bunch extends for $\zeta < 0$. We require that, for any longitudinal slice, $\int_0^\infty g_{\perp}(r;\zeta) r dr = \int_0^\infty g_{\perp}(r;\zeta = 0) r dr$, so the bunch current profile only depends on the choice of $g_{\|}(\zeta)$, and this can be arbitrary. In this case the perturbed wakefield is axisymmetric, i.e., $\mathbf{W}_{\perp} = W_r \hat{\mathbf{r}}$, $\hat{\mathbf{r}}$ being the transverse radial versor, and within the bunch region we have

$$\frac{W_r}{E_0} = \frac{k_p r}{2} - Z_i \frac{m}{M_i} \frac{n_{b,0}}{n_0} \frac{k_p^3}{r} \int_{\zeta}^{0} d\zeta' (\zeta - \zeta') g_{\|}(\zeta') \times \int_0^r g_{\perp}(r';\zeta') r' dr'. \quad (3)$$

The first term in Eq. (3) is the unperturbed wake due to uniform ions (see Eq. (1)). The second term is the wake perturbation due to the bunch-induced ion motion. The analytical expression Eq. (3) is valid for $\Gamma \lesssim 1$ (i.e., non-relativistic ion motion limit).

For a bunch with an uniform longitudinal current profile, i.e., $g_{\|}(\zeta) = 1$ for $-L_b \le \zeta \le 0$, and zero elsewhere, and with an uniform transverse density profile with a slice-dependent bunch radius, i.e., $g_{\perp}(r;\zeta) = [R_0/R(\zeta)]^2 \Theta[R(\zeta) - r]$, where $R(\zeta)$ is the $\zeta-$dependent bunch radius, $R_0 = R(\zeta = 0)$, and where $\Theta(\cdot)$ is the Heaviside function, we have

$$\frac{W_r}{E_0} = \frac{k_p r}{2} \left\{ 1 - Z_i \frac{m}{M_i} \frac{n_{b,0}}{n_0} \frac{k_p^2}{r^2} \times \int_{\zeta}^{0} d\zeta' (\zeta - \zeta') \left[\frac{R_0}{R(\zeta')} \right]^2 \min[r^2, R^2(\zeta')] \right\}. \quad (4)$$

We note that within the bunch and for a bunch with a decreasing radius from head to tail (i.e., $\partial_\zeta R \ge 0$) Eq. (4) simplifies to

$$\frac{W_r}{E_0} = \frac{k_p r}{2} \Lambda^2(\zeta), \quad (5)$$

where

$$\Lambda^2(\zeta) = 1 + \frac{\Gamma}{L_b^2} \int_{\zeta}^{0} d\zeta' (\zeta' - \zeta) \left[\frac{R_0}{R(\zeta')} \right]^2, \quad (6)$$

and so, even in presence of ion motion, the focusing wake retains a linear dependence on the transverse coordinate, but the focusing gradient increases going from the head towards the tail of the bunch ($\partial_\zeta \Lambda^2 \le 0$). For a constant bunch radius, namely $R(\zeta) = R_0$, we have $\Lambda^2(\zeta) = 1 + Z_i (m/M_i)(n_{b,0}/n_0)(k_p \zeta)^2/2$.

BUNCH EQUILIBRIUM SOLUTION WITH ION MOTION

Owing to the slice-dependent nature of the ion-motion-induced wake perturbation, achieving matched bunch propagation in presence of ion motion requires a proper longitudinal tailoring of the transverse phase space properties of the bunch [11]. More specifically, at any longitudinal location, the transverse phase-space distribution of the bunch must be a stationary solution of the Vlasov equation.

Denoting by $f_{\perp}(\mathbf{r}, \mathbf{u}; \zeta, z)$ the slice-dependent transverse phase-space bunch distribution ($\mathbf{u}$ is the electron momentum normalized to mc), the Vlasov equation reads $df_{\perp}/dz = \partial_z f_{\perp} + \{f_{\perp}, H_{\perp}\} = 0$. Here $H_{\perp} = (u_x^2 + u_y^2)/2\gamma_b + eU_{\perp}/mc^2$ is the slice-dependent transverse Hamiltonian of the bunch, where the potential $U_{\perp}$ satisfies $\mathbf{W}_{\perp} = \nabla_{\perp} U_{\perp}$, and $\{\cdot,\cdot\}$ are the Poisson brackets. Note that since the bunch is highly relativistic the particle transverse motion in each slice of the bunch is decoupled from the motion in other slices (no longitudinal particle slippage). Stationary solutions ($\partial_z f_{\perp} = 0$) to the Vlasov equation are, by construction, distributions of the form $f_{\perp}(\mathbf{r}, \mathbf{u}; \zeta) \propto F[H_{\perp}(\mathbf{r}, \mathbf{u}; \zeta)/H_0(\zeta)]$, where F is any positively defined function such that $\mathcal{N}(\zeta) = \iint f_{\perp}(\mathbf{r}, \mathbf{u}; \zeta) d^2\mathbf{r} d^2\mathbf{u} < +\infty$, and $H_0(\zeta)$ is a $\zeta-$dependent scale parameter used to control the local properties of the distribution. The phase-space moments at any slice are $\overline{x}^2 = \mathcal{N}^{-1} \iint x^2 f_{\perp}(\mathbf{r}, \mathbf{u}; \zeta) d^2\mathbf{r} d^2\mathbf{u}$, $\overline{u}_x^2 = \mathcal{N}^{-1} \iint u_x^2 f_{\perp}(\mathbf{r}, \mathbf{u}; \zeta) d^2\mathbf{r} d^2\mathbf{u}$, and $\overline{x u_x} = 0$ (because of the symmetry properties of the stationary distribution function). The slice emittance is then $\epsilon_n^2(\zeta) = \overline{x}^2(\zeta) \overline{u}_x^2(\zeta)$. We require that the slice emittance is constant along the bunch, namely $\overline{x}^2(\zeta)\overline{u}_x^2(\zeta) = \overline{x}^2(\zeta = 0)\overline{u}_x^2(\zeta = 0)$. This can be enforced by properly choosing the scale parameter $H_0(\zeta)$. We note that since the knowledge of the phase-space distribution relies on the knowledge of the Hamiltonian, which depends on the wakefield that, in turns, is determined by the bunch distribution and ion dynamics via Maxwell's equations, obtaining an explicit expression for the bunch and wake quantities requires solving self-consistently the coupled set of Maxwell-Vlasov equations.

As a completely analytically tractable example we will study the matched solution obtained taking $F(q) = \delta(q - 1)$, where $\delta(\cdot)$ is the Dirac delta function. Furthermore, we consider an uniform longitudinal current profile. Generalization to arbitrary profiles is straightforward. By projecting the phase-space distribution onto physical space and taking into account the proper normalization we find that the transverse bunch density distribution at any given longitudinal location

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is uniform within a disk, namely

$$g_\perp(r;\zeta) = \frac{R_0^2}{R^2(\zeta)}\Theta\left[H_0(\zeta) - \frac{eU_\perp(r;\zeta)}{mc^2}\right], \quad (7)$$

where the slice-dependent bunch radius, $R(\zeta)$, is implicitly defined by the equation $eU_\perp[R(\zeta);\zeta]/mc^2 = H_0(\zeta)$ [as before, $R_0 = R(\zeta = 0)$], and the transverse wake within the bunch including ion motion is given by Eq (4).

In order to simplify the calculations, we make the assumption that the bunch equilibrium radius in presence of ion motion is decreasing going from the head to the tail of the bunch (this assumption is consistent with the findings presented in Ref. [11]). In this case, using Eq. (5) we have $eU_\perp(r;\zeta)/mc^2 = [k_p r\Lambda(\zeta)]^2/4$, and so $H_0(\zeta) = [k_p R(\zeta)\Lambda(\zeta)]^2/4$. With this assumption, and by projecting the phase-space distribution onto momentum space, we find that at any location within the bunch the transverse particle momentum distribution is isotropic and the momentum of each particle is such that $|\mathbf{u}| = u_0(r;\zeta)$, where $u_0^2(r;\zeta) = (\gamma_b/2)\Lambda^2(\zeta)k_p^2[R^2(\zeta) - r^2]$.

Evaluation of the slice-dependent second-order phase-space moments of the bunch yields, $\overline{x}^2(\zeta) = R^2(\zeta)/4$, and $\overline{u}_x^2(\zeta) = \gamma_b[k_p R(\zeta)\Lambda(\zeta)]^2/8$. Imposing a constant slice emittance along the bunch gives $\epsilon_n^2 = \gamma_b[k_p R^2(\zeta)\Lambda(\zeta)]^2/32 = \gamma_b[k_p R_0^2]^2/32$, and this requires the bunch radius to be tapered according to

$$R(\zeta) = R_0/\Lambda^{1/2}(\zeta)\,. \quad (8)$$

Even though $\Lambda(\zeta)$ depends on $R(\zeta)$ (see Eq. (6)), an explicit solution to Eq. (8) can be obtained recursively starting from the head of the bunch ($\zeta = 0$), and then progressively computing the value of the bunch radius for slices gradually closer to the tail of the bunch. Note that since $\partial_\zeta\Lambda^2 \le 0$, the assumption of a decreasing bunch radius going from the head towards the tail of the bunch is certainly verified. For the matched bunch solution the total projected rms emittance differs from the slice emittance owing to the fact that the second order phase space moments are slice-dependent. An approximate solution to Eq. (8), valid for a mild tapering, is given by

$$\begin{aligned}\frac{R(\zeta)}{R_0} &\simeq \frac{3}{4} + \frac{1}{4}\cos\left[\left(Z_i\frac{m}{M_i}\frac{n_{b,0}}{n_0}\right)^{1/2} k_p\zeta\right] \quad (9)\\ &\simeq 1 - \frac{Z_i}{8}\frac{m}{M_i}\frac{n_{b,0}}{n_0}(k_p\zeta)^2.\end{aligned}$$

As an example, in Fig. 1 we show (black) a plot of the bunch radius along the bunch, $R(\zeta)/R_0$, for the case $n_{b,0}/n_0 = 500$, $k_p L_b = 2$ (Hydrogen ions background). The solid line is the numerical solution to Eq. (8), the dashed line is the approximate analytical solution Eq. (9). The red plot shows the slice-dependent rms momentum, $\left[\overline{u}_x^2(\zeta)/\overline{u}_x^2(\zeta = 0)\right]^{1/2}$.

CONCLUSION

We have presented analytical results describing the construction of a stationary equilibrium distribution for a bunch in a PA in presence of ion motion. This solution, valid in the limit of non-relativistic ion motion, is an exact stationary solution of the Vlasov equation and, hence, completely eliminates ion-motion-induced emittance growth. Generalization of these results to the nonlinear regime, where ion motion becomes relativistic, is straightforward but requires a numerical evaluation of the ion-motion induced wake perturbation.

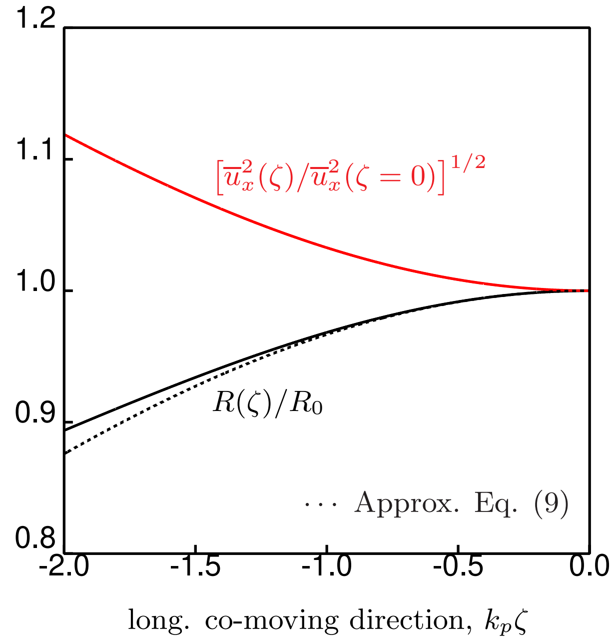

Figure 1: Example of matched equilibrium beam distribution. Black: bunch radius along the bunch, $R(\zeta)/R_0$, for the case $n_{b,0}/n_0 = 500$, $k_p L_b = 2$ (Hydrogen ions background). The solid line is the numerical solution to Eq. (8), the dashed line is the approximate analytical solution Eq. (9). Red: slice-dependent rms momentum, $\left[\overline{u}_x^2(\zeta)/\overline{u}_x^2(\zeta = 0)\right]^{1/2}$.

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