

SYNCHRONOUS PHASE SHIFT FROM BEAM LOADING ANALYSIS*

G. Bassi[†], J. Tagger, A. Blednykh, J. Rose, V. Smaluk, BNL, Upton, NY 11973-5000, USA.

INTRODUCTION

We discuss measurements done at the NSLS-II storage ring of the synchronous phase shift as a function of single bunch current as calculated from the total RF power delivered to the beam. An online application to monitor continuously the beam loading parameters has been developed and offers the opportunity for an alternative, competitive method to measure the synchronous phase and loss factor, compared to more direct measurements techniques [1]. The synchronous phase measurements from beam loading parameters, as discussed in this paper, require the storage of uniform multi-bunch configurations, and allow the estimation of the average synchronous phase of the beam.

EQUATIONS OF MOTION FOR A SYSTEM OF TWO RF CAVITIES

Under present standard operations, the NSLS-II storage ring accumulates an average current $I_0 = 300\text{mA}$. The nominal relative energy spread and energy loss per turn induced by the operational lattice are $\sigma_\delta = 0.00087$ and $U_0 = 680\text{keV}$ respectively. The present configuration of the RF system consists of two 500 MHz superconducting RF cavities, that we label cavity C and cavity D, operated at the same RF voltage $V_C = V_D = 1.5\text{MV}$, for a total voltage $V_T = 3\text{MV}$. With equal voltages the two cavity system can be equivalently described by one cavity with $V = V_T$ and phase $\phi = \phi_s$, where ϕ_s is the synchronous phase satisfying the condition $\sin \phi_s = U_0/(eV_T)$. Configurations with unequal RF voltages have been frequently used during the commissioning phases of the NSLS-II storage ring, an option that is still considered. To this end, we discuss next the longitudinal equations of motion for a two cavity system with arbitrary voltages V_C and V_D , subjected to suitable stability constrained. The two cavities are assumed to be independently beam loading compensated.

We review first the one cavity case. Letting $\varphi = \phi - \phi_s$ and $\delta = (E - E_0)/E_0$, where ϕ_s and E_0 are the phase and energy of the synchronous particle, the dynamics of a particle with phase ϕ and energy E is governed by

$$\begin{aligned} \dot{\varphi} &= -\omega_{rf}\eta\delta, \\ \dot{\delta} &= \frac{eV_{rf}}{E_0T_0}(\sin(\varphi + \phi_s) - \sin \phi_s), \quad \sin \phi_s = \frac{U_0}{eV_{rf}}, \end{aligned} \quad (1)$$

where $\eta = 1/\gamma^2 - \alpha_c$, α_c is the momentum compaction factor, U_0 the energy loss per turn and T_0 the revolution period. Equivalently, using arrival time difference $\tau = \varphi/\omega_{rf}$, it follows

$$\dot{\tau} = -\eta\delta, \quad \dot{\delta} = \frac{eV_{rf}}{E_0T_0}(\sin(\omega_{rf}\tau + \phi_s) - \sin \phi_s). \quad (2)$$

Given the transition-gamma $\gamma_{tr} = 1/\sqrt{\alpha_c}$, stability requires $\eta > 0$ and $0 < \phi_s < \pi/2$ for $\gamma < \gamma_{tr}$, and $\eta < 0$ and $\pi/2 < \phi_s < \pi$ for $\gamma > \gamma_{tr}$. For $\omega_{rf}|\tau| \ll 1$, Eq.(2) describe harmonic oscillations with synchrotron frequency ω_s

$$\ddot{\tau} = -\omega_s^2\tau, \quad \omega_s = \omega_0\sqrt{\frac{h\eta eV_{rf} \cos \phi_s}{2\pi E_0}}. \quad (3)$$

In the case of two cavities with voltage V_C and V_D respectively, the equations of motion read

$$\begin{aligned} \dot{\tau} &= -\eta\delta, \\ \dot{\delta} &= \frac{e}{E_0T_0} \left(V_C \sin(\omega_{rf}\tau + \phi_C) \right. \\ &\quad \left. + V_D \sin(\omega_{rf}\tau + \phi_D) - \frac{U_0}{e} \right), \end{aligned} \quad (4)$$

and can be written in the form

$$\dot{\tau} = -\eta\delta, \quad \dot{\delta} = \frac{e}{E_0T_0} \left(V_T \sin(\omega_{rf}\tau + \phi_T) - \frac{U_0}{e} \right), \quad (5)$$

where V_T and ϕ_T satisfy

$$V_T^2 = V_C^2 + V_D^2 + 2V_C + V_D \cos(\phi_C - \phi_D), \quad (6)$$

$$\tan \phi_T = \frac{V_C \sin \phi_C + V_D \sin \phi_D}{V_C \cos \phi_C + V_D \cos \phi_D}. \quad (7)$$

It follows that for $\omega_{rf}|\tau| \ll 1$ Eq.(6) describe harmonic oscillations with frequency ω_T

$$\ddot{\tau} = -\omega_T^2\tau, \quad (8)$$

$$\omega_T = \omega_0\sqrt{\frac{h\eta eV_T \cos \phi_T}{2\pi E_0}} = \sqrt{\omega_C^2 + \omega_D^2}, \quad (9)$$

where

$$\omega_{C,D} = \omega_0\sqrt{\frac{h\eta eV_{C,D} \cos \phi_{C,D}}{2\pi E_0}}. \quad (10)$$

STEADY STATE BEAM LOADING COMPENSATION

Figure 1 illustrates the beam loading compensation scheme for an RF cavity system with a given RF voltage V_{rf} and synchronous phase ϕ_s . The beam loading parameters are for the present standard operations of the NSLS-II storage ring at the average current $I_0 = 300\text{mA}$, where the two cavities have equal voltage $V_C = V_D$, for a total voltage $V_T = V_{rf} = 3\text{MV}$. Here we briefly give a qualitative description of the beam loading scheme following [2], without giving the specific values of the parameters. In Fig. 1, the red phasors \mathbf{V}_{rf} , \mathbf{V}_g and \mathbf{V}_b represent the RF, generator and beam voltage respectively, and the green phasors \mathbf{V}_{gr} and

* Work supported by DOE contract DE-SC0012704

[†] gbassi@bnl.gov

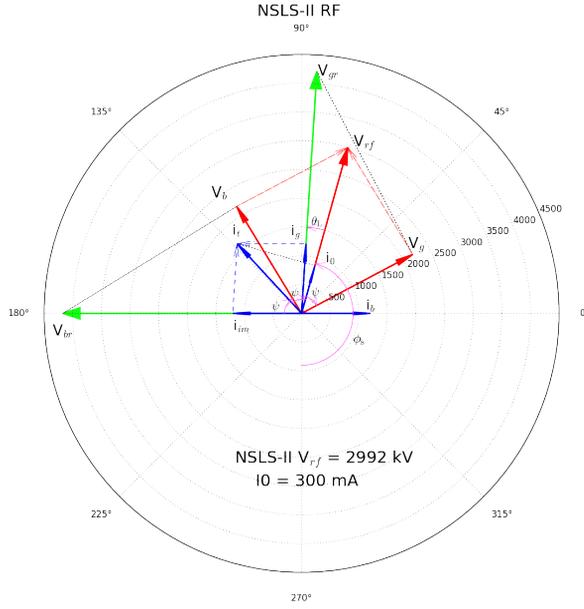


Figure 1: Phasor plot for steady state beam loading compensation. The beam loading parameters are for present standard NSLS-II operations at $I_0 = 300\text{mA}$: two cavities with equal voltage $V_C = V_D = 1.5\text{ MV}$, for a total voltage $V_{rf} = 3\text{MV}$.

V_{br} the generator and beam voltage at resonance, i.e.. for $\psi = 0$, where ψ is the detuning angle defined as

$$\tan \psi = 2Q_L \frac{\omega_r - \omega_{rf}}{\omega_{rf}}, \quad (11)$$

where ω_r and Q_L are the frequency and loaded quality factor of the cavity respectively. The blue phasors \mathbf{i}_t , \mathbf{i}_{im} , \mathbf{i}_g and \mathbf{i}_0 represent current phasors. \mathbf{i}_t , \mathbf{i}_{im} and \mathbf{i}_g correspond to the total, image, generator current respectively, and \mathbf{i}_0 corresponds to the generator current when the cavity is at resonance and there is no circulating beam. They are defined by the following equations

$$\begin{aligned} V_{gr} &= i_g R_L, \quad V_{br} = i_{im} R_L, \quad i_{im} = 2I_0, \quad i_0 = \frac{V_{rf}}{R_L}, \\ R_L &= \frac{R_s}{1 + \beta}, \quad Q_L = \frac{Q_0}{1 + \beta}, \end{aligned} \quad (12)$$

where R_s and Q_0 are the unloaded cavity shunt impedance and quality factor respectively, and β the beta-coupling. θ_L is the load angle and corresponds to the angle between the generator current and RF voltage phasors.

By projecting the current phasors along and perpendicular to the RF voltage phasor it follows

$$\tan \theta_L = \frac{i_0 \tan \psi - i_{im} \cos \phi_s}{i_0 + i_{im} \sin \phi_s}, \quad (13)$$

$$i_g = \frac{i_0 + i_{im} \sin \phi_s}{\cos \theta_L}. \quad (14)$$

From the definition of generator power

$$P_g = \frac{(1 + \beta)^2 V_{gr}^2}{8\beta R_s}, \quad (15)$$

and the cosine law for the triangle given by \mathbf{V}_g , \mathbf{V}_b and \mathbf{V}_{rf}

$$\begin{aligned} V_{gr}^2 &= V_{br}^2 + V_{rf}^2 (1 + \tan^2 \psi) \\ &- 2V_{br} V_{rf} (\tan \psi \cos \phi_s - \sin \phi_s), \end{aligned} \quad (16)$$

it follows

$$\begin{aligned} P_g &= \frac{R_s}{8\beta} [(i_0 + i_{im} \sin \phi_s)^2 + (i_0 \tan \psi - i_{im} \cos \phi_s)^2] \\ &= \frac{R_s}{8\beta} \frac{(i_0 + i_{im} \sin \phi_s)^2}{\cos^2 \theta_L}. \end{aligned} \quad (17)$$

The minimum generator power is clearly obtained for $\theta_L = 0$, thus imposing the in-phase condition between \mathbf{i}_g and \mathbf{V}_{rf} . From conservation of energy it follows that the reflected power P_r satisfies

$$P_r = P_g - P_c - P_b, \quad (18)$$

where the cavity power P_c and beam power P_b are given by

$$P_c = \frac{V_{rf}^2}{2R_s}, \quad P_b = \frac{1}{2} i_{im} V_{rf} \sin \phi_s. \quad (19)$$

For $\theta_L = 0$, from Eq.(11) and Eq.(13) it follows that the detuning frequency reads

$$\Delta\omega = \omega_r - \omega_{rf} = \frac{\omega_{rf} R_L i_{im} \cos \phi_s}{2Q_L V_{rf}}. \quad (20)$$

SYNCHRONOUS PHASE MEASUREMENTS

The application that has been developed at NSLS-II to monitor continuously the beam loading parameters calculates the synchronous phase as follows. From the forward and reflected RF cavity powers measured at pick-up locations, applying Eq.(18) and Eq.(19) to both cavity C and D , the cavity phases ϕ_C and ϕ_D can be calculated to determine the synchronous phase with the use of Eq.(6) and Eq.(7).

Measurements of the synchronous phase shift as a function of single bunch current at $V_{rf} = 3.4\text{MV}$ are shown in Fig. 2, where, at the same average current $I_0 = 100\text{mA}$, four different uniform filling patterns with number of bunches $M = 200$, $M = 100$, $M = 400$ and $M = 1200$ have been injected, corresponding to a single bunch current $I_b = 0.5\text{mA}$, $I_b = 1\text{mA}$, $I_b = 0.25\text{mA}$ and $I_b = 0.08\text{mA}$ respectively. Figure 3 shows the synchronous phases in the four cases, with the measurements lasting between 15–30 minutes. The faster decay of the average current at higher single bunch current is clearly visible as shown by the orange trace, indicating a lower beam life time at higher single bunch current. The average values of the synchronous phases as shown in Fig. 3, subtracted the extrapolated value of the synchronous

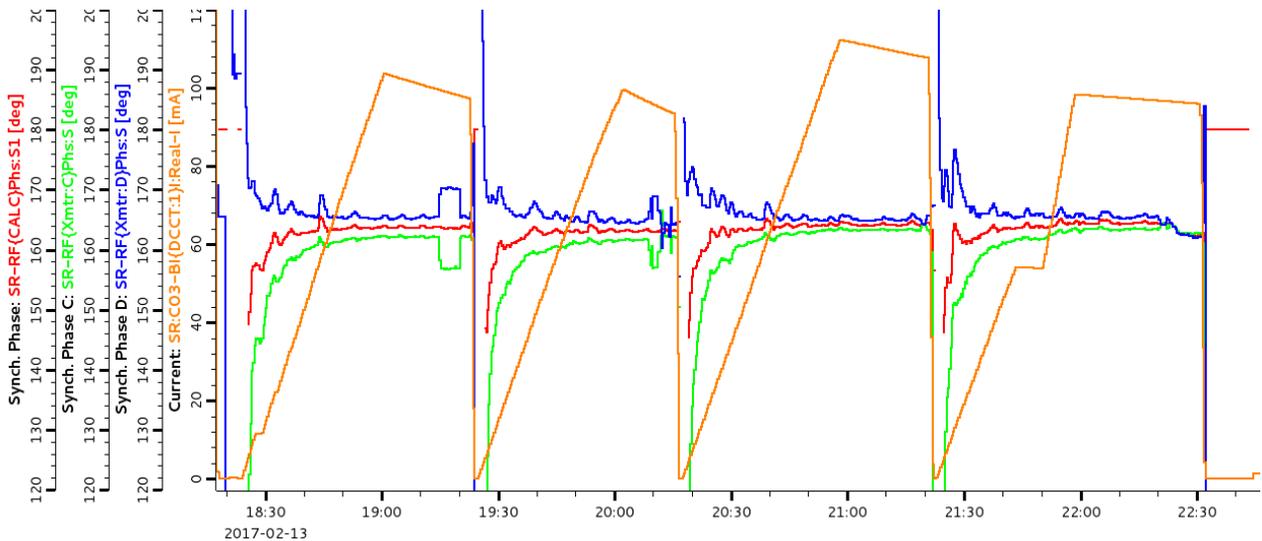


Figure 2: Synchronous phases from beam loading parameters as monitored during measurements at $I_0 = 100\text{mA}$ for four different uniform filling patterns with number of bunches $M = 200$, $M = 100$, $M = 400$ and $M = 1200$, corresponding to a single bunch current $I_b = 0.5\text{mA}$, $I_b = 1\text{mA}$, $I_b = 0.25\text{mA}$ and $I_b = 0.08\text{mA}$ respectively.

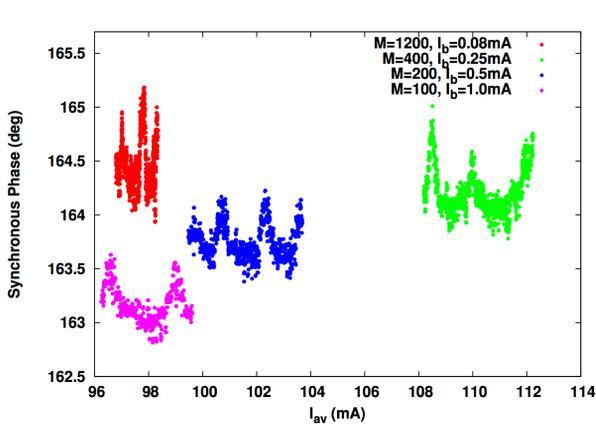


Figure 3: Synchronous phase measurements from beam loading parameters in the NSLS-II storage ring for different bunch configurations, as described in Fig. 2.

phase at zero current, and then multiplied by ω_{rf} to give the bunch centroid, are shown in Fig. 4, and compared with direct measurements from streak camera and oscilloscope at $V_{rf} = 3\text{MV}$ [1]. A very good agreement is found, within the precision of 1ps of direct measurements. This can be explained by the observation, confirmed by measurements [1], of a weak dependence of the synchronous phase shift on the RF voltage, as corroborated by the addition of two measure-

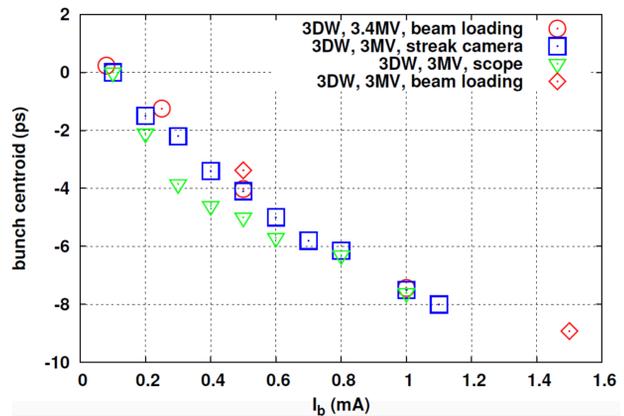


Figure 4: Comparison of synchronous phase measurements from beam loading parameters with streak camera and oscilloscope measurements in the NSLS-II storage ring.

ments at $V_{rf} = 3\text{MV}$ from beam loading analysis in Fig. 4.

REFERENCES

- [1] W. Cheng *et al.*, “Precise synchronous phase measurements” presented at IPAC’17, Copenhagen, Denmark, May 2017, paper MOPAB152, this conference.
- [2] K. Y. Ng, “The Physics of Intensity Dependent Beam Instabilities”, World Scientific, Singapore, 2006.