# NONLINEAR CORRECTION STRATEGIES FOR THE LHC USING RESONANCE DRIVING TERMS 

F. Carlier<br>CERN, Geneva, Switzerland and NIKHEF, Amsterdam, Netherlands<br>E.H. Maclean, R. Tomás, T.H. Persson<br>CERN, Geneva, Switzerland

## Abstract

The correction of nonlinearities in future colliders is critical to reach operational conditions and pose a significant challenge for commissioning schemes. Several approaches have been succesfully used in the LHC to correct sextupolar and octupolar sources in the insertion regions. Measurements of resonance driving terms at top energy in the LHC have improved and now offer a new observable to calculate and validate nonlinear corrections. This paper reports on measurements of resonance driving terms in the LHC and the relevant strategies used for nonlinear corrections.

## INTRODUCTION

The correction of nonlinear magnetic errors in the insertion regions of the High Luminosity LHC will be critical to achieve design perfomance [1]. Current correction schemes based on experience in the LHC rely on amplitude detuning measurement using the LHC AC dipoles [2], and measurements of secondary observables of feeddown to tune and coupling with crossing angles. These methods have achieved successfull corrections of $b_{3}, a_{3}$ and $b_{4}$ errors in the two high luminosity insertion regions of the LHC, IR1 and IR5 [3].

The method of resonance driving terms has been applied in various machines for nonlinear diagnostics [4-6] and has long been proposed to correct machine nonlinearities in the LHC $[4,7]$. Resonance driving terms have been measured in the LHC [8-10], but were until now not used for nonlinear correction optimisation. Improvements in measurement quality, noise reduction, as well as phase calculations have now made it possible to use resonance driving terms for direct measurements of the nonlinear content of the machine and have been used to validate previous corrections. Strategies involving resonance driving terms measurements using the LHC AC dipoles are now envisegable to determine local IR corrections of higher order multipolar errors.

The motion under forced oscillations with AC dipoles is considerably changed. Following the same procedure as in [11] the turn-by-turn evolution of the normalized horizontal variable ( $\xi_{x,-}$ ) under influence of an AC dipole can be given in normal form by

$$
\begin{aligned}
& \xi_{x,-} \approx 2 I_{x} e^{\mp i\left(2 \pi Q_{x} D^{\tau-} \eta_{x-}\right)} \\
& -\sum_{j k l m} 2 i j f_{j k l m}^{\prime}\left(2 I_{x}\right)^{(j+k-1)}\left(2 I_{y}\right)^{(l+m)} \\
& e^{i\left[(k-j+1)\left(2 \pi Q_{x} \tau \tau-\eta_{x,-}\right)+(l+m)\left(2 \pi Q_{y D} \tau-\eta_{y,-}\right)\right]} \text {, }
\end{aligned}
$$

with the AC dipole resonance driving terms defined as

$$
\begin{equation*}
f_{j k l m}^{\prime}=\frac{h_{j k l m}^{\prime}}{1-e^{i 2 \pi\left[-Q_{x}+(k-j+1) Q_{x D}+(l+m) Q_{y D}\right]}} \tag{2}
\end{equation*}
$$

where $Q_{x, y}$ are the natural tunes, $Q_{x, y D}$ are the AC dipole tunes, $h_{j k l m}^{\prime}$ are the Hamiltonian terms under AC dipole oscillations, $I_{x, y}$ are the motion invariants, and $j, k, l$ and $m$ are indices defining the order of the resonance driving terms.

It is important to note at this point that the magnitude of the resonance driving terms depend on the distance of the working point to the corresponding resonance, as is defined in Eq. (2). This characteristic can be used to improve measurements of resonance driving terms in the LHC. Resonance driving terms may be calculated from turn-by-turn transverse BPM data. The secondary spectral line amplitudes from spectra of the reconstructed complex signal are dependent on the resonance driving terms as follows:

$$
\begin{align*}
& H(k-j+1, m-l)=2 j \cdot\left|f_{j k l m}^{\prime}\right|\left(2 I_{x}\right)^{\frac{j+k-1}{2}}\left(2 I_{y}\right)^{\frac{m+l}{2}}  \tag{3}\\
& V(k-j, m-l+1)=2 l \cdot\left|f_{j k l m}^{\prime}\right|\left(2 I_{x}\right)^{\frac{j+k}{2}}\left(2 I_{y}\right)^{\frac{m+l-1}{2}}, \tag{4}
\end{align*}
$$

where $H(v, w)$ and $V(v, w)$ are the measured spectral line amplitudes. The phases of the driving terms at a specific location are related to the secondary line phases by

$$
\begin{align*}
\phi_{k-j+1, m-l}^{H}= & \phi_{j k l m}+(k-j+1) \psi_{b, x, 0} \\
& +(m-l) \psi_{b, y, 0}-\pi / 2  \tag{5}\\
\phi_{k-j, m-l+1}^{V}= & \phi_{j k l m}+(k-j) \psi_{b, x, 0} \\
& +(m-l+1) \psi_{b, y, 0}-\pi / 2, \tag{6}
\end{align*}
$$

where $\phi_{j k l m}$ is the phase of the driving terms, $\phi_{k-j+1, m-l}^{H}$ and $\phi_{k-j, m-l+1}^{V}$ are the phase of the secondary lines in the horizontal and vertical spectra respectively, and $\psi_{b, x, 0}$ and $\psi_{b, y, 0}$ are the betatron phases in the horizontal and vertical planes. The phases of the driving terms are calculated individually for each measurement and then averaged over the set of measurements, thus providing an estimate on the observational error of the phase term.

## ENHANCEMENT OF RESONANCE DRIVING TERMS

Measurements of resonance driving terms have the potential to correct multipolar errors through an order-by-order approach. By moving both the natural and AC dipole working points closer to resonances, the measured amplitude of

Figure 1: Measurement of $\left|f_{1210}^{\prime}\right|$ for two different working points, where working point 2 is closer to the $Q_{x}=Q_{y}$ resonance. For Beam 2 at 6.5 TeV and $\beta^{*}=40 \mathrm{~cm}$.
the resonance driving terms can be enhanced. To that effect, tests were made in 2016 at $\beta^{*}=40 \mathrm{~cm}$ to study the $f_{1210}^{\prime}$ driving term generated by the skew octupolar resonance $Q_{x}=Q_{y}$. Measurements with large diagonal AC dipole excitations were taken at the two working points reported in Table 1.

Table 1: Summary of Working Points

|  | $Q_{x}$ | $Q_{y}$ | $Q_{x}^{\mathrm{AC}}$ | $Q_{y}^{\mathrm{AC}}$ |
| :--- | :---: | :---: | :---: | :---: |
| Working point 1 | 0.28 | 0.31 | 0.27 | 0.324 |
| Working point 2 | 0.29 | 0.30 | 0.28 | 0.314 |

The first working point uses the nominal LHC injection unes as natural tunes, while for the second working point the natural tunes are moved closer to the coupling resonance $\left(Q_{x}=Q_{y}\right)$. The AC dipole tunes are always at the same distance to the natural working point; $Q_{x}^{\mathrm{AC}}=Q_{x}-0.01$ and $Q_{y}^{\mathrm{AC}}=Q_{y}+0.014$. Figure 1 shows the measured amplitude of $\left|f_{1210}^{\prime}\right|$ for the two different configurations. The amplitude of $\left|f_{1210}^{\prime}\right|$ is successfully increased by a factor $\sim 1.4$ when moving the working point closer to resonance. Correction strategies can be envisaged for higher-order sources by probing specific resonances to maximise the driving terms signals. The measured phases, $\phi_{1210}^{\prime}$, are shown in Fig. 2. The phase measurements show discontinuities in the insertion regions. These observations become clearer when looking at the real and imaginary parts of $f_{1210}^{\prime}$, as shown in Fig. 3. Clear steps in both real and imaginary parts are observed in the insertion regions where large phase changes in $f_{1210}^{\prime}$ occur.

## CORRECTIONS OF SKEW OCTUPOLAR ERRORS

In 2017, local corrections for skew octupolar errors in IR1 and IR5 were derived from measurements of feed down to coupling with crossing angle. The corrections in IR1 were implemented and validated with crossing-angle scans. It should be noted that the $a_{4}$ corrector right of IP1 is broken,


Figure 2: Measurement of $\phi_{1210}^{\prime}$ for two different working points, where working point 2 is closer to the $Q_{x}=Q_{y}$ resonance. For Beam 2 at 6.5 TeV and $\beta^{*}=40 \mathrm{~cm}$.


Figure 3: Measurement of $\operatorname{Re}\left[f_{1210}^{\prime}\right]$ and $\operatorname{Im}\left[f_{1210}^{\prime}\right]$ for two different working points, where working point 2 is closer to the $Q_{x}=Q_{y}$ resonance. For Beam 2 at 6.5 TeV and $\beta^{*}=40 \mathrm{~cm}$.
and that the single corrector magnet left of IP1 cannot fully correct both beams. Local IR5 corrections are not applied operationally due to increased AC dipole lossses after attempted correction. Measurements of the $f_{1210}^{\prime}$ driving term are used to verify the derived IR5 corrections. The measurements are presented for ATS optics at $\beta^{*}=30 \mathrm{~cm}$. Due to uncertainties about the polarity of the $a_{4}$ correctors left and right of IP5 the correction tests are done with both polarities. Three settings are compared in the following results; no correction, correction 1 (assuming positive polarity of correctors), correction 2 (assuming negative polarity of cor-


Figure 4: Measurement of $\left|f_{1210}^{\prime}\right|$ for the LHC without $a_{4}$ corrections and two $a_{4}$ corrections with different polarities. For Beam 2 at 6.5 TeV and $\beta^{*}=30 \mathrm{~cm}$.


Figure 5: Measurement of $\phi_{1210}^{\prime}$ for the LHC without $a_{4}$ corrections and two $a_{4}$ corrections with different polarities. For Beam 2 at 6.5 TeV and $\beta^{*}=30 \mathrm{~cm}$.
rectors). Figure 4 shows the measured amplitudes of $f_{1210}^{\prime}$ for the three cases. An increase by a factor 2 is observed for correction 2, while the amplitude of correction 1 remains unchanged with respect to the case without correction. A closer look at the phases reveals a significantly different pattern than previously shown in Fig. 2. This difference results from the change to ATS optics made in 2017 compared to the 2016 optics, where the integer parts of the tunes are more separated. (Fig. 5)

By comparing the difference in $\phi_{1210}^{\prime}$ between the measurements with corrections and the reference measurement without corrections, as presented in Fig. 6, a constant phase offset is observed in both cases. This indicates that the phases of the $a_{4}$ correctors in IR5 are not aligned with the error that is attempted to be corrected. The phase offset of the corrections are given in Table 2. The results show that it cannot be assumed that only local errors are corrected with the IR5 corrections. In fact, the phase offset strongly suggests that $a_{4}$ sources in IR1 are not fully corrected and propagate to IR5 hence spoiling the phase alignment of the correctors. Measurements with and without IR1 corrections should provide further insights in the success of local IR1 corrections and its effect on the phase offset in IR5.


Figure 6: Difference in driving term phases with $\Delta \phi_{1210}^{\prime}=$ $\phi_{1210, \text { no corr }}^{\prime}-\phi_{1210, \text { with corr, }}^{\prime}$, for two $a_{4}$ corrections with different polarities. For Beam 2 at 6.5 TeV and $\beta^{*}=30 \mathrm{~cm}$.

Table 2: Correction Phase Offsets for the Two Corrections

|  | Polarity | Phase Offset $\left[^{\circ}\right]$ |
| :--- | :---: | :---: |
| Correction 1 | +1 | $56 \pm 5$ |
| Correction 2 | -1 | $109 \pm 6$ |

Furthermore, the good quality of the measurements allow new strategies to find corrections for $a_{4}$ errors based on resonance driving terms by matching the real and imaginary parts of $\left|f_{1210}^{\prime}\right|$ with the simulated driving term responses of the $a_{4}$ correctors in the insertion regions. This approach will be a high priority in the 2018 commissioning campaign.

## CONCLUSIONS

The results presented in this paper have successfully demonstrated an enhancement of resonance driving terms amplitude by moving the working point closer to the driving term specific resonance. Furthermore, recent improvements in resonance driving terms phase calculations show clear indications of local $a_{4}$ sources in both IR1 and IR5. First tests of local $a_{4}$ corrections in IR5 have demonstrated a significant phase offset between the error and the corrector magnets in IR5 of $57 \pm 8^{\circ}$ and $109 \pm 11^{\circ}$, depending on the correction. This offset suggests that local sources in IR1 are not fully corrected and propagate to IR5, resulting in a phase offset. The results show the first successfull usage of resonance driving terms measurements in the LHC to verify local nonlinear errors in the insertion regions. Further developments in the measurements and analysis will aim to provide direct corrections from resonance driving terms measurements to locally correct nonlinearities in the LHC.

## ACKNOWLEDGEMENTS

The authors would like to thank the whole OMC team for their help with the measurements.
[1] F.S. Carlier et al., "Optics Measurements and Correction Challenges for the HL-LHC", Technical Report CERN-ACC-2017-0088, CERN, Geneva, Oct 2017.
[2] S. White, E. Maclean, and R. Tómas, "Direct amplitude detuning measurement with ac dipole", Phys. Rev. ST Accel. Beams, 16:071002, Jul 2013.
[3] E.H. Maclean, "New optics correction approaches in 2017", In Proc. 8th LHC Operations Evian Workshop, 2017.
[4] R. Bartolini and F. Schmidt, "Normal form via tracking or beam data", Part. Accel., 59(LHC-Project-Report-132. CERN-LHC-Project-Report-132):93-106. 10 p, Aug 1997.
[5] W. Fischer, .F Schmidt, and R. Tomás, "Measurement of sextupolar resonance driving terms in RHIC", In Proc. of 20th IEEE Particle Accelerator Conference (PAC'03), Portland, USA, May 2013.
[6] R. Bartolini, I. P. S. Martin, J. H. Rowland, P. Kuske, and F. Schmidt, "Correction of multiple nonlinear resonances in storage rings", Phys. Rev. ST Accel. Beams, 11:104002, Oct 2008.
[7] R. Tómas Garcia, "Direct measurement of resonance driving terms in the Super Proton Synchrotron (SPS) of CERN using beam position monitors", PhD thesis, Valencia U., València, 2003, Presented on 30 Mar 2003.
[8] M. Benedikt, F. Schmidt, R. Tomás, P. Urschütz, and A. FausGolfe, "Driving term experiments at CERN", Phys. Rev. ST Accel. Beams, 10:034002, Mar 2007.
[9] G. Vanbavinckhove et al., "First measurements of higher order optics parameters in the LHC", In Proc. of 2nd International Particle Accelerator Conference (IPAC'11), San Sebastian, Spain, September 2011, paper WEPC032, pp. 2073.
[10] F.S. Carlier, J.M. Coello de Portugal, E.H. Maclean, T. Persson, and R. Tomás, "Observations of resonance driving terms in the LHC during Runs I and II", In Proc. of 7th International Particle Accelerator Conference (IPAC'16), Busan, Korea, May 2016, paper THPMR037, pp. 3468.
[11] R. Tómas, "Normal form of particle motion under the influence of an ac dipole", Phys. Rev. ST Accel. Beams, 5:054001, May 2002.

