

MODELLING WAKE IMPEDANCE OF A ROUGH SURFACE IN APPLICATION TO THE FCC-hh BEAMSCREEN*

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Abstract

The inner surface of the future circular collider (FCC-hh) beamscreen is proposed to be laser-treated in order to mitigate the electron cloud build-up. However, the rough structure of the treated surface can result in unwanted impedance increase, potentially leading to the transverse mode coupling instability (TMCI). Three models have been adopted to estimate the wake impedance of a beamscreen with a rough surface. The models use the resistive wall formalism generalized for the case of an arbitrary surface impedance. The results apply to a beamscreen of a circular cross-section with the homogeneously rough inner surface for the case of ultrarelativistic particles. The free parameters of the models were fit into preliminary measurements of the surface resistivity, giving, as a result, a range of the real and the imaginary parts of the wake impedance.

INTRODUCTION

Electron cloud build-up inside the vacuum chamber (beamscreen) of a circular collider is considered to be one of the major limiting factors in reaching high beam intensity. There is evidence that making the internal beamscreen surface intentionally rough can reduce the secondary electron yield (SEY) [1,2] which helps to mitigate the electron cloud build-up. Laser ablation surface engineering (LASE), or laser-engineered surface structures (LESS) is proposed as a method to produce a rough beamscreen surface in application to the high luminosity large hadron collider (HL-LHC) and the future circular collider (FCC-hh). However, the increase in coupling impedance due to the roughness needs to be evaluated in order to estimate the potential negative impact on the beam stability.

Models for the impedance of a rough surface have been developed before, based on purely geometric considerations. Such models include the inductive model [3] and the resonator mode model [4]. These models assume that the surface is made of a perfectly conducting metal, thus considering the impedance change due to roughness independent of the bulk conductivity of the metal. This, however, might not be true if the impedance due to resistive losses is comparable to the geometric impedance, as in this regime the geometrical shape is altered by the skin effect. This regime is important in our case, as roughness caused by an electron cloud treatment is required to not dramatically increase the total impedance of the beamscreen.

In order to account for the interplay between the geometrical and the resistive sources of impedance, a generalization

of the resistive wall formalism ([5], page 40) is used below. If the wavelength in vacuum at the frequencies of interest is much larger than the characteristic size of roughness, the solution for the electromagnetic fields inside the vacuum region can be obtained using the surface impedance boundary condition. This approach effectively replaces the geometric irregularities with a plain smooth surface, with the effect of the irregularities hidden in the modified surface impedance. For an axially symmetric circular beamscreen, over a certain frequency range the *surface impedance* can then be related to the longitudinal and the transverse *coupling impedances* as follows

$$\begin{aligned} Z_{||}(\omega) &= \frac{L}{2\pi b} Z_{surf}(\omega), \\ Z_{dip}(\omega) &= \frac{Lc}{\omega\pi b^3} Z_{surf}(\omega), \end{aligned} \quad (1)$$

where L and b are the length and the inner radius of the beamscreen, and c is the speed of light in vacuum, and the beam is considered to be ultrarelativistic ($\beta \approx 1$).

It has to be stated that the problem with the actual laser-treated surface might be too hard to analyze. The reason is that the surface exhibits two types of roughness: large parallel grooves that trace the laser trajectory, and (sub)micron-level balls chaotically placed on top of the grooves (Fig. 1, top). The micron-level roughness is a result of condensation of the evaporated material and is a necessary tool for reducing the SEY.

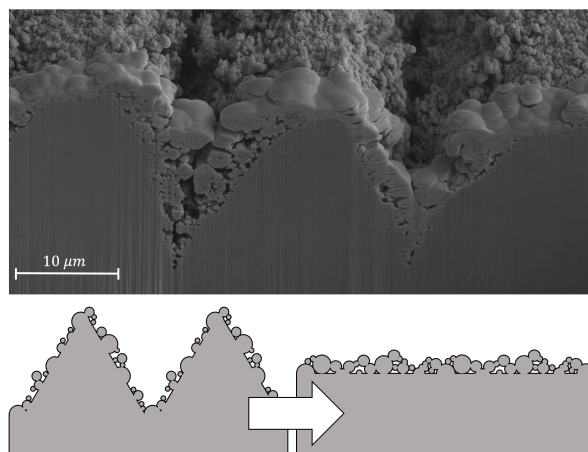


Figure 1: Top: a photo of a LESS treated sample. The lighter bubbles are the added layer of platinum to reduce the curtaining effect. Courtesy of Anite Perez Fontenla. Bottom: representation of the treated surface for the impedance models used below.

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In what follows, the effect of roughness due to the chaotically spaced features is studied without considering the periodic groove structure (Fig. 1, bottom). This, besides simplifying the problem, also reflects the fact that the grooves can be made parallel to the direction of the surface currents, minimizing their effect on the impedance. Additionally, the grooves are an artifact of the method and are not essential for reducing the SEY, and there is an effort to reduce their depth [1, 2].

Besides ignoring the periodic structure, the problem is further simplified by considering a circular axially symmetric beamscreen. In reality, however, the shape is not a circle and more importantly, the treatment might be applied to only a sector of the cross-section. In that case, contribution to the longitudinal coupling impedance should be calculated based on the average (over the angle) surface impedance. For the horizontal and the vertical transverse coupling impedances, the problem is more complex and depends on which sector is treated.

IMPEDANCE ESTIMATES

The problem is reduced to finding the surface impedance of a rough wall. Based on the approach from [6], the effect of roughness is accounted for by considering conductivity σ to be a function of depth into the wall $r - b$. Surface impedance can be defined if the ratio of the fields of a wave incident on the surface is the same for all incident angles. For this the local skin depth should be much smaller than the wavelength in vacuum [7], which means an allowed conductivity profile $\sigma(r)$ cannot slowly approach zero. Below we consider several different conductivity profiles that satisfy this condition (Fig. 2).

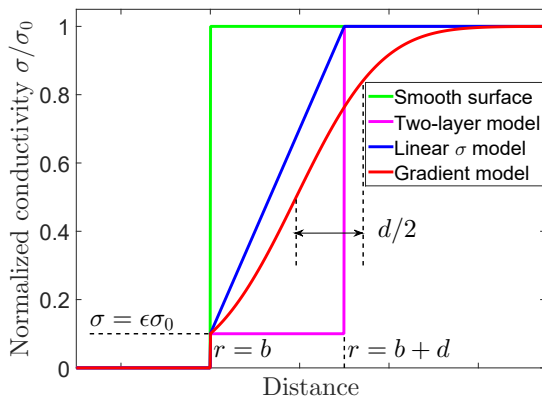


Figure 2: Normalized conductivity profiles used in the models of a rough surface.

Omitting here the derivation steps, the results for the considered conductivity profiles are presented below. We define d as the characteristic depth of the rough layer, and σ_0 as the bulk conductivity of the metal. The two dimensionless parameters are $\alpha = d\sqrt{\mu_0\omega\sigma_0}$ (the normalized roughness

depth) and $\epsilon = \sigma_{r=b}/\sigma_0$ (the normalized conductivity at the surface).

Smooth Surface

The impedance of a smooth surface is given by

$$Z_{surf}^{smooth} = \frac{2\pi b}{L} Z_{||} = \frac{\omega\pi b^3}{Lc} Z_{dip} = (1+i)\sqrt{\frac{\mu_0\omega}{2\sigma_0}} \quad (2)$$

Two-layer Model

For the two-layer model (magenta line in Fig. 2), the ratio of the impedance to that of a smooth surface is

$$\begin{aligned} \frac{Z_{surf}}{Z_{surf}^{smooth}} &= \frac{Z_{||}}{Z_{||}^{smooth}} = \frac{Z_{dip}}{Z_{dip}^{smooth}} = \\ &= \frac{1}{\sqrt{\epsilon}} \frac{1 - \Lambda e^{-\sqrt{2}(1+i)\sqrt{\epsilon}\alpha}}{1 + \Lambda e^{-\sqrt{2}(1+i)\sqrt{\epsilon}\alpha}} \end{aligned} \quad (3)$$

$$\text{where } \Lambda = \frac{1-\sqrt{\epsilon}}{1+\sqrt{\epsilon}}.$$

Linear Conductivity Model

For the linear conductivity model (blue line in Fig. 2), the ratio of the impedance to that of a smooth surface is

$$\begin{aligned} \frac{Z_{surf}}{Z_{surf}^{smooth}} &= \frac{Z_{||}}{Z_{||}^{smooth}} = \frac{Z_{dip}}{Z_{dip}^{smooth}} = \\ &= \frac{i}{\sqrt{\epsilon}} \frac{J_{1/3}(u_0) + CY_{1/3}(u_0)}{J_{-2/3}(u_0) + CY_{-2/3}(u_0)}, \end{aligned} \quad (4)$$

where J and Y are the Bessel functions of the first and the second kind, and the constants are defined as

$$C = -\frac{J_{1/3}(u_1) + iJ_{-2/3}(u_1)}{Y_{1/3}(u_1) + iY_{-2/3}(u_1)}, \quad (5)$$

$$u_0 = \frac{2i-1}{3} \frac{\alpha}{\sqrt{2}} \frac{1}{1-\epsilon} \epsilon^{3/2}, \quad u_1 = \frac{2i-1}{3} \frac{\alpha}{\sqrt{2}} \frac{1}{1-\epsilon} \quad (6)$$

Gradient Model

The gradient model first introduced in [6] is a more natural choice for the conductivity profile given by the random distribution of the rough features: $\sigma(r) = \frac{\sigma_0}{2} [1 + \text{erf}(\sqrt{2}x - \eta(\epsilon))]$ (red line in Fig. 2). Here erf is the error function, $x = (r-b)/d$, and $\eta(\epsilon) = \text{erf}^{-1}(1-2\epsilon)$. The resulting impedance can be written in a form of a numerical solution to a differential equation:

$$\frac{Z_{surf}}{Z_{surf}^{smooth}} = \frac{Z_{||}}{Z_{||}^{smooth}} = \frac{Z_{dip}}{Z_{dip}^{smooth}} = -\frac{1+i}{\sqrt{2}\mathcal{F}(\alpha, \epsilon)} \quad (7)$$

$\mathcal{F}(\alpha, \epsilon) = y|_{x=0}$, given that

$$\begin{cases} \frac{\partial y}{\partial x} = \alpha \left(-y^2 + i \frac{1 + \text{erf}(\sqrt{2}x - \eta(\epsilon))}{2} \right) \\ y(+\infty) = -\frac{1+i}{\sqrt{2}} \end{cases} \quad (8)$$

APPLICATION OF THE METHOD

Ideally, an actual conductivity profile can be obtained if enough measurement data is available (for example, by analyzing cross-sections of the material similar to Fig. 1). The method could then be applied to the obtained conductivity profile directly by numerically solving the corresponding differential equation. In practice, however, it may be easier to instead use one of the suggested models for conductivity profiles, each one using only two parameters. The parameters can be determined by a fit to the measured surface resistance, i.e. the real part of the surface impedance. Multiple experimental points necessary for the fit can be obtained by varying either the frequency or the bulk conductivity of the metal. The latter is possible, for example, for copper samples due to the strong dependence of its conductivity on temperature.

As an illustrative example, experimentally measured points for the proposed FCC-hh beamscreen sample are shown in Fig. 3. The real part of the surface impedance (surface resistance R_s) was measured by placing the sample in a cryogenic resonator and observing the change in the Q-factor. The measurements were done at the temperatures between 4 K and 16 K with little variation with temperature. No external magnetic field was applied.

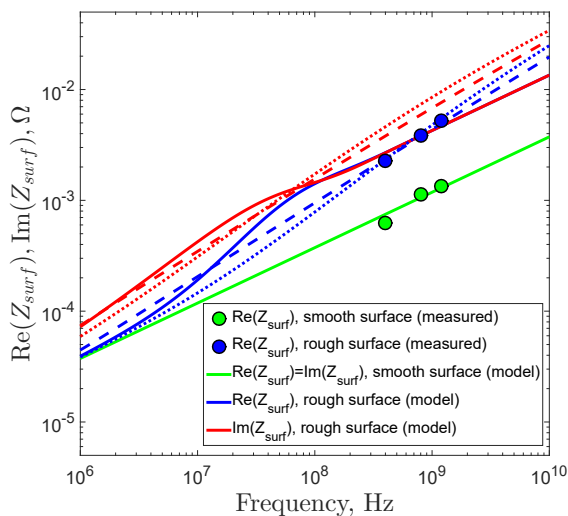


Figure 3: An example of fitting the models to the measured data (the data is a courtesy of Sarah Aull, June 2017). The bulk conductivity of metal is chosen to fit the green line to the green data points. The parameters d and ϵ are chosen to fit the blue data points: $d = 5 \mu\text{m}$, $\epsilon = 7.6 \times 10^{-2}$ for the two-layer model (solid curves), $d = 12 \mu\text{m}$, $\epsilon = 10^{-2}$ for the linear conductivity model (dashed curves), $d = 2.5 \mu\text{m}$, $\epsilon = 10^{-2}$ for the gradient model (dotted curves). The measured data points do not span a sufficient frequency range for the fit parameters to be unique, therefore example fits are shown.

By applying one of the developed roughness models, values of R_s at three different frequencies are used to reconstruct

both the real and the imaginary parts of the impedance in the entire frequency range (Fig. 3). The parameters of the models are adjusted to fit the experimental points. In order to select the appropriate conductivity model with unique parameters, more data points in a wider frequency range are required. Nevertheless, a qualitative assessment of the impedance increase at the relevant frequencies can be made. The increase in the imaginary part of impedance at the frequencies driving the single bunch instabilities is between 3.5 and 7, depending on the model. To make a definitive conclusion on the impedance impact of the FCC-hh beamscreen, more research on production and measurements of rough surfaces is required.

CONCLUSIONS

A method for evaluating the coupling impedances of a circular beamscreen with a rough surface is proposed. The method uses the relation between the surface impedance and the coupling impedance, valid in the entire frequency range of interest for the concerned beam instabilities. Three different models within the method are offered to represent roughness as a one-dimensional conductivity profile. The models predict the scaling of the impedance with the frequency, the depth of the rough layer, the bulk conductivity and the surface conductivity of the wall. The models can be checked against measurements of the surface resistivity versus frequency and can be used to estimate the imaginary part of impedance that is especially hard to measure.

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