# A POSSIBLE SCHEME FOR GENERATING HIGH-HARMONIC COHERENT RADIATION IN STORAGE RINGS\*

X. Wang <sup>1†</sup>, C. Feng, Z. Zhao, Shanghai Institute of Applied Physics, 201800 Shanghai, China <sup>1</sup>also at SLAC National Accelerator Laboratory, 94025 Menlo Park, USA

## Abstract

We propose a possible technique for generating highharmonic coherent radiation in storage rings. The technique takes advantage of a dogleg and a wave-front tilted seed laser to make "z-p-x" three-dimensional manipulation of the electron beam phase space. Compared with existing frequency up-conversion schemes, like coherent harmonic radiation (CHG) [1–3], echo-enabled harmonic generation (PEHG) [4, 5], phase-merging enhanced harmonic generation (PEHG) [6], the proposed scheme provides the minimum energy modulation amplitude, thus can significantly decrease the damping time to recover the quality of the electron beams. Simulation shows megawatt-scale level EUV pulses can be generated. This scheme offers an alternative option in SR-based light sources to enhance the average brightness.

## **INTRODUCTION**

Several frequency up-conversion schemes have been proposed, like coherent harmonic radiation (CHG) [1–3], echoenabled harmonic generation (EEHG) [4,5], phase-merging enhanced harmonic generation (PEHG) [6]. These beam manipulation techniques use lasers to modulate the electron beam so the electron beam is strongly bunched. Then the electron beam emits strong high-harmonic radiation in the undulator.

Generating high harmonic and strong coherent radiation in storage rings has been of great interests in the accelerator community [7, 8]. Therefore it is inevitable that these technologies are transplanted into the storage rings.

However, storage rings are self-stabilizing systems. They have specific requirements for the quality of electron beam. Transplanting frequency up-conversion schemes into storage rings requires minimal damage to the quality of electron beam, especially the energy change. Large energy modulation will cause distortion of the electron beam phase space due to the presence of residual  $R_{56}$  in the beam-line, or even worse cause beam loss. Then long damping time is required to recover the quality of the electron beam, which may decrease the repetition rate and average brightness of storage rings.

In this paper, we compare proposed frequency upconversion scheme with existing schemes, like CHG, EEHG, PEHG. We try to introduce as small energy modulation depth as possible in the modulation process. Optimization results

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show that the proposed scheme has the minimum energy modulation amplitude.

## THREE-DIMENSIONAL MANIPULATION OF ELECTRON BEAM PHASE SPACE

Figure 1 shows the schematic layout of the proposed scheme. The electron beam traverses a dogleg, and then interacts with a wave-front tilted laser "seed light 1" intersecting at an angle of  $\theta$  in "modulator 1", in which the electron beam is modulated with amplitude  $A_1 = \delta \gamma / \sigma_{\gamma}$ , where  $\delta \gamma$  is the modulation depth induced by "seed light 1",  $\sigma_{\gamma}$  is the initial rms beam energy spread. After electron beam passing through the chicane, it interacts with a normal wave-front forward laser "seed light 2" in "modulator 2", in which the electron beam is modulated with amplitude  $A_2$ . The definition of  $A_2$  is the same as  $A_1$ . After undergoing the second chicane, the electron beam in phase space will be strongly micro-bunched similar to EEHG scheme. Thus can generate coherent harmonic in the radiator.



Figure 1: Schematic layout of the beam line for the proposed technique.

The density modulation of the electron beam that contains high harmonic components is usually quantified by bunching factor. In the following, we'll derive the expression of bunching factor. To simplify the calculation process, we use the dimensionless energy deviation  $p = (\gamma - \gamma_0)/\sigma_{\gamma}$  and dimensionless horizontal position of a particle  $X = x/\sigma_x$ as main variables, where  $\gamma_0$  is the average Lorentz factor,  $\sigma_x$  is the intrinsic horizontal rms beam size. Normally, the definition of bunching factor can be written as:

$$b = \frac{1}{N} \iint dX dP f(X, p) < e^{-i\alpha\xi} >$$
(1)

where *N* is the total number of electrons,  $\xi = k_1 z$  means the longitudinal phase,  $k_1$  is the wave number of the seed light 1,  $< \cdots >$  means taking average of the parameter, f(X, P) is the function of beam distribution:

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$$f(X,P) = \frac{N}{2\pi} e^{-\frac{X^2}{2}} e^{-\frac{P^2}{2}}$$
(2)

The manipulation process of the electron beam in the dogleg, modulator 1, first chicane, modulator 2, second chicane can be respectively written as:

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<sup>&</sup>lt;sup>†</sup> wangxiaofan@sinap.ac.cn

$$X' = X + DP \tag{3}$$

$$P' = P + A_1 \sin(\xi \cos \theta + TX') \tag{4}$$

$$\xi' = \xi + B_1 P' \tag{5}$$

$$P'' = P' + A_2 \sin(K\xi' + \Phi)$$
 (6)

$$\xi'' = \xi' + B_2 P'' \tag{7}$$

9th International Particle Accelerator Conference IP ISBN: 978-3-95450-184-7 1000 IP ISBN: 978-3-95450-184-7 X' = X + DP (3)  $P' = P + A_1 \sin(\xi \cos \theta + TX')$  (4)  $\xi' = \xi + B_1 P'$  (5)  $P'' = P' + A_2 \sin(K\xi' + \Phi)$  (6)  $\xi'' = \xi' + B_2 P''$  (7) Where  $D = \frac{\eta \sigma_{\gamma}}{\sigma_x \gamma}$  is the dimensionless dispersive strength in the horizontal,  $B_{1,2} = R_{56}k_{1,2}\frac{\sigma_{\gamma}}{\gamma}$  is the dimensionless strength of the chicane,  $T = k_1 \sigma_x \sin \theta$  is defined as a di-mensionless wave-front tilt parameter.  $\eta, k_{1,2}, R_{56}$  are re-spectively the dispersion in the dogleg, wave number of the seed light 1 or 2, the momentum compaction in the chicane. After the electron beam traversing the beam line, a strong beam density can be obtained before the entrance of the radiator, shown in Figure 3[a]. We can see that the phase

 $\frac{1}{2}$  radiator, shown in Figure 3[a]. We can see that the phase space is very similar to the EEHG mechanism. The bunching radiator, shown in Figure 3[a]. We can see that the phase factor of the electron beams can be calculated:

$$b_a = \sum_{n,m} e^{-\frac{n^2 T^2}{2}} \cdot e^{-\frac{(C-nTD)^2}{2}} \cdot J_n(-CA_1) \cdot J_m(-aA_2B_2)$$
(8)

distribution of this work where  $a = n \cos \theta + mK$  is the integer referred to as a harmonic order,  $K = k_2/k_1$  is the ratio of wave number of the second seed laser to the first seed light,  $C = n \cos \theta (B_1 + C)$ 2018).  $B_2$ ) + mKB<sub>2</sub>. Because  $\theta$  is normally on the order of  $10^{-3}$ magnitude, n and m can be considered as integers. O

Schemes Comparison In the following, for the sake of the proposed scheme as "TEHG". In the following, for the sake of simplicity, we abbreviate

B We compared CHG, EEHG, PEHG and proposed scheme TEHG to demonstrate which scheme has the ability to achieve minimum energy modulation amplitude.

We use a nominal parameters of a  $3^{rd}$  generation light We use a nominal parameters of a  $3^{rd}$  generation light source as a representative example. The parameters of the electron beam and the seed laser are listed in Table 1. Opti-2 mization and simulation are carried out under these parame-

ters. We set achieving no less than 5% bunching factor for the 20th harmonic as our goal, in the mean time we want to have minimum energy modulation amplitude. We also set þ some limitations, like dispersion of the dogleg  $\eta < 0.1$ m,  $\stackrel{>}{\equiv}$  momentum compaction in the chicane  $R_{56} < 0.1$ m. Under work these limitations, we have B < 2371, D < 10. In the following, unless we specifically define it, the definitions of the Content from this variables are the same as that already defined in this paper.

For CHG, the bunching factor at the *n*-th harmonic is given by:

$$b_n = e^{-\frac{n^2 B^2}{2}} \cdot J_n(-nAB)$$
 (9)

For PEHG, the bunching factor at the *n*-th harmonic is given by:

$$b_n = e^{-\frac{n^2 T^2}{2}} \cdot e^{-\frac{n^2 (B+TD)^2}{2}} \cdot J_n(-nAB)$$
(10)

where T is the dimensionless gradient parameter of TGU.

For EEHG, the bunching factor at the *a*-th harmonic is given by:

$$b_a = \sum_{n,m} e^{-\frac{C^2}{2}} \cdot J_n(-CA_1) \cdot J_m(-aA_2B_2)$$
(11)

where a = n + mK,  $C = nB_1 + (n + mK)B_2$ .

After detailed mathematical analysis, we get the maximum bunching factor as a function of one major constraint for every scheme, as shown in Figure 2.



Figure 2: Maximum bunching factor  $b_{20}$  as functions of energy modulation amplitude A, energy modulation amplitude  $A_1$  in the 1<sup>st</sup> modulator, dimensionless gradient parameter of TGU T, dimensionless wave-front tilt parameter T, respectively for CHG, EEHG, PEHG and TEHG.

For CHG, the main constraint comes from the energy modulation amplitude of the electron beams in the modulator A. If wen want to achieve no less than 5% bunching factor for 20th harmonic, this parameter should not smaller than 13.87. This is an unacceptable quantity, not only from practically reason that huge energy modulation amplitude demands super high brightness seed laser, but also such high

Table 1: Nominal Parameters of Electron Beam and Seed Laser

Parameters	Magnitude
Energy	3.5 GeV
Wavelength of Seed Laser	265 nm
Peak current	100 A
Energy Spread $\frac{\sigma_{\gamma}}{\gamma}$	$10^{-3}$
Transverse Beam Size $\sigma_x$	$10^{-5}$

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energy modulation amplitude will cause phase space distortion of the electron beam because of residual momentum compaction  $R_{56}$  existing in the beam line.

For EEHG, the main constraint comes form the energy modulation amplitude  $A_1$  of the electron beams in the first modulator. If wen want to achieve no less than 5% bunching factor for 20th harmonic, this parameter should not smaller than 0.8. By controlling other variables, we can reduce the energy modulation amplitude  $A_2$  of the electron beams in the second modulator.

For PEHG, the main constraint comes form the dimensionless gradient parameter T of the TGU. If wen want to achieve no less than 5% bunching factor for 20th harmonic, this parameter should not larger than 0.08. Combined with the limitation of the dispersion of the dogleg D no more than 100, we calculate the minimum modulation depth of PEHG, which is 1.4.

For TEHG, the main constraint comes form the dimensionless parameter T. If wen want to achieve no less than 5% bunching factor for 20th harmonic, this parameter should not larger than 1.25.

Table 2: Schemes Comparison (× means the parameter is not existing in the corresponding scheme)

Sch.	D	Т	$A_1$	$B_1$	$A_2$	$B_2$	$b_{20}$
CHG	Х	×	13.87	0.08	×	×	6.76%
EEHG	×	×	0.8	2324	0.01	116	5.39%
PEHG	10	-0.08	1.4	0.79	×	×	6.91%
TEHG	10	1.25	0.15	2336	0.01	116	6.38%

Table 2 shows the optimization results of these four schemes. As we can see from Table 2, compared with other schemes, TEHG has the minimum energy modulation amplitude. This is because TEHG combines the techniques of other schemes and the complex overall design provides more margin to adjust parameters. Thus can achieve minimum energy modulation amplitude by controlling more variables.

In Figure 3, left figure shows the phase space of the electron beam before the entrance of the radiator, which is very similar to EEHG scheme. The right picture shows the bunching factor as a function of the integer of the harmonic number. We can see that at the 20th harmonic, the bunching factor is about 6.4%, same as the result shown in Table 2.



Figure 3: (a) P- $\xi$  phase space before the radiator. (b) Bunching factor as a function of the integer of the harmonic number.

02 Photon Sources and Electron Accelerators A05 Synchrotron Radiation Facilities It is worth emphasizing here that when we replace the wave-front tilted laser modulation section to TGU, we can obtain the same bunching factor after optimization. However, the scheme we proposed in this paper shows more practical potential to generate high harmonic radiation because high gradient TGU is not easily obtained under current technology.

Simulation is carried out with softwares: Matlab and Genesis. We use Genesis to simulate the radiation process, while other process is done by Matlab. Figure 4 shows the growth of radiation power with the wavelength of 13.25nm as a function of the radiator distance. The output peak power at the exit of the radiator approaches 1.6 MW.



Figure 4: Radiation power growth at the radiator.

Combining the advantages of the small distortion of electron beam phase space brought by small energy modulation amplitude, and high radiation pulse power, this technique may be an alternative option in SR-based light sources to enhance the average brightness at short wavelength.

## CONCLUSION

The technique proposed in this paper may be considered as a modified version of EEHG scheme, where a dogleg and a wave-front tilted seed laser is added to make "z-p-x" three-dimensional manipulation of the electron beam phase space. Compared with existing frequency up-conversion schemes, like CHG, EEHG, PEHG, the proposed scheme can provide the minimum energy modulation amplitude, thus can significantly decrease the damping time to recover the quality of the electron beams. Simulation shows megawattscale level EUV pulses can be generated. The scheme offers an alternative option in SR-based light sources to enhance the average brightness. And it may also be used to enhance the frequency up-conversion efficiency of a seeded FEL. Further investigations on this topic are ongoing.

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