TWISS PARAMETER MEASUREMENT AND APPLICATION TO SPACE CHARGE DYNAMICS

K. Ohmi*, N. Kuroo[1], S. Igarashi, Y. Sato, T. Toyama, KEK, Tsukuba, Japan
H. Harada, S. Hatakeyama, JAEA/J-PARC, Tokai-mura, Japan
A. Wegscheider, R. Tomas, CERN, Geneva, Switzerland
[1] also University of Tsukuba, Tsukuba, Japan

Abstract

We are looking for feasible and quantitative method to evaluate space charge induced beam loss in J-PARC MR. One possible way is space charge simulation and theory based on measured Twiss parameter. Twiss parameter measurement using turn-by-turn monitors is presented. Resonance strengths of lattice magnets and space charge force are estimated by the measured Twiss parameters. Emittance growth and beam loss under the resonance strengths are discussed.

INTRODUCTION

Beam loss and emittance growth due to space charge force is caused by resonance excitation due to nonlinear components. We use one turn map to study nonlinear effects in circular accelerators ($\boldsymbol{x}(s+C) = \mathcal{M}\boldsymbol{x}(s)$). Nonlinear components in accelerators are nonlinear magnets such as sextupoles and octupoles, and space charge force. One turn map including the space charge force is expressed as follows,

$$\mathcal{M}(s) = \prod_{i=0}^{N-1} e^{-:H_I(\boldsymbol{x},s_i):} M(s_i, s_{i+1}), \quad (1)$$

where H_I is generator of transformation of nonlinear element, magnets or space charge. where $M(s_i, s_{i+1})$ is a linear operator for transformation from s_i to s_{i+1}. Note that operator product is done left to right different from matrix product.

$$\begin{aligned}\mathcal{M}(s) &= \left[\prod_{i=0}^{N-1} M^{-1}(s_i, s) e^{-:H_I(\boldsymbol{x},s_i):} M(s_i, s)\right] M(s), \\ &= \left[\prod_{i=0}^{N-1} e^{-:H_I(M(s,s_i)\boldsymbol{x},s_i):}\right] M(s), \quad (2)\end{aligned}$$

where $M(s)$ is linear operator for one revolution at s.

$$\mathcal{M}(s) \approx \exp\left[- : \oint H_I(M(s,s')\boldsymbol{x}, s')ds' :\right] M(s) \quad (3)$$

In one turn map, nonlinear components can be integrated with transferring the variables to locations of nonlinear elements.

We now consider sextupole magnets,

$$H_I = \frac{K_2}{6}(x^3 - 3xy^2). \quad (4)$$

x is represented by variables at reference position s.

$$x(s') = \sqrt{2J_x\beta_x(s')}\cos(\theta_x + \phi_x(s')) + \eta_x(s')\delta \quad (5)$$

where $\phi(s')$ is betatron phase difference from s. y is represented by similar form with $x \to y$.

The integral is expressed by

$$\oint H_I ds' = \frac{1}{6}G_{30}J_x^{3/2} + \frac{1}{2}G_{12}J_x^{1/2}J_y, \quad (6)$$

$$\begin{aligned}G_{30}(s) &= \int_0^s ds' K_2(s')\beta_x^{3/2}(s')e^{3i\phi_x(s')}, \quad (7)\\ G_{1\pm2}(s) &= \int_0^s ds' K_2(s')\beta_x^{1/2}(s')\beta_y(s')e^{i(\phi_x(s')\pm2\phi_y(s'))}.\end{aligned}$$

Lattice of J-PARC MR has three fold symmetry. We have to take into account the symmetery in one turn map. When the tune is slightly deviate from nonstructural resonance, Eq.(3) gives a finite nonlinear term even in perfect three hold symmetry is kept. Thus we have to consider map of 1/3 revolution.

$$\mathcal{M}_{\frac{p}{3}}(s) \approx \exp\left[- : \int_{\frac{(p-1)L}{3}}^{\frac{pL}{3}} H_I(M(s,s')\boldsymbol{x}, s')ds' :\right] M_{\frac{p}{3}}(s) \quad (8)$$

where $p = 1, 2, 3$.

J-PARC MR is operated at tune $(\nu_x, \nu_y) = (21.35, 21.45)$. Considering negative tune shift due to the space charge force, (1) $3\nu_x = 64$, (2) $\nu_x + 2\nu_y = 64$ and (3) $\nu_x - 2\nu_y = 21$ are resonances closed to the tune operating point. (1) and (2) are nonstructural resonance, while (3) is structure differential resonance. Parameter list are shown in Table 1.

Table 1: Parameter List of J-PARC MR

	injection	extraction
Circumference C (m)	1567	
Energy E (GeV)	3	30
Bunch population	3×10^{13}	
# bunches/harmonics	8/9	
Emittance (10^{-6}) m	50	5
Tune (x/y)	21.35/21.45	

MEASUREMENT OF BETATRON MOTION

Betatron motion is excited by kickers in the horizontal and/or vertical directions. Beam position is measured at

* ohmi@post.kek.jp

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186 BPM's in horizontal and vertical turn-by-turn. The positions are stored in 230 turns. Betatron frequency is determined by Fourier Transformation of the turn-by-turn data. Betatron phase is determined by real/imaginary part of the Fourier coefficient at the frequency peak. Figure 1 shows an example of horizontal betatron phase (top) and beta function (bottom) along MR. The fluctuation of the phase is $\sigma_{\phi_x} = 0.0028$ and 0.0025 for the two shots. Phases of two shots, red and blue lines, are plotted in the figure. The two lines do not coincide each other. Power supply for main magnets fluctuate with frequency of 100Hz-100kHz, that results in shot-by-shot tune fluctuation 0.005. Therefore the phase difference between two shots seems to be real. Horizontal beta function in bottom is calculated by the phase using the three BPM method. The standard deviations from setting values, $\sigma_{\Delta\beta/\beta}$ are 3.3% and 3.7% for the two shots.

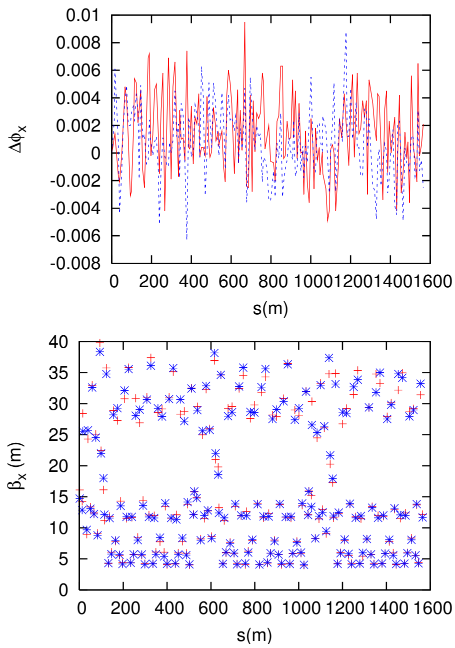

Figure 1: Measured horizontal betatron phase difference from a setting lattice and beta function evaluatd by the phase.

RESONANCE STRENGTH

Resonance terms, $G_{30}, G_{1\pm2}$ are evaluated by Eq.(7) using measured beta/phase and sextupole strengths. Figure 2 shows real and imaginary parts of G_{30} along MR. G for the two shots are plotted with red(real), magenta(imag.) and blue(real), cyan(imag.) lines. G_{30} jumps at sextupole location. The variation of G_{30} reflects 3 fold symmetry of the ring. Phase advance of each 1/3 ring is $\nu_x = 21.35/3$. The G_{30} is integrated with phase $3\phi_x$. The phase $3\phi_x$ advances by 21.35 for each 1/3 ring. Space charge force defocus particles in the beam depending on the amplitude. The tune becomes $21.\dot{3}$ at an amplitude. Phase variation in G_{30} at the amplitude is $21.\dot{3}$ for each 1/3 ring. When G_{30}'s for every 1/3 ring are equal, integral of G_{30} whole ring is cancelled. When tune is deviate from $21.\dot{3}$, even G_{30}'s for every 1/3 ring are equal, integral of G_{30} whole ring has finite value. Nonlinear component in one turn map in Eq.(2) does not give strength of resonances for MR with 3 fold symmetry. Nonlinear component of each 1/3 ring is evaluated as shown in Fig. 3. Magenta points are given for design lattice. Three points have the same value ($G_{30} = 0.1 - 0.37i$). Red and blue points are given for two measurements shown in Fig. 1. The same analysis was done for G_{1+2} as shown in Fig. 4.

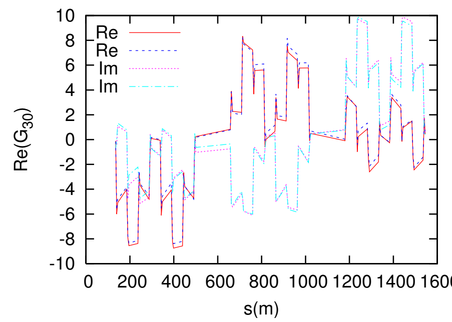

Figure 2: G_{30} given by measured betatron phase.

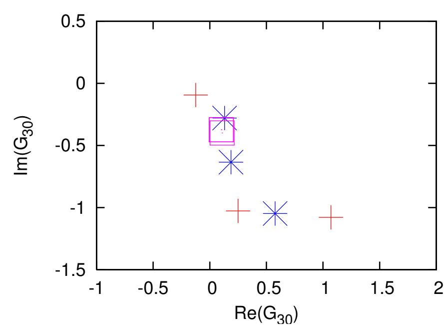

Figure 3: G_{30} for every 1/3 ring.

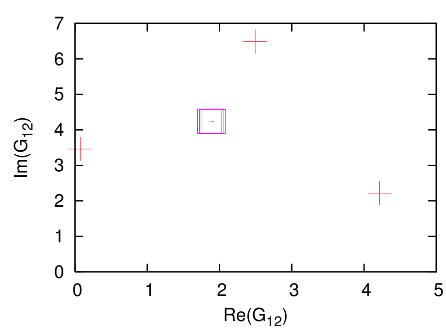

Figure 4: G_{1+2} for every 1/3 ring given by measured betatron phase.

BPM ERRORS

Beam position monitor has a reading error of 300 μm for turn-by-turn mode [1]. The error is transferred to betatron phase error. Monte Carlo simulation for the BPM error are performed for 5 samples of random seeds. The phase error is $\sigma_\phi = 0.0013$ and 0.0011 for horizontal and vertical. The phase error is 1/2 of measurement: i.e. the measured phase error in Fig. 1 is meaningful error.

Figure 5 shows G_{30} and G_{1+2} for the samples. The spread of G_{30} and G_{12} is narrower than the measurement in Figs.3 and 4. When the reading error is 100 μm, G_{30} and G_{12} distribute 1/3 area; that is, the measurement error is negligible.

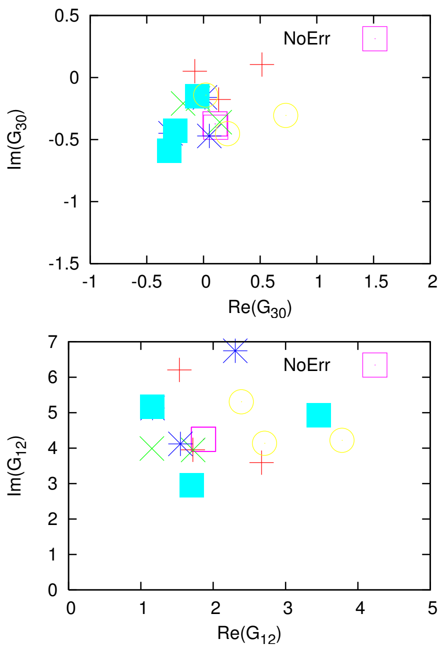

Figure 5: G_{30} and G_{1+2} given by Monte-Carlo simulation for BPM errors.

EFFECTS OF THE RESONANCES

We discuss emittance growth for the measured resonance strengths. Nonlinearity in phase space is characterized by the resonance width. The resonance width [2] is measure of the growth rate.

$$\Delta J_x = 4m_x\sqrt{\frac{U\boldsymbol{m}}{\Lambda}} \qquad \Delta J_y = 4m_y\sqrt{\frac{U\boldsymbol{m}}{\Lambda}}. \tag{9}$$

where Λ is determined by nonlinear force/potential,

$$\Lambda \equiv m_x^2\frac{\partial \nu_x}{\partial J_x} + m_x m_y\frac{\partial \nu_x}{\partial J_y} + m_y^2\frac{\partial \nu_y}{\partial J_y}\bigg|_{\boldsymbol{J}=\boldsymbol{J}_R}. \tag{10}$$

where $\boldsymbol{J}_R = (J_{x,R}, J_{y,R})$ is betatron amplitude in which the resonance condition is satisfied. The nonlinear motion satisfied the following relation,

$$\frac{\Delta J_x}{m_x} = \frac{\Delta J_y}{m_y}. \tag{11}$$

This relation shows that differential resonances are not serious compare with sum resonances.

$$U_{30} = \frac{G_{30}}{6}J_x^{3/2}\bigg|_{J_x=J_{x\,R},} \qquad U_{1+2} = \frac{G_{1+2}}{2}J_x^{1/2}J_y\bigg|_{\boldsymbol{J}=\boldsymbol{J}_R} \tag{12}$$

G's in Eq.(12) are the values at $\boldsymbol{J}_R$, in which the resonance condition is satisfied. G's seen in Figs. 3 and4 are the values at the operating point (21.35,21.45). G's at the resonance condition depends on how tune/betatron phase shifts along the ring due to space charge force. Here we assume betatron phase changes $1/9\times 2\pi$ radian in each 1/3 ring to evaluate G_{30}. G_{30}'s in Fig. 3 are taken summation with phase difference $\Delta 3\phi_x = 2\pi/3$ in each 1/3 ring. For G_{1+2}, $\Delta(\phi_x + 2\phi_y)$ is assumed $2\pi/3$ in each 1/3 ring. For the structure resonance, G_{1-2}, G's are taken summation without phase.

$$\begin{aligned} G_{30} &= G_{30,1/3} + e^{i2\pi/3}G_{30,2/3} + e^{i4\pi/3}G_{30,3/3} \\ G_{1+2} &= G_{1+2,1/3} + e^{i2\pi/3}G_{1+2,2/3} + e^{i4\pi/3}G_{1+2,3/3} \\ G_{1-2} &= G_{1-2,1/3} + G_{1-2,2/3} + G_{1-2,3/3} \end{aligned} \tag{13}$$

G's are given for the red points in Figs. 3 and 4 as follows,

$$G_{30} = 0.63{+}1.63i \quad G_{1+2} = -7.0{-}3.9i \quad G_{1-2} = -39{-}8.9i \tag{14}$$

The resonance width for G's are

$$\Delta J_{x,30} = 1.1\mu\text{m} \quad \Delta J_{x,1+2} = 2.4\mu\text{m} \tag{15}$$

where $\partial\nu/\partial J \approx 2\times 10^4$ m^{-1} [3]. $J_{xy,R} = 20$ μm, which depends on the operating point and beam intensity, is assumed. For BPM error 300μm, $|G_{30}| \sim 0.5$ and $|G_{1+2}| \sim 3.5$ are induced. BPM resolution 100 μm is better to estimate the resonance width.

CONCLUSIONS

In J-PARC MR, third order resonances are cancelled by the three fold symmetry of the lattice. Third order resonances are induced by errors of optics. Betatron function and phase are measured using turn-by-turn monitor. Resonance strength induced by sextupole magnets are evaluated by the measured beta and phase. Resonance width under the space charge tune spread is about 5-10% of the emittance.

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