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THEORETICAL MODELING OF ELECTROMAGNETIC FIELD FROM ELECTRON BUNCHES IN PERIODIC WIRE MEDIUM*

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Abstract

The interaction of relativistic electrons with periodic conducting structures results in radiation via a number of mechanisms. In case of crystals one obtains parametric X-ray radiation, its frequency is determined by the distance between crystallographic planes and the direction of electron beam. If instead of a crystal one considers a periodic structure of metallic wires with the spacing of the order of mm, it is plausible to expect the emission of radiation of a similar nature ("diffraction response") at THz frequencies. Additionally, a "long-wave" radiation will occur in this case with wavelengths much larger then structure periods.

In this contribution, we present different theoretical approaches for describing the electromagnetic radiation field from prolonged electron bunch propagated in the lattice of metallic wires. The validity of these analytical descriptions is checked by numerical simulations. We discuss the possible applications of aforementioned structure as sources of coherent THz radiation.

INTRODUCTION

The sufficiently short electron bunch traversing the lattice of parallel conducting (metallic) wires can have wide enough frequency spectrum containing wavelengths λ comparable with the structure periodicity. The portion of the electromagnetic (EM) radiation related to these short enough wavelengths ("diffraction response" or "short-wave response") can be described using Bragg's diffraction theory formalism, similarly to the parametric X-ray radiation (PXR) in real crystals [1]. Under the described conditions, the metallic wire assembly can be referred to as a "wire crystal".

If the array spacing is on order of millimeters, the resulting quasi-Cherenkov radiation wavelength is in the THz frequency range. This range is of significant interest during last decade due to its prospective applications in various areas. The portion of EM radiation related to the low frequency part of spectrum (with relatively long wavelengths) should be described using the "effective medium" formalism, where the discussed wire assembly is usually referred to as "wire medium". This "long-wave" response is of essential interest due to its non-divergent properties predicted analytically [2]. In addition, analytical approach based on vibrator antenna theory can be useful for describing the wire structure with finite length wires.

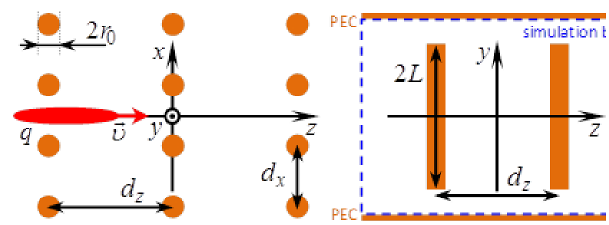

Figure 1: Geometry of the wire structure and notations. PEC plates used in simulations are indicated.

In this paper, we first present the numerical simulations (via CST PS code [3]) for the Gaussian bunch propagating through the wire crystal (see Fig. 1) and show that both "long-wave" and "short-wave" responses are generated. It should be noted, that the simulation in CST requires the inclusion of conducting plates in the model. These plates result in parasitic reflections of the generated radiation, therefore altering the results. To mitigate this effect, we increased the transverse size of the simulation area. We then present results obtained via the aforementioned theoretical approaches for describing the produced EM radiation field and compare the analytical predictions with the numerical simulations.

THEORETICAL MODEL

We consider a Gaussian electron bunch with the charge density ρ,

$$\rho = q[\sqrt{2\pi}\sigma)]^{-1}\delta(x)\delta(y)\exp\left(-(z-vt)^2/(2\sigma^2)\right), \qquad (1)$$

moving along z-axis of the conducting wire array the with length $2L$ and periods d_x and d_z; see Fig. 1. Here σ is the bunch length, $v = \beta c$ is the bunch velocity, c is the light speed; corresponding charge current is $\mathbf{j} = v\rho\mathbf{e}_z$.

"Short-Wave" Response

Since the wire structure is periodic, it is similar to conventional solid state crystals where the scattering occurs. It is shown in Ref. [1] that for $L \to \infty$ in case of Bragg's condition the wire structure is identical to diffraction crystal, and a recipe to calculate the parameters of such "crystal" (periodic coordinate-dependent "permittivity" $\varepsilon(\mathbf{r})$, $\mathbf{r} = \{x, y, z\}$) is given. The resulting quasi-Cherenkov EM radiation can be described by a set of microscopic Maxwell's equations

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with the material relation $\mathbf{D} = \varepsilon(\mathbf{r})\mathbf{E}$ [4]. First, all values are presented in reciprocal space (as Fourier transforms over wave vector $\mathbf{k}$ and frequency ω), e.g. $\mathbf{E} = \int \mathbf{E}(\mathbf{k},\omega)e^{i\mathbf{kr}-i\omega t}d\mathbf{k}d\omega$, resulting in the following relation:

$$\mathbf{D}(\mathbf{k},\omega) = \varepsilon_0\mathbf{E}(k,\omega) + \sum\nolimits_{\mathbf{g}\neq 0}\chi^{(-\mathbf{g})}\,\mathbf{E}\,(\mathbf{k}+\mathbf{g},\omega), \quad (2)$$

where ε_0 is the "host" permittivity, $\chi^{(-\mathbf{g})}$ and $\chi^{(+\mathbf{g})}$ are crystal susceptibilities, $\mathbf{g}$ are reciprocal vectors of the crystal. When Bragg's condition is satisfied for a particular reciprocal vector $\mathbf{g} = \mathbf{h}$, $h^2 = 2\mathbf{kh}$, one can use two-wave approximation and neglect all terms in the sum in Eq. (2) excluding those for $\mathbf{g} = \pm\mathbf{h}$. After a series of manipulations, the final expressions for $\mathbf{E}(\mathbf{k},\omega)$ can be obtained.

To return to coordinate-time space (and compare the obtained results with CST simulations), the reversed Fourier integrals should be calculated. This work is still in progress and will be reported elsewhere. It should be also noted that calculating the effective susceptibilities for the wire crystal is an independent and rather complicated problem.

"Long-Wave" Response

As mentioned above, in the long-wave limit, $\lambda \gg d_x, d_z, r_0$, the structure shown in Fig. 1 for $L \to \infty$ is considered as a homogeneous medium described by the following dielectric permittivity tensor possessing both frequency and spatial dispersion:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\parallel & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \varepsilon_\parallel = 1 - \frac{\omega_p^2}{\omega^2 - c^2k_y^2}. \quad (3)$$

Here $\omega_p^2 = 2\pi c^2\left[d_xd_zu\right]^{-1}$, u depends in a complicated manner on structural parameters. For the square lattice, $d_x = d_z = a$ and $u = \ln(a/r_0) - C$, where $C \sim 1$ is some constant.

Field components are determined in standard way by solving macroscopic Maxwell's equations with tensor (3). The details can be found in Ref. [2], where the nondivergent nature of the generated radiation is discussed. Below we compare these analytical results with CST simulations.

Vibrator Antenna Approach

If suppose that each wire of finite length is excited by the field of flying charged bunch only, and is not affected by the diffracted field, then the corresponding Hallen's problem for the current $I(y)$ of each wire can be solved. In the simplest quasistatic approximation and for $\beta \to 1$, we obtain for the wire with coordinates $z = z_{lm} = md_z$, $x = x_{lm} = ld_x$:

$$I_{lm}(y) = cU_{lm}(y)[2\Omega(y)]^{-1}$$
$$U_{lm} = A\sin(k_0y) + C_1(y)\sin(k_0y) + C_2(y)\cos(k_0y), \quad (4)$$

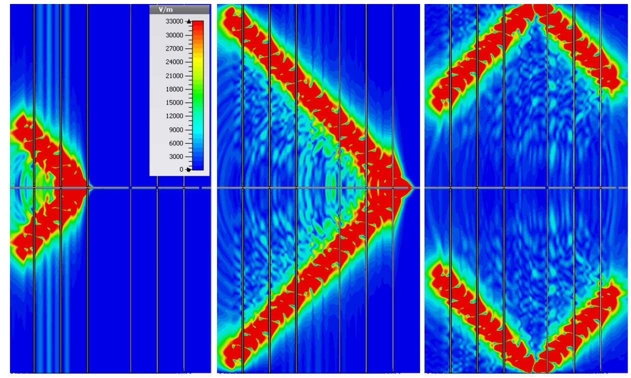

Figure 2: Two-dimensional distribution of the absolute value of electric field in the yz-plane (CST simulated result). Structure and bunch parameters: $d_x = d_z = \sigma = 1$cm, $r_0 = 1$mm, $q = 1$nC, $E = 34$ MeV.

where

$$C_1 = \frac{2iC_0}{\pi k_0}\int_0^y \frac{\cos(k_0\xi)\xi}{\xi^2 + x_{lm}^2}d\xi$$
$$C_2 = \frac{2C_0}{\pi i k_0}\int_0^y \frac{\sin(k_0\xi)\xi}{\xi^2 + x_{lm}^2}d\xi$$
$$\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{-L}^{L}\frac{d\phi d\xi}{\sqrt{(y-\xi)^2 + 4r_0^2\sin^2(\phi/2)}},$$

in turn with the definitions $C_0 = qk_0c^{-1}\exp(ik_0z_lm - \omega^2/\omega_\sigma^2)$, $\omega_\sigma = \sqrt{2}c/\sigma$, $k_0 = \omega/c$, and constant A is determined from the boundary condition $U_{lm}(L) = 0$. After that we obtain surface current $\mathbf{j}^s_{lm} = I_{lm}/(2\pi r_0)$ and can calculate EM field produced by the wire using convolution integral with Green function. Note that in the described simplest approximation we have the following limitation for the wire length: $L \ll \pi\sigma/\sqrt{2}$. Below we will compare this result with corresponding CST result.

NUMERICAL SIMULATIONS

Numerical simulations were performed in CST Particle Studio [3]. A Gaussian bunch propagates along z-axis, as shown in Fig. 1, or the bunch trajectory can be rotated for some angle in the xz-plane. Figure 2 shows two-dimensional field distribution in the yz plane for the case of bunch propagation along z-axis. Bunch length σ is chosen so that "short-wave" part of the spectrum is strongly depressed. One can clearly see that intensive EM field is generated near metallic wires at the time of electron bunch flight near each wire. These pulses correspond to "long-wave" response. Figure 3 shows comparison between CST simulations and the analytical result obtained with the theory discussed above in the corresponding subsection. As one can see, for long enough bunch (with the spectrum containing mainly long wavelengths) analytical results are in very good agreement

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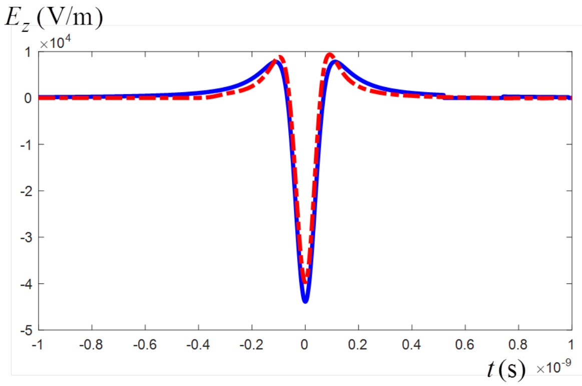

Figure 3: Comparison between "long-wave" theory and CST simulations for the behavior of the longitudinal electric field E_z over time. Red line - CST result, blue line - analytical result. Observation point is $x = 10$ cm, $z = d_z/2$, $y = 0$, other parameters are the same as in Fig. 2.

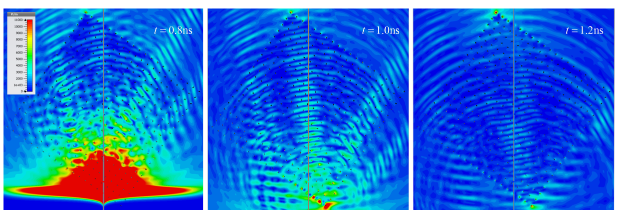

Figure 4: Two-dimensional distribution of the absolute value of the electric field in the xz-plane illustrating generation of the "short-wave" response during inclined bunch flight through the wire structure. Structure parameters are the same as in Fig. 2. Bunch length $\sigma = 3$ mm, angle of bunch trajectory inclination is 34.6 grad.

with simulated ones. Hereafter we consider E=34 MeV electron bunch, which corresponds to the value of $\beta = 0.9999$ in CST. The electron beam energy corresponds to the nominal operational energy of FAST electron injector at Fermilab motivating the prospective experimental attempt [5].

CST simulated results for the generation of "short-wave" response are shown in Fig. 4. To illustrate this effect we have rotated the bunch trajectory over certain angle and decreased bunch length σ so that bunch spectrum contains wavelengths comparable with periods of the wire crystal. One can see strong EM field concentration over several lines formed behind the bunch due to diffraction. However, EM field at these lines is several orders of magnitude weaker compared to the "long-wave" response and the self field of the bunch. Unfortunately, these results are difficult for interpretation from the point of view of PXR-like theory: due to the wide spectrum of the bunch field it is difficult to indicate the specific "crystal plane" responsible for the mentioned diffraction effects. Note that all simulations discussed above were performed for the wires attached to the PEC planes (see Fig. 1).

To clarify the process of radiation generation, we have performed CST simulations for a single wire (disconnected from the PEC planes) excited by the bunch moving along

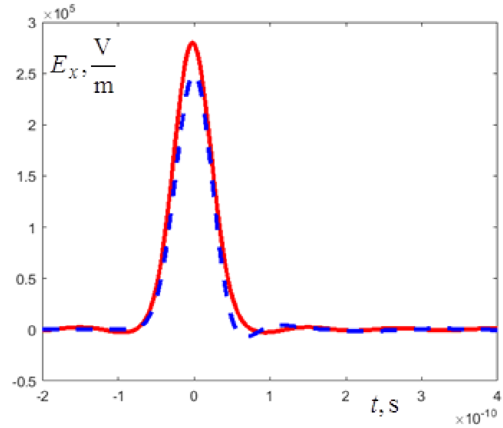

Figure 5: Comparison between simulation and vibrator antenna theory for the dependence of E_x field on time for the case of a single wire with coordinates $z = 0$, $x = d_x$. Blue line - CST result, red line - analytical result. Other parameters: $\sigma = 7$ mm, $L = 4$ mm, $d_x = d_z = 1.5$ mm, $r_0 = 0.05$ mm, E =34 MeV. Bunch trajectory goes along z-axis. Observation point: $z = y = 0$, $x = 1.5d_x$.

z-axis. Figure 5 shows the comparison between simulated results and that obtained with the vibrator antenna described above. Note, that due to the discussed limitation on wire length, only a short vibrator can be analyzed at the moment. In the future this constraint will be lifted by using a more detailed theory. As one can conclude, presented curves are in reasonable agreement. In a similar wave a string of wire vibrators and two-dimensional lattice of vibrators can be considered.

SUMMARY

We developed an initial analytical approach for calculation of the quasi-Cherenkov radiation field in the wire crystals using conventional two-wave approximation. We also implemented a numerical model of the wire crystal in CST and compared the simulations against the vibrator antenna approach. We note a very good agreement between the theoretically predicted and simulated Cherenkov field at $E = 34$ MeV beam energy. Further analytical considerations of the problem will be reported in the near future.

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