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MEASUREMENT AND ANALYSIS OF SYNCHROTRON TUNE VARIATION WITH BEAM CURRENT IN BEPCII*

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Abstract

Coherent synchrotron frequency shift is observed during machine studies in BEPCII (Beijing Electron Positron Collider Upgrade). The results show that the synchrotron frequency varies parabolicly with the increase of the beam current. This phenomenon is supposed to be induced by the interaction of the beam with the fundamental mode of the accelerating cavity. In order to explain this phenomenon, a simple physical model is developed from the couple bunch instability theory. The analytical estimations based on the physical model show good agreement with the measurements.

INTRODUCTION

BEPCII is an electron positron collider with a design luminosity of 1×10^{33} cm^{-2}s^{-1} at beam energy of 1.89 GeV [1]. To achieve high luminosity, multi-bunch collision with double ring is applied. Electron and positron beam circulate in separate storage rings, while colliding at the common interaction point. The beam instability issues are important topics for high beam performance. Superconducting RF cavity of 499.8 MHz is used for its advantage of high accelerating gradient and well-damped HOMs.

During the machine studies, quadratic growth of synchrotron tune versus beam current were observed in both electron and positron rings. The main parameters of BEPCII relevant to the machine studies are listed in Table 1, where BER represents the electron storage ring and BPR represents the positron storage ring. In this paper, measurements of frequency shift for the lowest synchrotron sideband with beam current in BER and BPR are first introduced. The phenomena are then explained by the coupled bunch instability model with the fundamental RF resonator. A simplified physical model is developed to explain the experiments.

MEASUREMENTS ON THE SYNCHROTRON FREQEUNCY TUNE SHIFT

During the operation of BEPCII, some current dependent phenomena were observed. Measurements of the frequency shift for the lowest synchrotron sideband with beam current was recently performed in BER and BPR. During the measurements, each beam consists of 120 bunches with bunch spacing of 6 ns. Figures 1 and 2 show the measured synchrotron oscillation frequency as a function of total beam current in BER and BPR, respectively. The dots represent the measured data, and the solid lines are the parabolic fits of the data. The fitting curves show excellent agreement with the measured data. The parabolic increase of the synchrotron frequency with beam current is supposed to be induced by the fundamental mode of the RF cavity, whose resonant frequency is often tuned along with the variation of the beam current. More details will be given in the following sections.

Table 1: Main Parameters Relevant to the Machine Studies

Parameters, Unit	BER	BPR
Beam energy E, GeV	1.89	
Circumference C, m	237.53	
Revolution freq. f_0, MHz	1.2621	
Harmonic number h	396	
RF frequency f_{rf}, MHz	499.8	
Bunch number n_b	120	
Momentum compaction α_p	0.0171	0.170
Synchrotron freq. f_{s0}, kHz	37.959	36.823
RF voltage V_c, MV	1.52	1.52
R/Q of the RF cavity, Ω	47.65	47.65
Q of the RF cavity	5.4×10^8	9.6×10^8

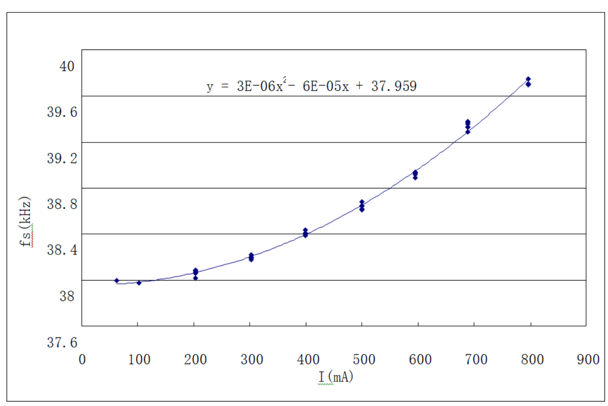

Figure 1: Variation of the synchrotron oscillation tune with beam current for BER ("◆": measurement data, solid line: fitting curve).

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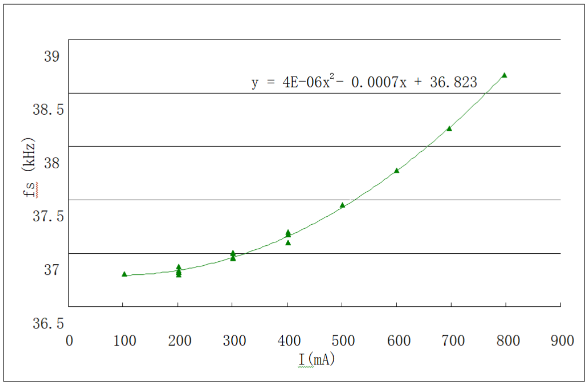

Figure 2: Variation of the synchrotron oscillation tune with beam current for BPR ("▲": the measurement data, solid line: fitting curve).

ANALYTICAL MODEL

When a beam pass through a narrow resonance, the interaction between the two can increase the amplitude of the beam oscillation and cause beam unstable. On the other hand, the interaction can also perturbs the frequency of synchrotron oscillations and induce a frequency shift. In the longitudinal plane, the complex coherent frequency shift of the nth oscillation mode for uniform beam filling with n_b bunches can be expressed as [2]

$$\Delta\Omega_{\|n} = i\frac{\alpha n_b I_b}{4\pi\nu_s E/e}\sum_{p=-\infty}^{\infty}\omega_{np} Z_{\|}(\omega_{np}) e^{-\omega_{np}^2\sigma_t^2}, \quad (1)$$

where $\omega_{np} = (pn_b+\mu+\nu_s)\ \omega_0$ is the coupled bunch oscillation mode frequency with ω_0 denotes the revolution angular frequency, $\mu = 0, \pm1, \pm2, \ldots, n_b-1$ is the synchrotron oscillation mode number, ν_s is the synchrotron oscillation tune, I_b is the average single bunch current, E is the beam energy, $\sigma_t = \sigma_z/c$ is the rms bunch length for Gaussian distribution bunch in time, and $Z_{\|}$ is the longitudinal impedance. The real part of $\Delta\Omega_{\|n}$ gives the coherent synchrotron frequency shift, which can be written as

$$\mathrm{Re}\,\Delta\Omega_{\|n} = -\frac{\alpha n_b I_b}{4\pi\nu_s E/e}\sum_{p=-\infty}^{\infty}\omega_{np}\,\mathrm{Im}\,Z_{\|}(\omega_{np}) e^{-\omega_{np}^2\sigma_t^2}, \quad (2)$$

Assume that the synchrotron oscillation modes are excited by a resonator characterized by the impedance

$$Z_{\|}(\omega) = \frac{R_s}{1+iQ\left(\frac{\omega_R}{\omega}-\frac{\omega}{\omega_R}\right)}, \quad (3)$$

where ω_R is the resonant frequency, R_s is the shunt impedance, and Q is the quality factor. If we further take both the width of the impedance $\omega_R/2Q$ and the synchrotron frequency ω_s are much less than ω_0, only the coupled bunch mode frequencies $(\pm n+\nu_s)\ \omega_0$, which are close to the resonant frequency ω_R, will contribute to the overall coherent frequency shift. In this case, the coherent synchrotron oscillation frequency shift can be simplified to

$$\mathrm{Re}\,\Delta\Omega_{\|n} = -\frac{\alpha n_b I_b}{4\pi\nu_s E/e}\left(\omega_1\,\mathrm{Im}\,Z_{\|}(\omega_1)e^{-\omega_1^2\sigma_t^2} + \omega_2\,\mathrm{Im}\,Z_{\|}(\omega_2)e^{-\omega_2^2\sigma_t^2}\right), \quad (4)$$

where $\omega_1=(n+\nu_s)\omega_0=\omega_n+\omega_s$, and $\omega_2=(n-\nu_s)\omega_0=\omega_n-\omega_s$. From the expression we can see that the coherent synchrotron oscillation frequency shift is proportional to the total beam current.

Since normally $\omega_s << \omega_n$, and $\omega_n\sigma_t<1$ for short bunches, Eq. (4) can be further simplified. If we define $\Delta\omega$ as the difference between the resonant frequency and the nearest harmonic of the revolution frequency, i.e. $\Delta\omega = \omega_R-\omega_n$, and assume $|\Delta\omega| << \omega_n$, $|\Delta\omega-\omega_s|>>\omega_n/2Q$, the synchrotron tune shift can be written as

$$\mathrm{Re}\,\Delta\Omega_{\|n} = \frac{\alpha I_b n_b R_s}{4\pi\nu_s Q E_k/e}\frac{\Delta\omega\omega_n^2}{\Delta\omega^2-\omega_s^2}e^{-\omega_n^2\sigma_t^2}, \quad (5)$$

From the above expression we can see that the frequency shift is proportional to the total beam current, R_s/Q, and $\Delta\omega/(\Delta\omega^2-\omega_s^2)$. When $\Delta\omega << \omega_s$, we have $\Delta\omega^2-\omega_s^2 \approx -\omega_s^2$ and the frequency shift is also proportional to $\Delta\omega$. Therefore, In order to explain the quadratic growth of the synchrotron tune with beam current, the resonant frequency of the resonator must be a function of beam current.

THEORETICAL ANALYSIS ON THE MEASUREMENTS

One of the main impedance sources which has the property of current dependancy is the fundamental mode of the RF cavity. During the current ramping, the frequency of the fundamental RF cavity mode is often detuned to compensate the beam loading effect. The detuning frequency can be expressed as

$$\Delta f = -\frac{I_0\sin\varphi_s}{V_c}\frac{R}{Q}f_{RF}, \quad (6)$$

Here, f_{RF} is the RF frequency, $I_0=n_bI_b$ is the average beam current, φ_s is the synchrotron phase advance, V_c is the RF voltage, R/Q is defined by

$$\frac{R}{Q} = \frac{V_c^2}{2\omega_r U}. \quad (7)$$

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where U is the stored energy, and $\omega_r=2\pi(f_{RF}+\Delta f)$ is the resonant angular frequency of the RF cavity. We can further define the detuning rate of the resonant frequency as

$$k=\frac{\Delta f}{I_0}=-\frac{\sin\varphi_s}{V_c}\frac{R}{Q}f_{RF}\,. \tag{8}$$

For superconducting RF cavities, the assumptions used in Eq. (5) can be easily fulfilled. By inserting Eq. (8) into Eq. (5), we get the dependence of the coherent synchrotron oscillation frequency on beam and RF parameters

$$\mathrm{Re}\,\Delta\Omega_{\|n}=\frac{\alpha R_s\omega_n^2}{8\pi^2\nu_s QE_k/e}\frac{kI_0^2}{(kI_0)^2-f_{s0}^2}e^{-\omega_n^2\sigma_t^2}\,, \tag{9}$$

Based on the parameters listed in Table 1, the dependence of the coherent synchrotron frequency on beam current induced by the fundamental RF mode is calculated with Eq. (2) and Eq. (9). The results are shown in Fig. 3 and Fig. 4, for BER and BPR respectively. The simplified formula shows excellent agreement with the accurate expression. By comparing with the measured data, we find that the numerical results show reasonably good agreement with the measurements. Slight disagreement between the measurements and calculations indicates that the actual parameters of the RF cavity differ from what we assumed in Table 1.

If we refitting the experimental data with the following expression

$$f_s=P_0+P_1\frac{I_0^2}{I_0^2-P_2^2}\,, \tag{10}$$

where P_0, P_1 and P_2 are fitting coefficients.

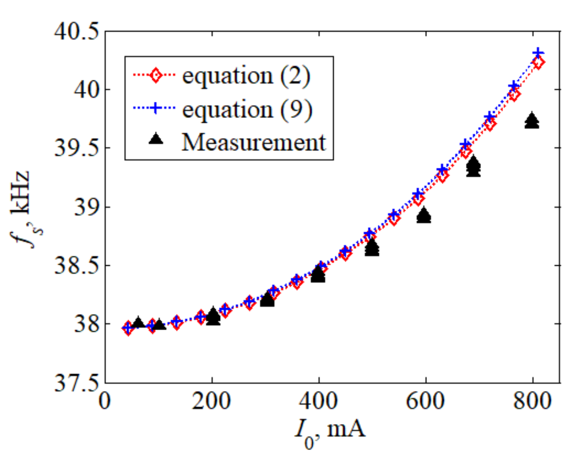

Figure 3: Theoretical estimation on the coherent synchrotron oscillation frequency versus beam current, and compared with the measured data (BER).

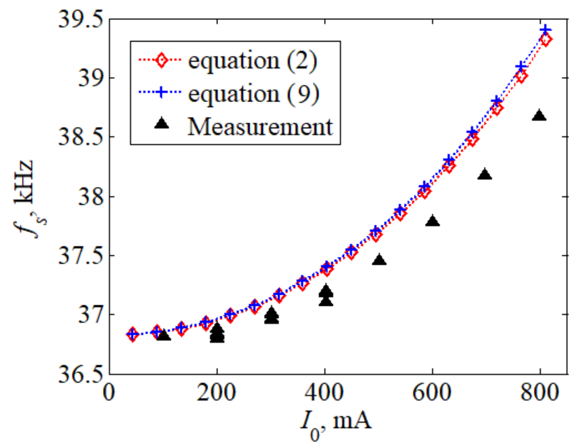

Figure 4: Theoretical estimation on the coherent synchrotron oscillation frequency versus beam current, and compared with the measured data (BPR).

By comparing the above expression with Eq. (9), we can get the relation between the coefficients and the machine parameters. The new fitting curve for BPR shows excellent agreement with the measurements and gives f_{s0}=36.732 kHz, k=−18.25 Hz/mA, R_s/Q=31.9Ω.

CONCLUSION

During the machine studies on BEPCII, quadratic growth of coherent synchrotron oscillation frequency versus beam current was measured in both electron and positron rings. A simplified physical model is also developed to explain the measurement. The phenomena are explained by the coupled bunch instability model with impedance of the fundamental mode of the RF cavity. The numerical calculations show reasonably good agreements with the measurements. The slight differences between the calculations and the measurements are explained by the uncertainly of the RF parameters.

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