# CORRECTOR LAYOUT OPTIMIZATION USING NSGA-II FOR HALS* 

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## Abstract

In this paper, we present a method to find the global optimum correctors layout based NSGA-II when the number of correctors is limited to be equal to the number of BPMs. We prove that this method works well with HALS.

## INTRODUCTION

The Hefei Advanced Light Source (HALS) project [1], as a soft X-ray diffraction limited storage ring (DLSR), was proposed by National Synchrotron Radiation Laboratory with a beam energy of 2.4 GeV and the detailed lattice design is still in progress. The detailed parameters of HALS can refer to [2].

In order to achieve an ultra-low emittance, HALS adopts strong quadrupoles to depress dispersion function and strong sextupoles to perform chromaticities correction in a multi-bend achromat (MBA) cell. Feeddown in such strong magnets gives rise to changes in the optics, dispersion and beam coupling, etc. Therefore, centering of the closed orbit in the small aperture magnets necessitates a proper beam diagnostics and correction layout.

According to the Closed Orbit Distortion (COD) formula,

$$
\begin{equation*}
u(s)=\theta \frac{\sqrt{\beta\left(s_{0}\right) \beta(s)}}{2 \sin \pi v} \cos \left(\pi v-\left|\psi(s)-\psi\left(s_{0}\right)\right|\right) \tag{1}
\end{equation*}
$$

the correctors at higher $\beta$ function theoretically have greater abilities to control the closed orbit. However, where a maximum $\beta_{x}$ is taken, a minimum $\beta_{y}$ is also taken and vice versa. It is impossible that a global correction scheme has optimal correction capability in two transverse directions simultaneously. NSGA-II is introduced ro resolve the conflict.

NSGA-II is a highly effective randomly searching algorithm, which can globally search a set of solutions over a domain [3]. These solutions are superior to the rest of solutions when considering all of the objectives, which are called Pareto-optimal solutions.
In this paper, we develop a method to correct the closed orbit using NSGA-II. The goal is to control the residual closed orbit within an acceptable level in two directions. A reasonable optimization objective is proposed and its validity is carried out through statistical analysis and error analysis.

## DIPOLE ERRORS ESTIMATION AND BEAM POSITION MONITORS LAYOUT

The magnet elements of a storage ring can never be placed at their ideal positions. To simulate a real ma-

[^0]chine, we have to assume a statistical variation of their positions. The orbit distortion is caused by dipole errors which can be produced by bending magnets tilt, bending magnets strength or length error and transverse misalignment of quadrupoles, etc. Taking technology limitation into account, Table 1 summarizes all dipole errors used in the simulation.

Table 1: Dipole Error Sheet for Magnets. All Values are RMS, and the Truncation is $3 \sigma$

|  | $\Delta X / Y$ <br> $(\mu \mathrm{~m})$ | $\Delta Z$ <br> $(\mu \mathrm{~m})$ | $\Delta \theta_{z}$ <br> $(\mathrm{mrad})$ | FSE <br> $\left(10^{-3}\right)$ | Multi <br> $\left(10^{-4}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Girder | 50 | 150 | 0.1 | - | - |
| Dipole | 30 | 100 | 0.2 | 0.5 | 2.0 |
| Quad | 30 | 100 | 0.2 | 1.0 | 2.0 |
| Sext | 30 | 100 | 0.2 | 10 | 2.0 |

Once a closed orbit is established, it is measured by a large number of Beam Position Monitors (BPM) and small corrector magnets are used to correct the closed orbit towards the ideal orbit. The present HALS optics and BPM locations are shown in Fig. 1. There are 5 girders for each


Figure 1: One arc section of the newest HALS lattice with a schematic view of the magnets at the bottom. In the bottom, the blue, red and cyan squares represents defocussing combined function dipoles, pure quadrupoles and sextupoles respectively. The black dots on the base line represents BPMs.
of the 32 storage ring cells. Each girder supports 6 or 7 magnets. The BPMs are placed as following the guidelines. Two BPMs are at both ends of each insertion straight section, and two BPMs are close to the ends of the middle dipole magnet in order to provide local COD adjustment. The rest four BPMs are as close as possible to the quadrupoles or the sextupoles, where misalignments are sources of orbit distortion or dynamic aperture reduction. As a result, the set of 8 BPMs per cell, as shown in Fig. 1 and Fig. 2,
has been adopted, in order to minimize their total number $\ddot{\oplus}(8 \times 32=256)$.


Figure 2: Phase advance in one cell. The left is phase advance as a function of longitudinal position, and the BPMs are visualized as inverted triangles. The right is phase advance between a BPM and its previous one.

## OPTIMIZATION OBJECTIVE AND ALGORITHM DESCRIPTION

It is nature to use the maximum COD along the ring as the optimization objective. However, to ensure the optimization validity, a statistical anslysis must be carried out which will take so much time that it is impossible to modify some parameters in the algorithm.

## Optimization Objective

To find a reasonable optimization objective, review of correction algorithm is needed. In this paper, only Singular Value Decomposition (SVD) theory is considered [4]. SVD decomposes the corrector to BPM response matrix into a product of two unitary matrices and a diagonal matrix,

$$
\begin{equation*}
\mathbf{R}=\mathbf{U} \cdot \mathbf{W} \cdot \mathbf{V}^{\mathbf{T}} \tag{2}
\end{equation*}
$$

The columns of the matrices $\mathbf{U}$ and $\mathbf{V}$ are the orthogonal basis vectors of BPMs space and correctors space. The main diagonal elements of the matrix $\mathbf{W}$ are called eigenvalues The smaller an eigenvalue is, the less effect of the corresponding vector in correctors space has. In accelerator simulation tools, such as Elegant and AT, too small eigenvalues are eliminated to prevent correctors strength exceeding threshold. In other words, the smaller of the ratio is,

$$
\begin{equation*}
\varepsilon_{m}=\frac{\min (\mathbf{W})}{\max (\mathbf{W})} \tag{3}
\end{equation*}
$$

the fewer eigenvalues in closed orbit correction are used. Therefor, the residual COD are bigger. In our case, we will use the same set of correction scheme to correct both horizontal and vertical directions. As a result, $\varepsilon_{m x}$ and $\varepsilon_{m y}$ are used as optimization objective and a multiobjective genetic algorithm is applied.

## Coding

A corrector could be a dedicated magnet or a correction coil equipped on multipole magnets. From Fig. 1, every drift apart from the long straight section is so short enough that the change of phase advance in two directions is very slow. There is no difference in correction ability when a corrector is placed at any position in a short drift. A corrector combined a sextupole can also be represented by its

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adjacent drifts. Considering the limitation of the installation space, only drifts longer than 0.1 m will be considered. In conclusion, there are 32 optional corrector locations. We use a 32 bits data to describe a set of correctors. " 1 " means that there is a corrector located at the corresponding position, while " 0 " means empty on the contary. Number of correctors or bit " 1 " is 8 , the same as the number of BPMs.

## Mutation

A mutation operation is defined as moving a corrector randomly selected from one place to another. That is exchange one bit " 0 " with one bit " 1 " in a 32 bits binary number. An example is listed below for illustration.

## Before: 00100000010001000100001110001000 After : 00100000010001000100011100001000

Here, the red bits are mutation positions.

## Crossover

To ensure that the two sets of correction scheme are still effective after crossing, the choice of intersection points is not arbitrary. From view of binary number, the total number of bits " 1 " after crossing are still the same. Another example is listed below for illustration too.

$$
\begin{aligned}
& 01000000001100010100000101001000 \\
& 00000010001001101010000000100010
\end{aligned}
$$

Here, every red bit is a possible intersection point. To avoid invalid operation, every continuous red slice is considered one intersection point and red bits at both ends are considered in the same slice. Every slice is chosen randomly. In the above instance, there are 7 possible intersection points in total.

## OPTIMIZATION RESULTS

Parameters used in our optimization are listed in Table 2. The size of the final pareto front is 31 , and the front is shown

Table 2: Parameters Used In NSGA-II

| parameter | value |
| :---: | :---: |
| population size | 250 |
| evolution time | 100 |
| mutation rate | 0.05 |
| crossover rate | 0.90 |

in Fig. 3. We choose 3 typical sets of correctors scheme to verify the correction effect. The selected scheme are labeled red in Fig. 3, and are numbered from "\#1" to "\#3" respectively. The corresponding lattice layouts are shown in Fig. 4.

## SIMULATION RESULTS

All of the simulation are done with elegant [5]. Under the error set shown in Table 1, the statistical results of the maximum closed orbit distortion all over the ring are shown in

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Figure 3: The final pareto front distribution in optimization objective space.


Figure 4: Lattice layout of different set of correction scheme. The blue dotted line is where correctors locate. The other colored squares are the same as in Fig. 1. The number of correctors are the same with the BPMs.

Fig. 5. Up to 2000 groups of random errors are used. Without correction, the maximum horizontal COD is around 5 mm , while the vertical is around 8 mm . The both are all


Figure 5: Max-COD before correction.
beyond the current HALS dynamical aperture. After correction, the statistical results are shown in Fig. 6. All three sets of correction scheme meet the requirements of orbit correction. The CODs in two directions are both well controlled and the kick angles of correctors are appropriate. However, their correction capacities are different. The \#1 scheme is most suitable for the correction of the horizontal direction.

## REFERENCES

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Figure 6: The maximum COD and the maximum corrector strength after correction. The same 2000 groups of random errors in Fig. 5 are used.
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