

# LHC/FCC-BASED MUON COLLIDERS\*

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## Abstract

In recent years, three schemes for producing low-emittance muon beams have been proposed: (1)  $e^+e^-$  annihilation above threshold using a positron storage ring with a thin target, (2) laser/FEL-photon back-scattering off high-energy proton beams circulating in the LHC or FCC-hh, (3) the Gamma factory concept where partially stripped heavy ions collide with a laser pulse to directly generate muons. The Gamma factory would also deliver copious amounts of positrons which could in turn be used as source for option (1). On the other hand the top-up booster of the FCC-ee design would be an outstanding  $e^+$  storage ring, at the right beam energy, around 45 GeV. After rapid acceleration the muons, produced in one of the three or four ways, could be collided in machines like the SPS, LHC or FCC-hh. Possible collider layouts are suggested.

## INTRODUCTION

The Large Hadron Collider (LHC) at CERN currently collides protons at a centre-of-mass (c.m.) energy of 13 TeV, which defines the “energy frontier”. The hadron version of the Future Circular Collider (FCC-hh) would be a 100 TeV collider, based on new 100 km tunnel infrastructure [1]. As an initial step, the same tunnel could host a high-luminosity electron-positron collider with c.m. energies up to about 370 GeV. A similar double lepton/hadron collider project is considered in China, under the name CEPC/SppC [2]. As a future extension, muon colliders offer a tantalizing path towards fundamental collisions at the 10–100 TeV energy scale [3]. In the following we study how the LHC/FCC (or CEPC/SppC) complex could be upgraded into a muon collider, using one of the recently proposed novel muon production schemes [4–7] and the circulating high-energy high-intensity positron, proton, and/or heavy-ion beams.

Muons at rest decay exponentially,  $(1/N_\mu)dN_\mu/dt = -1/\tau_0$ , with a lifetime  $\tau_0$  of 2.2  $\mu$ s. The muon lifetime  $\tau$  increases with the Lorentz factor  $\gamma$ ,  $\tau = \tau_0\gamma$ .

## MUON PRODUCTION

While past concepts required ionization cooling, e.g. [8], recent schemes [4–7] produce the muons with low emittance.

1. Colliding the photons from an X-ray Free Electron Laser with a multi-TeV proton or deuteron beam circulating in the LHC or FCC-hh generates pion beams via photo-production [4, 9, 10]. These pion beams from proton-photon ( $p-\gamma$ ) collisions are produced in a Lorentz boosted frame. They are characterized by a low emittance and TeV energies. The pions will soon decay into muons. The higher energy

of the FCC compared the LHC greatly facilitates the FEL parameters [4]. In case of the FCC, the optimal photon energy of the FEL is about 3 keV ( $\lambda_{\text{FEL}} = 0.4$  nm), which is “easily achievable by a CW SC GeV-class linac driving the FEL radiator” [4]. We consider  $2 \times 10^{11}$  protons per bunch (similar to HL-LHC, and  $10\times$  lower than in Ref. [4, 9]),  $10^{14}$  photons per pulse (with an energy above  $\pi$  threshold), an rms spot size of 7  $\mu$ m, and a bunch/FEL repetition rate of 10 MHz. Considering collisions at the LHC, about 3% of the muons produced are in the highest “energy band,” around 2500 GeV, with a transverse normalized emittance of 6.5  $\mu$ m, and 3% longitudinal momentum spread.

2. Laser-pulse collisions with a partially stripped heavy-ion (PSI) beam circulating in the LHC or FCC-hh can be used to produce intense bursts of gamma rays. By sending gamma rays of suitable energy (a few 100 MeV) onto a few-mm thick tungsten target, the so-called Gamma Factory (G.-F.) [6, 7] can be a copious source of muons. If the power limit of the extracted gamma beam is limited to about 1 MW per collision point, as is likely to be required for target survival, the number of muon pairs per second would be of the order of  $4 \times 10^{11}$  [11]. For the canonical operation of the Pb beam at the LHC, with a bunch spacing of 100 ns, this corresponds to  $4 \times 10^4$  pairs per bunch. Note that these numbers are limited only by the assumed limit on the extracted gamma ray beam power and correspond to a case where each ion converts laser photons to gamma rays about 10 times per turn [11]. The rms kinetic energy would be close to 100 MeV, with a total energy spread around 30 MeV. If the target is located between 10 and 100 m from the laser-beam collision point, the beam size at the target is of the order of 1 cm. Considering a transverse momentum spread equal to about 10% of the average longitudinal momentum, at an average kinetic energy of 100 MeV, the normalized emittance is  $\sim 2$  mm.

3. In addition to producing muons, the G.-F. can also be used to generate a high flux of positrons, at a rate of up to  $10^{17}$   $e^+$  per second. This powerful positron source could be combined with a scheme of muon production through positron annihilation, which will be introduced next.

4. This alternative approach generates a low-emittance muon beam by positron annihilation [5]. For greatest efficiency a thin target in a storage ring is preferred. The threshold energy for  $\mu$  production in positron annihilation is

$$E_{e^+, \text{thr}} = (4m_\mu^2c^4 - 2m_e^2c^4)/(2m_e c^2) \approx 43.7 \text{ GeV} . \quad (1)$$

Serendipitously, this energy is very close to the 45.5 GeV beam energy required for Z-pole operation of the 100-km FCC-ee, or CEPC, the FCC-ee top-up booster synchrotron, or a LEP3 Z factory in the LHC tunnel [12]. The exact choice of beam energy will be the result of a careful trade off, since both the production cross section and the muon

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transverse emittance increase as the positron energy is raised above the threshold [5]. Only a tiny fraction of the positron-target interactions results in a muon pair: For a 3 mm thick internal target made from beryllium, the probability that a positron annihilates into a muon pair is about  $10^{-7}$  in a single passage (or  $10^{-6}$  after multiple traversals) [13]. The average energy of the muons produced is around 22 GeV and the rms energy spread of order 2 GeV, or 10%. The latest design [13], including accumulator rings, provides a muon rate of about  $10^{11}$  per second, with about  $5 \times 10^7$  muons per bunch at a bunch rate of 2 kHz. This requires a positron rate of  $10^{17}$  per second. Though this is 4 to 5 orders of magnitude higher than for the SLC  $e^+$  source, it could be attainable at a G.-F. [6, 7]. On the other hand the present FCC-ee injector is designed to produce and accelerate to 45 GeV about  $3 \times 10^{12}$  positrons per second [14]. It provides bunches of about  $2 \times 10^{10}$  positrons spaced by 15 ns. With a conversion efficiency of  $10^{-6}$  this would amount to  $3 \times 10^6$  muons per second.

Parameters from the four different muon sources are compared in Table 1. The Gamma factory offers the best performance for muon production, especially if not used for generating muons directly, but for producing positrons in combination with the positron annihilation scheme (profiting from the small emittance available in this scenario).

Table 1: Comparison of muon production schemes, applied to the existing or planned LHC/FCC-ee/FCC-hh infrastructure. The  $e^+$  annihilation scheme could be based either on the maximum positron rate provided by the FCC-ee injector complex (second to last column) or it could use the higher positron flux from a Gamma factory (last column).

scheme	$p\text{-}\gamma$	G.-F. $\mu$	$e^+$	G.-F. $e^+$
base	LHC/FCC-hh	FCC-ee	FCC	
rate $\dot{N}_\mu$ [GHz]	1	400	0.003	100
$\mu$ /pulse [ $10^4$ ]	0.01	4	0.2	6,000
p. spacing [ns]	100	100	15	15
energy [GeV]	2.5	0.1	22	22
rms en. spread	3%	10%	10%	10%
n. emit. [ $\mu\text{m}$ ]	7	2000	0.04	0.04
$\dot{N}_\mu/\varepsilon_N$ [ $10^{15} \text{ m}^{-1} \text{ s}^{-1}$ ]	0.1	0.2	0.1	3,000

## MUON RECIRCULATION

For stacking and acceleration it would be convenient to recirculate the muons in a ring, containing dipole magnets with a field  $B$ . We only consider the case  $v \approx c$ . If we denote the dipole filling factor of this ring by  $F_{\text{dip}}$  the revolution time  $T_{\text{rev}}$  at energy  $E = \gamma m_\mu c^2$  is

$$T_{\text{rev}} = \frac{2\pi\rho}{cF_{\text{dip}}} = \frac{2\pi E}{c^2 B e F_{\text{dip}}} = \frac{2\pi\tau m_\mu}{\tau_0 B e F_{\text{dip}}}, \quad (2)$$

from which the lifetime can be expressed as

$$\frac{\tau}{T_{\text{rev}}} = \frac{eB\tau_0 F_{\text{dip}}}{2\pi m_\mu}, \quad (3)$$

which depends only on the effective field ( $B F_{\text{dip}}$ ), as was already noticed by Budker [15]. Considering a dipole filling factor of  $F_{\text{dip}} = 0.7$ , the average muon survival time with LHC-type 8.3 T magnets amounts to 1680 turns; for a field of 16 T it is 3360 turns; for 24 T 5040 turns. Hence, the high-field accelerator magnets under development for FCC-hh (16 T) and CEPC (12–24 T) offer optimum conditions.

Equation (3) considers a constant magnetic field. In a case of acceleration this would imply a spiraling orbit, as in a cyclotron. For a synchrotron we should replace  $B$  by  $E/(\rho e c)$ :

$$\frac{\tau}{T_{\text{rev}}} = \frac{E}{\rho} \frac{\tau_0 F_{\text{dip}}}{2\pi m_\mu c} \quad (4)$$

At a constant bending radius  $\rho$ , a higher energy  $E$  is preferred. This argues for high injection energy and a limited swing.

## MUON ACCUMULATION

Muon stacking is indispensable. Suppose on successive turns, in intervals  $\Delta t_\mu$ , we inject  $N_{\mu,0}$  muons into the same bucket of a recirculator ring, i.e.  $T_{\text{rev}} = \Delta t_\mu$ . The lifetime in number of turns is maximized by choosing the highest possible magnetic field. The stacking-ring beam energy is

$$E = \Delta t_\mu c^2 B e F_{\text{dip}} / (2\pi). \quad (5)$$

For example, at a field of 16 T, for  $\Delta t_\mu = 15$  ns the energy is 2.4 GeV, for  $\Delta t_\mu = 100$  ns it is 16 GeV. The ring circumferences are 4.5 m and 30 m, respectively.

After the first turn the muons injected at time zero will be reduced by a factor  $\lambda \equiv \exp(-T_{\text{rev}}/\tau) < 1$ . Summing the geometric series, after  $n$  turns the muon bunch intensity becomes  $N_{\mu,0} \sum_0^n \lambda = N_{\mu,0} (1 - \lambda^{n+1}) / (1 - \lambda)$ , approaching

$$N_{\mu,\text{max}} = N_{\mu,0} / (1 - \lambda) \approx N_{\mu,0} \tau / T_{\text{rev}}. \quad (6)$$

For efficient stacking we require  $\lambda \approx 1$  or  $T_{\text{rev}} \ll \tau$ .

## MUON ACCELERATION

We parametrize the muon acceleration as  $\gamma(t) \approx (\gamma_0 + at)$ , where  $a$  relates to the accelerating gradient  $G$  [V/m] via

$$a = eGF_{\text{acc}} / (m_\mu c) \quad (7)$$

and  $F_{\text{acc}}$  the effective filling factor of accelerating structures. Considering an initial energy  $E_0$  and integrating the equation

$$(1/N_\mu) dN_\mu/dt = -1/(\tau_0(\gamma_0 + at)), \quad (8)$$

yields the fraction of muons surviving at energy  $E$

$$N_\mu(E)/N_{\mu,0} = (E/E_0)^{-1/(\tau_0 a)}. \quad (9)$$

Requiring the decay be less than a factor  $1/e$  ( $\sim 1/3$ ) yields

$$a > 1/(\tau_0) \ln(E/E_0). \quad (10)$$

If acceleration takes place in a storage ring of circumference  $C$  Eq. (10) yields the minimum ring rf voltage:

$$V_{\text{rf}} > m_\mu c^2 C / (e c \tau_0) \ln(E/E_0). \quad (11)$$

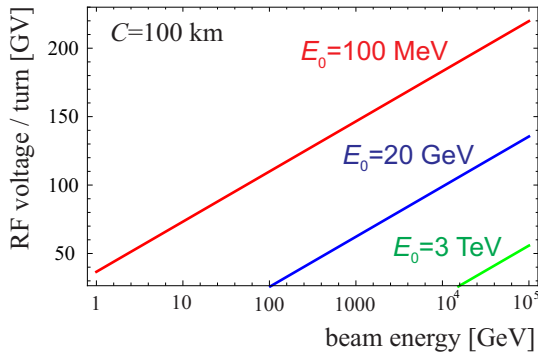


Figure 1: Minimum rf voltage as a function of final beam energy, for three different injection energies, at  $C = 100$  km.

Figure 1 shows this minimum rf voltage as a function of final muon beam energy for three different injection energies in an FCC type collider with a circumference of 100 km.

Muons could be accelerated in the 26.7 LHC tunnel with about 5 km of 16 T SC magnets and 20 km of  $\pm 3.5$  T pulsed magnets (ramping in less than 0.1 s) plus an additional 7 GV of new pulsed SC rf [16, 17]. Within about 1000 turns the beam can reach 7 TeV. Acceleration in the FCC tunnel up to 50 TeV requires an rf voltage of  $\sim 50$  GV. See also [18].

## COLLIDER SCHEMES

Figure 2 sketches two example configurations for converting the LHC-FCC complex into a muon collider, based on the Gamma factory concept applied to PSI ions stored in one of the FCC-hh rings. The gamma bursts from the laser-ion-beam collision would be used either for immediate muon production or for positron production followed by muon production through annihilation in a 45 GeV storage ring with internal target. The storage ring could be FCC-ee or its top-up booster, already optimized for Z pole operation at the same energy, and/or a LEP3 ring in the LHC tunnel. The muon beams would be collided in the other of the two FCC rings. A similar configuration for a 14 TeV collider may be based on the LHC [3].

## LUMINOSITY SCALING

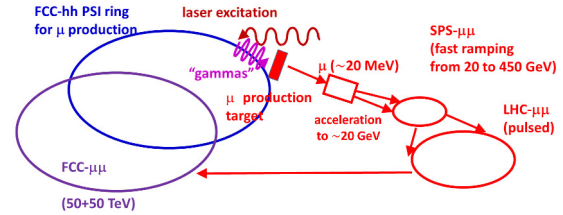
Considering a chain of three circular accelerators, each losing a factor 1/3 in muon intensity, round beams colliding at a focal point with beta function  $\beta^*$  and efficient accumulation, the maximum conceivable luminosity is

$$L \approx f_{\text{rev}} \dot{N}_\mu \frac{\dot{N}_\mu}{\varepsilon_N} \frac{1}{3^6} \gamma \tau^2 \frac{1}{4\pi\beta^*} \quad (12)$$

$$= \frac{1}{3^6} \left\{ \left( \frac{eF_{\text{dip}}}{2\pi m_\mu} \right)^3 \frac{\tau_0^2}{4\pi c^2} \right\} [B^3 C^2] \left[ \dot{N}_\mu \frac{\dot{N}_\mu}{\varepsilon_N} \right] \frac{1}{\beta^*}.$$

Intriguingly, the muon collider luminosity scales with the third power of the magnetic field and the square of the circumference, underlining the great potential of the FCC or CEPC/SppC complex for a future muon collider. The quantities  $\dot{N}_\mu$  and  $\dot{N}_\mu/\varepsilon_N$  are determined by the muon production

100 TeV  $\mu$  collider FCC- $\mu\mu$  with FCC-hh PSI  $\mu^\pm$  production



100 TeV  $\mu$  collider FCC- $\mu\mu$  with FCC-hh PSI  $e^+$  & FCC-ee  $\mu^\pm$  production

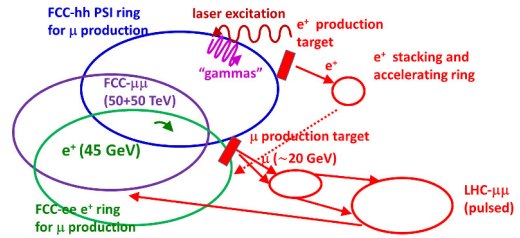


Figure 2: Example variants of a 100 TeV muon collider based on the FCC complex [3].

scheme (see Table 1). The interaction point (IP) beta function  $\beta^*$  depends on the free length from the IP, and on the magnet technology. Extrapolating from FCC, we may expect a value of order 1 centimetre.

Putting everything together and using  $\dot{N}_\mu$  from the direct muon production at a Gamma factory (third to last column in Table 1), a 100 TeV muon collider in the 100 km FCC tunnel could have a luminosity of order  $3 \times 10^{30} \text{ cm}^{-2} \text{ s}^{-1}$ . If instead we use the Gamma factory to produce positrons for muon generation through annihilation in one of the FCC-ee rings (the last column in Table 1), the value of the product  $\dot{N}_\mu \dot{N}_\mu/\varepsilon_N$  could be about  $7 \times 10^3$  times higher, corresponding to a luminosity above  $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ . Pushing  $\beta^*$  down to 1 mm would yield a further factor of ten.

The possible beneficial effect of synchrotron radiation is not yet accounted for. With 50 TeV muon beam energy in the 100 km FCC ring, the transverse emittance damping time from synchrotron radiation will be  $\sim 4$  seconds, almost comparable to the corresponding muon lifetime of about 1 second. At higher beam energies and higher fields the damping effect will dominate and provide a natural cooling.

## CONCLUSIONS AND THANKS

The upgrade of the existing and proposed facilities LEP(3)/LHC, FCC-hh/ee or CEPC/SppC into a muon collider should be seriously pursued. Several key features of these facilities (positron beam energy, positron flux, magnetic field, availability of intense beams of high-energy protons and partially-stripped heavy ions, etc.) exactly match the requirements of a future highest-energy muon collider complex.

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