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PARTICLE TRACKING SIMULATION OF COLLECTIVE MODES - PARAMETRIC LANDAU DAMPING OFF COUPLING RESONANCE

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Abstract

Employing Synergia simulations with the DMD method we investigate the Landau damping of space charge modes in bunched beams. The simulations reveal two instances of the parametric damping mechanism in bunched beams. The first example occurs in the proximity of coupling resonance and is due to the oscillation of particles' amplitudes in the transverse plane. This oscillation modulates the mode-particle coupling with particle dependent trapping frequency. The second example is due to the modulation of the mode-particle coupling in one transverse plane by the oscillatory motion in the other plane.

INTRODUCTION

Landau damping (LD) gives rise to the stabilization of collective modes in plasma and accelerator beams. The damping is caused by the energy transfer from the collective mode to the particles in resonance with the mode. The damping rate is, therefore, determined by the number of the particles capable of resonating with the mode. Conventionally, LD requires the coherent resonance frequency to lie within the incoherent spectrum, *i.e.*, to be located within the continuous frequency spectrum of the individual particles.

In a recent paper [1] we introduced the parametric LD which occurs when the mode-particle coupling has an extended frequency spectrum. In this case the damping is determined by the interplay of both particles and mode-particle coupling spectra. The existence of parametric LD was demonstrated by simulation of transverse coherent modes of bunched accelerator beams with space charge (SC) at the coupling resonance (CR). The parametric damping at CR is caused by the oscillation of particles' amplitudes in the transverse plane. This oscillation modulates the mode-particle coupling with particle dependent trapping frequency. Here we present another instance of parametric damping mechanism which occurs away of CR due to the modulation of the mode-particle coupling in one transverse plane by the oscillatory motion in the other plane.

We employ the Synergia accelerator modeling package [2, 3] to simulate the propagation of a single Gaussian beam through a linear lattice. The modes are extracted from the transverse displacement density using the dynamic mode decomposition (DMD) technique [4, 5]. DMD is a data-driven algorithm used for modal analysis in both linear and nonlinear systems.

After introducing the concept of parametric LD mechanism we proceed by investigating the Landau damping in a simplified 2-dimensional (2D) model for accelerator beams. While this model is useful for understanding the damping mechanism and interpreting the simulations, the numerical results presented in this paper are based only on the tracking simulations of a Gaussian beam through an OFORODO lattice.

PARAMETRIC LANDAU DAMPING

The LD mechanism results from the interaction of the collective mode with the individual particles. Using the simple harmonic oscillation approximation [6], the equation of motion for the particle i interacting with the mode $\bar{x}$ can be written as

$$\ddot{x}_i + \omega_i^2 x_i = -K_i \bar{x}(t), \qquad (1)$$

where x_i represents the particle displacement, ω_i the particle frequency, K_i the mode-particle coupling and $\bar{x}(t)$ the collective mode.

In systems with conventional LD, K is either time independent or its oscillation frequency is particle independent. The resonance condition is $\omega_i = \omega_c$, where ω_c is the $\bar{x}(t)$ frequency, *i.e.*, $\bar{x}(t) \propto \exp(-i\omega_c t)$. The damping rate is proportional to the spectral density at the resonant frequency,

$$\lambda \propto \rho(\omega_c) = \sum_i \delta(\omega_i - \omega_c). \qquad (2)$$

Nevertheless, it may happen that the mode-particle coupling is characterized by a frequency spectrum, *i.e.*, $K_i(t) \propto \exp(-i\mu_i t)$ and μ_i is particle dependent. The resonance condition in this case is $\omega_i = \omega_c \pm \mu_i$. The damping rate is proportional to the number of particles which fulfill the resonance condition,

$$\lambda \propto h(\omega_c) = \sum_i \delta(\omega_c - \omega_i \pm \mu_i). \qquad (3)$$

In this case the damping is determined by the interplay of both particles and mode-particle coupling spectra.

MODE-PARTICLE COUPLING FOR SPACE CHARGE MODES

Let's consider a 2D model and assume a transverse mode $\bar{x}(t)$ and a SC potential $V(x - \bar{x}, y)$. The equation of motion of a particle in the horizontal plane reads

$$\ddot{x} + \omega_{0x}^2 x = -\frac{\partial V}{\partial x}(x, y) + \frac{\partial^2 V}{\partial x^2}(x, y)\bar{x}. \qquad (4)$$

One way to determine the particle's tune shift [7] and the mode-particle coupling is to write

$$x = \sqrt{2J_x/\omega_{0x}} \sin \Phi_x, \quad y = \sqrt{2J_y/\omega_{0y}} \sin \Phi_y, \qquad (5)$$

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in equation (4) and assume $\omega_x \approx \dot{\Phi}_x$ and $\omega_y \approx \dot{\Phi}_y$ and slowly varying J_x and J_y. Only the terms oscillating with a frequency close to ω_x are retained in the first order perturbation, since the effect of higher order harmonics averages to zero.

Following this procedure, depending on the bare tunes ω_{0x} and ω_{0y}, in most cases one gets the following equation of motion

$$\ddot{x} + \left(\omega_{0x} - \delta\omega_x(J_x, J_y)\right)^2 x = -K(J_x, J_y)\bar{x}. \quad (6)$$

The situation described by Eq.(6) was presented in detail in Ref [1]. The behavior of $K(J_x, J_y)$ might yield conventional or/and parametric Landau damping. The conventional damping happens off-resonance, when the particle's action coordinates J_x and J_y are in a good approximation constant of motion. The frequency of the mode-particle coupling $K(J_x, J_y)$ is particle independent and the resonant energy exchange occurs when the particles frequency matches $\bar{x}(t)$ frequency, *i.e.*, $\omega_x \approx \omega_c$. The parametric damping happens in the vicinity of CR, *i.e.*, when $\omega_{0x} = \omega_{0y}$. For particles trapped at CR, J_x and J_y are oscillating with a trapping frequency ω_t which is particle dependent. The oscillation of the particle action coordinates modulates $K(J_x, J_y)$ with ω_t frequency. The resonant energy exchange occurs when the the particle's frequency plus its trapping frequency matches the $\bar{x}(t)$ frequency *i.e.*, $\omega_x + \omega_t \approx \omega_c$.

The numerical simulations presented in the next section show a different example of parametric LD, occurring away from CR. The conditions for the resonant energy exchange mechanism in this case can be understood by considering the following term in the expansion of the SC potential,

$$V(x, y) = \alpha x^2 y^2. \quad (7)$$

According to Eq.(4) the equation of motion is

$$\ddot{x} + \omega_x^2 x = 2\alpha y^2 \bar{x} \quad (8)$$

The mode-particle coupling K is modulated by the vertical motion via y^2. Since $y^2 = 2J_y(1-\cos 2\Phi_y)/2\omega_{0y}$ modulates K with $2\omega_y$ frequency, the condition for resonant energy exchange is, $\pm\omega_x = \pm\omega_c \pm 2\omega_y$. In our simulation we find resonant particles satisfying

$$2\omega_y \approx \omega_x + \omega_c. \quad (9)$$

Of course, for a particular region in the parameter space defining the accelerator beam more than one resonant energy exchange conditions can be fulfilled. One damping mechanism does not excludes others.

SIMULATION AND RESULTS

The simulations are done by employing the particle tracking code Synergia [2]. A lattice made by 10 identical OFORODO cells is chosen as in Refs. [1,8]. At every turn the transverse displacement density is calculated. The modes' properties are extracted using the DMD [4, 5] technique. Application of Synergia and DMD to beam dynamics is described in [8]. The beam distribution is longitudinally and transversely Gaussian with equal vertical and horizontal emittances. The chromaticity is zero. The SC parameter is defined as $q = \delta Q_{sc\ max}/Q_s$ where $\delta Q_{sc\ max}$ is the SC tune shift at the center of the bunch and Q_s is the synchrotron tune.

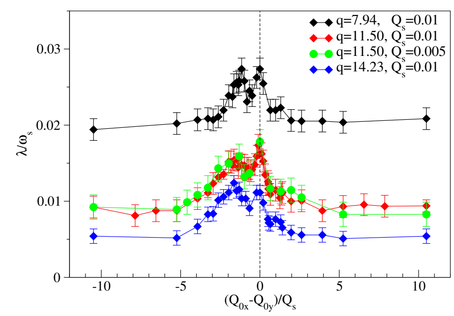

Figure 1: Landau damping λ/ω_s of the horizontal first SC mode versus $\Delta Q_0/Q_s = \left(Q_{0x} - Q_{0y}\right)/Q_s$ for different values of the SC parameter q and the synchrotron tune Q_s where $\omega_s = \omega_0 Q_s$ is the synchrotron frequency. The enhanced damping region is asymmetric with respect the to $\Delta Q_0 = 0$, extending predominantly on the negative side of ΔQ_0. Two maxima of the damping rate can be noticed.

In Fig. 1 we plot the damping of the horizontal first SC mode versus $\Delta Q_0/Q_s = (Q_{0x} - Q_{0y})/Q_s$. Around $Q_{0x} = Q_{0y}$ one can see a region with about $\approx 1.5 \sim 2$ times larger damping. The enhanced damping region is asymmetric with respect the to $\Delta Q_0 = 0$, extending predominantly ($\approx 80\%$) on the negative side of ΔQ_0. We notice two maxima of the damping rate in the enhanced damping region. One is at $\Delta Q_0 = 0$ and the other at negative ΔQ_0. Both maxima are caused by parametric damping, but the mechanism corresponding to these two cases is different, as discussed below.

Calculation of the coherent tune ν of the SC modes was presented in Ref. [8]. Due to their synchrotron motion, the particles *see* the first SC mode at the $\omega_0(\nu \pm Q_s)$ frequency [1]. The proportionality factor ω_0 between the tune and frequency is the revolution frequency of the synchronous particle.

To understand damping mechanism we investigate the tune properties of the LD responsible particles, *i.e.*, the particles involved in the resonant energy exchange with the mode. These particles are the ones with the largest change in their energy between the end and the beginning of the simulation [1].

In Fig. 2 (a) we plot the bunch footprint when $Q_{0x} \gg Q_{0y}$. The tunes of the LD responsible particles is located at the coherent tune, $\nu - Q_s$, as can be seen in Fig. 2 (b). The damping mechanism is conventional.

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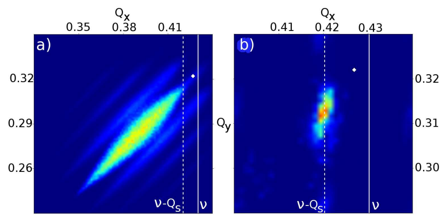

Figure 2: (a) Bunch tune footprint at off-resonance, $Q_{0x} - Q_{0y} = 11Q_s$. (b) Tune footprint of the LD resonant particles. The tunes are in the proximity of the coherent tune $Q_x = \nu - Q_s$. The damping mechanism is conventional.

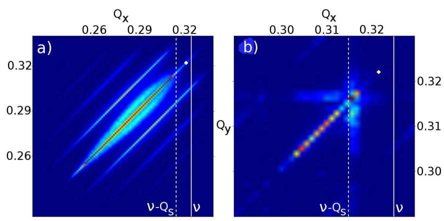

Figure 3: (a) Bunch tune footprint at CR, $Q_{0x} = Q_{0y}$. (b) Tune footprint for the LD responsible particles. Large part of the spectral weight is along the resonance line $2Q_x - 2Q_y = 0$, with the horizontal tune well below $\nu - Q_s$.

The bunch tune density at CR is plotted in Fig. 3 (a). Enhanced spectral weight is observed along the CR line $2Q_x - 2Q_y = 0$, consequence of resonance trapping. As illustrated in Fig. 3 (b) the tune of most LD-responsible particles is not at $\nu - Q_s$ but extends well below $Q_x = \nu - Q_s$ on the CR line. In fact we find that the tune of the LD-responsible particles satisfies $Q_x + Q_t \approx \nu - Q_s$, where $\omega_0 Q_t$ is the trapping frequency. The trapping frequency of each particle is extracted from the Fourier spectrum of $J_d(t) = J_x(t) - J_y(t)$, as described in Ref. [1]. Therefore the parametric damping mechanism is caused by the modulation of the mode-particle coupling with the oscillation of J_x and J_y.

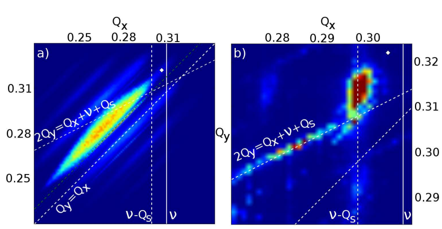

Figure 4: (a) Bunch tune footprint when $Q_{0x} - Q_{0y} = 1.75Q_s$. (b) Tune footprint for the LD responsible particles. Beside the spectral weight at $\nu - Q_s$ characteristic to conventional damping, a significant spectral weight is seen on the line $2Q_y = Q_x + (\nu + Q_s)$.

A different example of parametric damping is shown in Fig. 4. The bare betatron tunes satisfy $Q_{0x} - Q_{0y} = 1.75Q_s$, which corresponds approximately to the peak at $\Delta Q_0 < 0$ in the enhanced LD region shown in Fig. 1. The tune of most LD responsible particles is in the vicinity of $\nu - Q_s$ pointing to conventional damping. However, the tune of a significant fraction of LD responsible particles satisfies $2Q_y = Q_x + (\nu + Q_s)$, as can be seen from Fig. 4 (b). This condition is in agreement with Eq.(9) derived for the simplified model. The parametric damping mechanism is a consequence of mode-particle coupling being modulated by the motion in the vertical plane, $K \propto y^2 \bar{x}$.

CONCLUSIONS

Employing Synergia simulations with the DMD method we calculate the Landau damping of the first SC mode in bunched beams. In the parameter space defined by the bare betatron tunes, we find a region with enhanced damping in the vicinity of CR. This region is asymmetric with respect to $Q_{0x} - Q_{0y}$, with one maximum at CR and the other at $Q_{0x} < Q_{0y}$. By investigating the tune of the particles responsible for LD we find two different illustrations of parametric damping mechanism. The first case occurs at CR and is due to the oscillation of particles' amplitudes in the transverse plane. This oscillation modulates the mode-particle coupling with particle dependent trapping frequency. The second case is due to the modulation of the mode-particle coupling in one transverse plane by the oscillatory motion in the other plane. This second example explains the asymmetry of the enhanced damping region.

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