

PRELIMINARY STUDY OF BEAM DYNAMICS COMPENSATION FOR THE ELLIPTICALLY POLARIZED UNDULATOR AT THE HLS-II

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Abstract

An elliptically polarized undulator (EPU) was installed at the upgraded Hefei Light Source, HLS-II, for special users. Due to that the area of good field of the EPU is not large enough, the resulting beam dynamics is serious. At present, the lattice is changed to lower beta functions at the EPU to solve this problem. However, the compensation for the EPU is necessary for better operation of the machine in the future. In this paper, we used the surface fitting method to extract the Hamiltonian of the EPU from the real surface magnetic field data. Thus, we can obtain the effective Hamiltonian of the ring, which can be analyzed using normal form or other techniques. Then the beam dynamics effects resulting from the EPU can be compensated by optimizing the nonlinear quantities with striplines.

INTRODUCTION

The elliptically polarized undulator EPU104 is to obtain high-brightness coherent synchrotron radiation for the users of ARPES beam-line. Hence study of the nonlinear effects of the undulator on the particle beam is a realistic issue. The first step towards study of EPU104 is to rebuild the internal space magnetic field numerically. A Radia software package was applied to conduct the simulation of EPU104 because of its faster and more accurate computation of the magnetic field compared with Finite Element Methods (FEM) [1]. The Radia package was specially designed and optimized for solving problems of insertion devices. The main parameters of EPU104 are listed in Table 1 and a simulation model of EPU104 with 5 periods is shown in Figure 1.

With the resulting magnetic field data from the Radia package, surface fitting method was used to extract the vector potential and compute the exact Lie transfer map. Such an approach, taking all effects of the EPU into account, can guarantee that the results not only satisfy Maxwell equations exactly but also are insensitive to the errors in the surface data [2]. Thus, this method is more precise than other methods based on numerical differentiation such as Finite-Difference Method and Differential Algebra Method [3]. In addition, the results from surface method are convenient and easy for the implementation of Lie transfer map. And Lie transfer map can be implemented through symplectic integrators. Symplectic integrators aim at splitting the Hamiltonian into several simple terms to produce an approximation of

the desired order [4]. Symplectic integration keeps the symplectic property of Lie transfer map and plays an important role in long-term beam dynamic study. As for the beam dynamic compensation, striplines, instead of shimming, are applied to compensate the errors of magnetic field integrals and correct the multipole field. Compared with shimming method, stripline method is more efficient and easier for operation.

Table 1: Parameters of EPU104

Type	APPLE-II
Period length	104 mm
Period number	31
Structure length	3224 mm
Vertical/Horizontal gap	30~80 mm/2.8 mm
Remanence	1.25 T
Magnetic material	NdFeB N38SH
Standard magnetized block	32mm x 32mm x 26mm
End-magnetized block 1	32mm x 32mm x 6.5mm
End-magnetized block 2	32mm x 32mm x 19.5mm

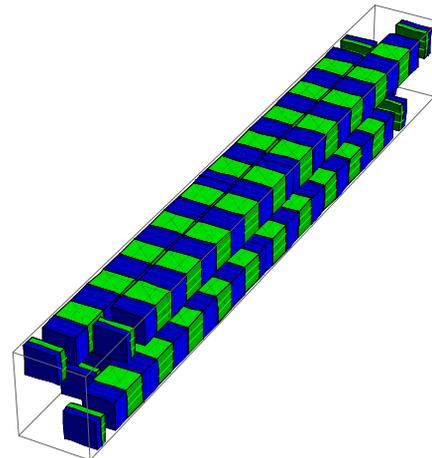


Figure 1: A simulation model of EPU104 with 5 periods.

EXTRACTION OF HAMILTONIAN

Surface fitting method is to make use of magnetic field data on a given surface which surrounds the design orbit and extends into the fringe-field region, to characterize the magnetic vector potential and compute the Hamiltonian with a collection of functions called general on-axis gradients [5]. The impacts on beam dynamic resulting from nonlinear fringe-field and high-order-multipole effects are implied in the surface magnetic field data, so the Hamiltonian extracted by surface fitting method is more accurate. Expansions of

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vector potential with Coulomb gauge can be expressed as:

$$\begin{aligned}
 A_x &= \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^l \frac{m!}{2^{2l+1} l! (l+m+1)!} [C_{m,s}^{[2l+1]}(z)(x^2+y^2)^l \\
 &\quad \text{Re}[(x+iy)^{m+1}] - C_{m,c}^{[2l+1]}(z)(x^2+y^2)^l \text{Im}[(x+iy)^{m+1}]] \\
 A_y &= \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^l \frac{m!}{2^{2l+1} l! (l+m+1)!} [C_{m,c}^{[2l+1]}(z)(x^2+y^2)^l \\
 &\quad \text{Re}[(x+iy)^{m+1}] + C_{m,s}^{[2l+1]}(z)(x^2+y^2)^l \text{Im}[(x+iy)^{m+1}]] \\
 A_z &= \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^l \frac{m!}{2^{2l} l! (l+m)!} [C_{m,c}^{[2l]}(z)(x^2+y^2)^l \\
 &\quad \text{Im}[(x+iy)^m] - C_{m,s}^{[2l]}(z)(x^2+y^2)^l \text{Re}[(x+iy)^m]]
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 C_{m,\alpha}^{[n]}(z) &= \frac{i^n}{2^m} \int_{-\infty}^{\infty} dk e^{-ikz} k^{m+n} G_{m,\alpha}(k) \\
 \alpha &= s \text{ or } c
 \end{aligned} \tag{2}$$

General on-axis gradients have different expressions for different geometrical cylinder surface, and are insensitive to the noise in the original data [6,7]. In this paper, general on-axis gradients for EPU104 are worked out with magnetic field data on surface of rectangular cylinder (60mm x 24mm x 4000mm) under conditions of horizontal polarized mode and minimum working vertical gap. The results are shown in Figure 2 and Figure 3. Theoretically, $C_{1,s}^{[0]}$ is equal to the on-axis vertical magnetic field B_y . Figure 4 shows the errors of vertical magnetic field obtained by two methods: surface method and Radia software package calculation.

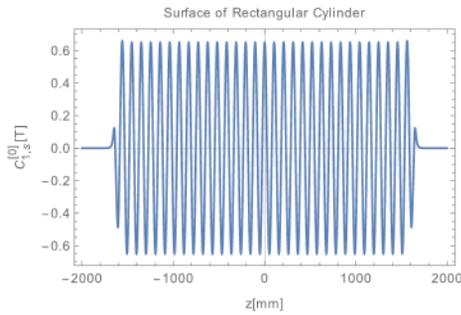


Figure 2: General on-axis gradient $C_{1,s}^{[0]}(z)$.

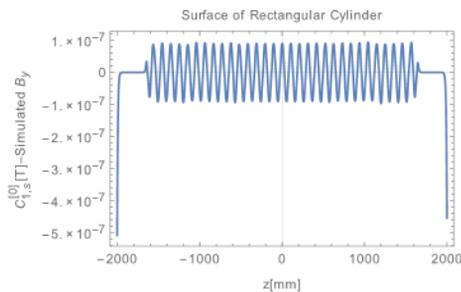


Figure 3: Errors between general on-axis gradient $C_{1,s}^{[0]}$ and vertical magnetic field calculated by Radia ($x=y=0$).

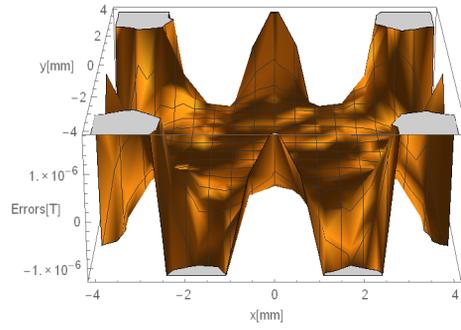


Figure 4: Errors of vertical magnetic field ($z=0$) obtained by surface method and Radia software package calculation.

IMPLEMENTATION OF TRANSFER MAP

As for implementation of transfer map, symplectic property and accuracy are the key point to be focused on. Symplectic integrators are those methods making an approximation to the exact Lie transfer map in such a way that the new Lie map has an explicit form and the transformation can be represented as finite power series [8]. Y. K. Wu developed an explicit symplectic integration technique by extending phase space, and such a method is suitable for s-dependent case [9]. The transformations of 800-MeV electron are shown in Figure 5 and Figure 6. Figure 7 and Figure 8 show the comparisons of results obtained by symplectic integration technique and explicit Runge-Kutta method. The initial conditions are given: $x=y=p_x=p_y=0$.

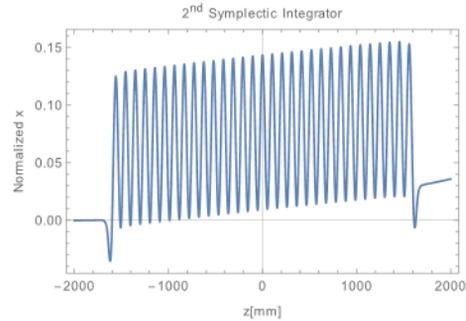


Figure 5: Normalized motion trajectory of horizontal direction of 800-MeV electron with 2nd order symplectic integrator.

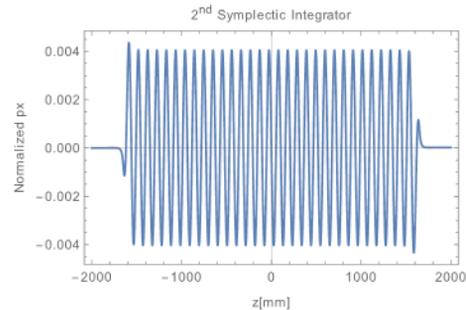


Figure 6: Normalized momentum trajectory of horizontal direction of 800-MeV electron with 2nd order symplectic integrator.

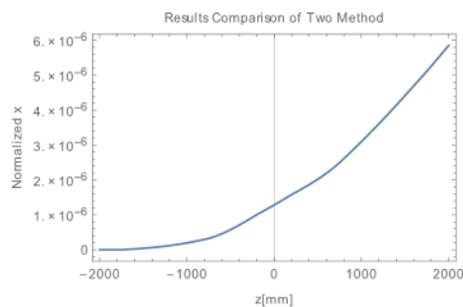


Figure 7: The comparison of normalized motion trajectory (obtained by 2nd order integrator and 4th order eRK method) of horizontal direction of 800-MeV electron.

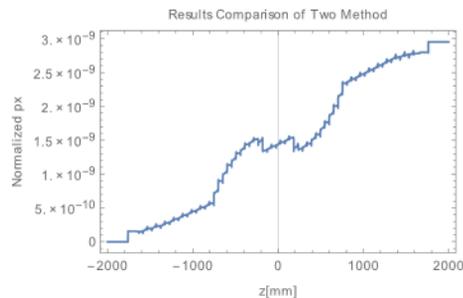


Figure 8: The comparison of normalized momentum trajectory (obtained by 2nd order integrator and 4th order eRK method) of horizontal direction of 800-MeV electron.

COMPENSATION WITH STRIPLINES

In the area of insertion devices, shimming method is widely used to correct magnetization errors, but this method is too complicated and inefficient in the calculation and implementation process. Stripline method shows its great advantages in these aspects. Back to EPU104, eight striplines are distributed averagely and symmetrically above and below the vacuum chamber for beam dynamic compensation. Striplines can produce specific 3D magnetic field with different currents, which should be adjusted in accordance with the objective functions of the nonlinear effects of EPU104, such as magnetic field integrals and dynamic aperture. Intelligent algorithms are adopted to choose the optimal parameters for the objective functions. Preliminarily, magnetic field integrals are set as the objective functions to be optimized. Stripline is 4mm x 2mm x 4000mm in volume, and the gap between striplines on the same horizontal plane is 1 mm. The trajectories after current compensation are shown in Figure 9 and Figure 10.

SUMMARY

We are carrying out the study on the dynamic effects of EPU104 step by step. From establishing the simulation of EPU104 to the striplines compensation for field integral, study of EPU104 goes more and more in-depth. In our future work, dynamic aperture will be the main key point. Along with intelligent optimization algorithm, stripline method is used to compensate those quantities resulting in the shrink of dynamic aperture. We attempt to accomplish

the compensation of nonlinear effects to enlarge DA.

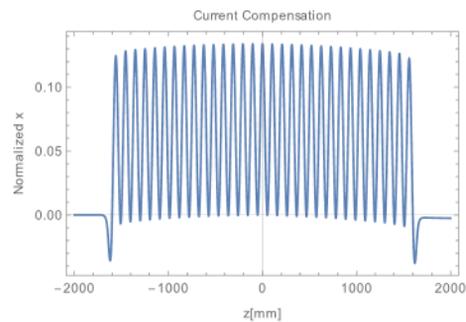


Figure 9: Normalized motion trajectory of horizontal direction of 800-MeV electron after current compensation.

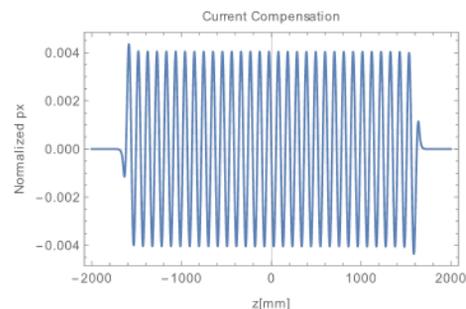


Figure 10: Normalized momentum trajectory of horizontal direction of 800-MeV electron after current compensation.

ACKNOWLEDGEMENT

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