ISBN: 978-3-95450-184-7    doi:10.18429/JACoW-IPAC2018-THPAK099

# THE INFLUENCE OF HIGHER ORDER MULTIPOLES OF IR MAGNETS ON LUMINOSITY FOR SuperKEKB

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*Abstract*

In SuperKEKB, the value of beta function is extremely small at interaction point (IP) to achieve higher luminosity. As magnets in interaction region (IR) have very strong focusing effect, they make a large disturbance to beams. Higher order multipoles and their skew components of IR magnets are located at a very high beta section with $\pi/2$ phase difference from IP. Since the beta in IR magnets has largest value, multipole components of x-y coupling can give critical effect to beam dynamics at IP and reduce luminosity on SuperKEKB design. In this study, we calculated beam dynamics for the nonlinear effect of multipole components and estimated the influence on luminosity deteriorated by them. The present result suggested that the unknown luminosity loss calculated by the SAD library can be explained by the contribution of the skew sextupole term.

## INTRODUCTION

SuperKEKB is an asymmetric energy electron-positron circular collider, which had been upgraded from KEK B-factory and adopted the nano-beam scheme that is a key technology for beam collision [1]. First commissioning of SuperKEKB had been operated from February until June 2016 to test performance as low emittance storage rings. Now, SuperKEKB is running under Phase-2 commissioning. Objectives of this operation are tests for Belle-II detector and beam collision. Beta function at interaction point (IP) will be focused in stages during Phase-2.

The nonlinear problem of this study is essentially the difference of beta between at multipole magnets in interaction region (IR) and at IP. To realize extremely small beam size and the beta function at IP, the intensity of magnetic fields is very strong which is provided by various magnets in IR (see Fig. 1). On the one hand values of beta at multipoles in IR are largest in the storage ring, but on the other hand the smallest beta is located at IP. Increasing beta to focus the beta at IP makes nonlinear effects of IR more emphasized.

Skew components of multipole magnets influence coupling between x and y direction on beam dynamics. Because of x-y coupling, closed orbit distortion (COD) and betatron oscillation amplitude in x direction makes them in y direction larger [2]. This strong nonlinear coupling was observed by calculations for positron storage ring called low energy ring (LER) of SuperKEKB. So we calculated nonlinearity of each magnets for beam at IP, and the impact of them on luminosity degradation by comparison of each results. Because canonical variables used for evaluating the kick force includes the normalization factor, namely beta at IP ($\beta^*_{x,y}$), the smaller we make $\beta^*_{x,y}$, the more important this study is. Therefore, it is very meaningful to study this problem in SuperKEKB which challenges unexplored luminosity with small beta.

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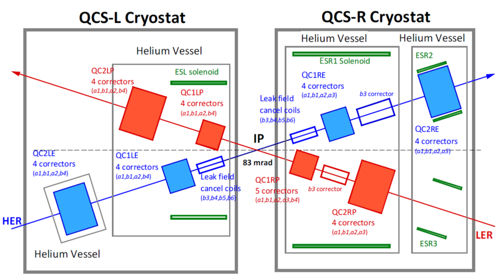

Figure 1: Constitution of the SC magnets in IR section [3].

This paper consists of following topics. At first, we studied the influence of beam properties at IP by calculating higher order of Hamiltonian, particularly skew components. We could find where the particles are kicked excessively and the effect of one-turn by integrating the contribution along the orbit. Subsequently, the effect including the hard-edge fringe of the quadrupole magnet was calculated. Finally, we checked contributions of nonlinearity by comparison with the luminosity calculated by SAD. We plotted the figures focusing on the IR section.

## SKEW COMPONENTS OF MULTIPOLES

Target nonlinear terms are sextupole components of LER, so we picked up the normal and skew effect for one-turn. To get knowledge of influences on beam dynamics at IP, coordinates transferred from IP using the following transfer matrix : **T** (which is 4th order square matrix) and returned with the continuing transfer matrix.

$$\begin{pmatrix} x^* \\ p_x^* \\ y^* \\ p_y^* \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{pmatrix} \begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix} \quad (1)$$

where $(x, y, p_x, p_y) = \vec{x}$ and $(x^*, y^*, p_x^*, p_y^*) = \vec{x}^*$ are canonical variables on phase space at each magnets and IP respectively. The vector notation allows us to write Eq.(1) as $\vec{x}^* = \mathbf{T}\vec{x}$.

The kick to be calculated is obtained from Hamiltonian. The one-turn map is the periodic function of s-parameter and circumference $(L)$ : $x(s + L) = \mathcal{M}x(s)$. Then, the matrix

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for one-turn : $\mathcal{M}$ can be transformed and represented by Hamiltonian [e.g., Eq. (2)].

$$\begin{aligned}\mathcal{M}(s) &= \prod_{i=0}^{N-1} e^{-\mathcal{H}(x,s_i)} M(s_i, s_{i+1}) \\ &\approx e^{-\oint \mathcal{H}(M(s,s')x,s')ds'} M(s) \end{aligned} \tag{2}$$

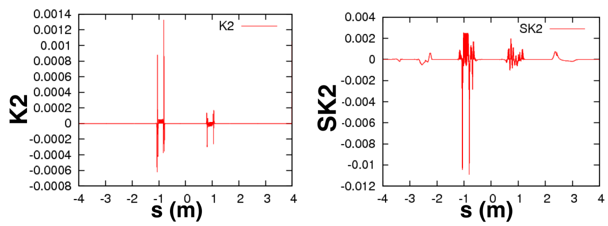


Figure 2: Position of $K_2$ (: left) and $SK_2$ (: right) in IR and its value.

In order to see nonlinearity at IP, Hamiltonian for each magnets are represented by canonical variables at IP ($x^*$ or $p_x^*$ and $y^*$ or $p_y^*$). Therefore, the action of kick is calculated by integral of Hamiltonian. Hamiltonian of sextupole magnetic fields (third order term) is $\mathcal{H}_{sextupole} = \mathcal{H}_1 + \mathcal{H}_2$, and then $\mathcal{H}_1$ and $\mathcal{H}_2$ are for normal and skew fields respectively.

$$\mathcal{H}_1 = \frac{K_2}{6}(\bar{X}^3 - 3\bar{X}\bar{Y}^2) \tag{3}$$

$$\mathcal{H}_2 = \frac{SK_2}{6}(3\bar{X}^2\bar{Y} - \bar{Y}^3) \tag{4}$$

where $K_2$ is the normal sextupole component and $SK_2$ is the skew sextupole component. The positions where $K_2$ and $SK_2$ have significant values in IR is shown in Fig. 2. The symbol :$\bar{X}$ represents $X$ or $P_X$, and the symbol :$\bar{Y}$ represents $Y$ or $P_Y$ which are normalized canonical variables :$(X, Y, P_X, P_Y) = \vec{X}$ by the product of $\sqrt{\beta^*_{x,y}}$ or $1/\sqrt{\beta^*_{x,y}}$ [e.g., Eq. (5)].

$$\begin{aligned} X &= x^*/\sqrt{\beta_x^*}, \quad & P_X &= p_x^*\sqrt{\beta_x^*} \\ Y &= y^*/\sqrt{\beta_y^*}, \quad & P_Y &= p_y^*\sqrt{\beta_y^*} \end{aligned} \tag{5}$$

Normalized form is written by $\vec{X} = \bar{\beta}\vec{x}^* = \bar{\beta}\mathbf{T}\vec{x}$ in the vector notation if we use $\bar{\beta}$ to symbolize Eq. (5).

Thus the more drastically the beta function is stopped down, the more serious this problem becomes. SuperKEKB has extremely low beta (Table 1), and so this problem is conspicuous.

Table 1: Emittance and beta at IP for SuperKEKB design

| | horizontal : x | vertical : y |
|---|---|---|
| emittance :$\varepsilon_{x,y}$ | 3.2 nm | 8.64 pm |
| beta at IP : $\beta^*_{x,y}$ | 32 mm | 270 $\mu$m |

Normalized canonical variables are nearly $\sqrt{\varepsilon_{x,y}}$ at IP ($X, P_X \simeq \sqrt{\varepsilon_x}$, and $Y, P_Y \simeq \sqrt{\varepsilon_y}$). The magnitude of the impact can be evaluated by comparison with the emittance.

There are ten combinations of canonical variables :($X$ or $P_X$ and $Y$ or $P_Y$) for sextupole fields. Among them, it was found that the influence of the $P_X^2 P_Y$ term was the largest and can not be ignored. From the canonical equation, the evaluation of the kick force is performed by following equations [e.g., Eq. (6) and (7)]. The result for $X^2Y$ and $P_X^2 P_Y$ is shown in Fig. 3. This result is calculated by SAD script which means that calculation routines are made by ourself but the computing language is SAD script due to use lattice properties.

$$For\ "X^2Y", \quad \begin{aligned} \bar{P_X} &= P_X - 2C_5XY \\ \bar{P_Y} &= P_Y - C_5X^2 \end{aligned} \tag{6}$$

$$For\ "P_X^2P_Y", \quad \begin{aligned} \bar{X} &= X - 2C_{10}P_XP_Y \\ \bar{Y} &= Y - C_{10}P_X^2 \end{aligned} \tag{7}$$

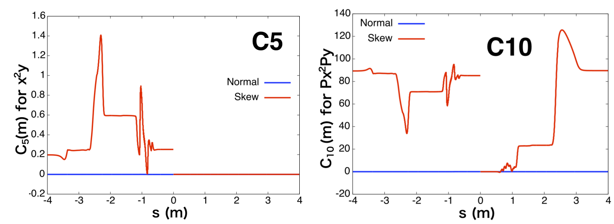


Figure 3: Coefficient of normal and skew sextupole fields for $X^2Y$ (: left) and $P_X^2 P_Y$ (: right).

In Fig. 3, vertical axises denote coefficients "$C_5$" or "$C_{10}$", and the horizontal axis denotes s-position in storage ring. The blue line is a result for normal components ($\mathcal{H}_\infty$) and the red line is for skew components ($\mathcal{H}_\in$). Especially, the value of $\bar{Y} = Y - C_{10}P_X^2$ ($C_{10}$ is shown in the right side of Fig. 3) has a large impact ($\Delta Y = \bar{Y} - Y \sim 0.1\sqrt{\varepsilon_y}$ for $P_X^2 \simeq \varepsilon_x$). Since beta function at magnet positions in IR is $\pi/2$ phase difference from IP, it is thought that the dominant contribution affects momentum. Figure 3 demonstrated that our idea is probable.

The skew sextupole component ($SK_2$) is also produced by octupole fileds and vertical COD [4], so the contribution of them on beam property at IP will be mentioned later discussion. In the subsequent studies, results are performed by adding further factors based on the term : $P_X^2 P_Y$.

## EFFECT OF OCTUPOLE FIELDS AND QCS HARD-EDGE FRINGE

In the same way, the influence on $P_X^2 P_Y$ coming from octupoles was calculated and compared with the case of only sextupoles [e.g., Fig. 4]. According to Fig. 4, octupole fields works to reduce the effect of nonlinear kick. Luckily in SuperKEKB, we could find that the influence of the pure-sextupole skew component is alleviated in higher order multipoles. Regarding higher order multipole fields than the octupole, we could not find any contributions.

It is also known that contribution to skew sextupole term "$SK_2$" is coming from hard-edge fringe of final focus quadrupole magnet systems (QCS). The contribution of QCS hard-edge fringe is shown in Fig. 5. It makes the impact

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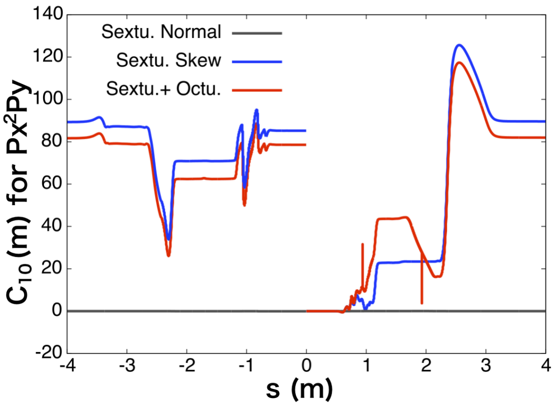


Figure 4: Coefficient of $P_X^2 P_Y$ caused by skew sextupole ($SK_2$) and octupole ($K_3 + SK_3$) fields.

of nonlinearity at IP nearly two times larger. Thus hard-edge fringe fields of QCS are important sources of nonlinearity in IR.

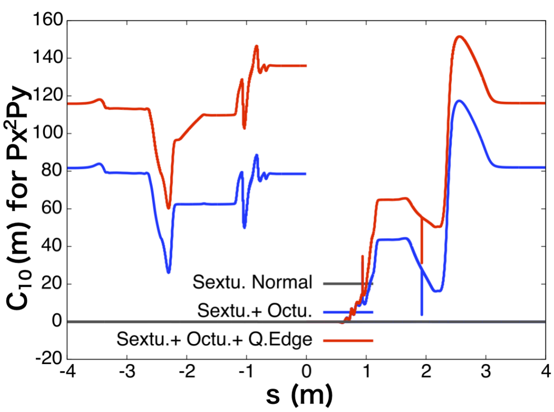


Figure 5: Coefficient of $P_X^2 P_Y$ for sextupole and octupole ($SK_2 + K_3 + SK_3$) and quadrupole hard-edge fringe ($SK_2 + K_3 + SK_3 + Q.edge$) fields.

We calculated the other skew sextupole terms which is other combinations of canonical variables ($X, Y, P_X, P_Y$) in the same way, but it turned out that most terms are negligible in SuperKEKB. This result agrees well with previous studies of nonlinear maps for LER [2].

## INFLUENCE ON LUMINOSITY

We have already simulated the luminosity used by the SAD library which is a detailed particle tracking calculation. Since this simulation includes many implicit nonlinear effect, it is useful to get knowledge for just luminosities, but we can not know the specific contributions of each element.

The calculation result of luminosity degradation is indicated in Fig. 6. The black line is the design luminosity which is target value. The red plot is a result of beam beam weak-strong simulation. Nonlinear effect of sextupoles are not included in this simulation. When we calculate luminosities,

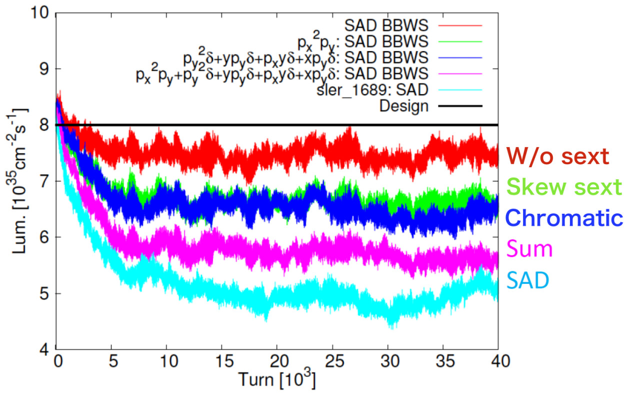


Figure 6: Luminosities for sextupole term (: $P_X^2 P_Y$), chromatic twiss, and SAD.

this simulation source is used with nonlinear terms obtained by the method of previous discussion. The green plot is including only skew sextupoles components which is calculated in this study (sextupoles + octupoles + QCS edges). The blue plot is come from chromatic x-y coupling without skew sextupoles. The summation of these contributions is indicated by pink plot ( = green + blue). As comparison with SAD tracking result (sky blue plot), most of contributions of nonlinearity is considered in this case (including factors we still have not found).

## CONCLUSION

Our data suggested that some part of the luminosity loss is coming from the contribution of skew sextupole components "$SK_2$". Regarding the intensity of effective nonlinearity, contributions of the kicks by skew sextupoles and chromatic twiss (x-y coupling included) were about the same. In summary, the luminosity degradation due to skew sextupole components and chromatic x-y coupling almost explains that of detailed simulation with SAD.

However, this result is given by an ideal calculation. In actual operation, it is possible that beams are deorbited larger than that of our estimation, so it is inferred that the higher luminosity requires very fine adjustment for QCS and other multipole magnets. As future plans, we will study the effects of space charge.

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