THREE DIMENSIONAL WAKE FIELD FOR ELECTRONS MOVING IN UNDULATOR

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Abstract

Electro-magnetic field for given trajectory of an electron is calculated by Lienard-Wiechert potential. The field near the electron moving in an undulator is presented. The field is regarded as a wake field in the undulator. We calculate the wake field based on the integrated Green function, which is used to analyze a bunch motion..

INTRODUCTION

Charged particles moving with the position and velocity $(\boldsymbol{x}'(t'), \boldsymbol{v}'(t'))$ given as a function of time t' induce an electromagnetic field in space-time $(\boldsymbol{x}, t)$ as follows,

$$\boldsymbol{E} = \frac{e}{4\pi\varepsilon_0}\left[\frac{\boldsymbol{n}-\boldsymbol{\beta}'}{\gamma^2\kappa^3R^2} + \frac{\boldsymbol{n}\times((\boldsymbol{n}-\boldsymbol{\beta}')\times\boldsymbol{\alpha}')}{\kappa^3R}\right] \quad (1)$$

$$\boldsymbol{B} = \frac{1}{c}\boldsymbol{n}\times\boldsymbol{E}. \quad (2)$$

We call a moving charged particle that induces an electromagnetic field as a source particle. $\boldsymbol{R}$ is a vector from the position of the source particle $(\boldsymbol{x}')$ to the position $(\boldsymbol{x})$ to observe the electromagnetic field, R and $\boldsymbol{n}$ are its norm and unit vector.

$$\boldsymbol{R} = \boldsymbol{x} - \boldsymbol{x}' \qquad R = |\boldsymbol{R}| \qquad \boldsymbol{n} = \frac{\boldsymbol{R}}{R}. \quad (3)$$

$\kappa = 1 - \boldsymbol{n}\cdot\boldsymbol{\beta}'$ and $\boldsymbol{\alpha}' = d\boldsymbol{\beta}'/d(ct')$. The relation between the time at which the source particle is moving (t') and the observed time (t) is given by

$$t = t' + \frac{R}{c}. \quad (4)$$

We are interested in motion of the beam. Another charged particle (called observation particle) is placed in the observation position $(\boldsymbol{x}, t)$ The position to observe the electromagnetic field is very close to the source particle. The observed particles which move at a speed $\boldsymbol{\beta}$, experience Lorentz force.

We use the position along beam line, s as time variable. Longitudinal variable z is difference of arrival time for light emitted at $s = 0$, $z = c(t_0 - t) = s - ct$, where $t = 0$ is arrival time of the light, $s = ct_0$. Canonical momentum for z is $\Delta E/E_0$. The transverse axis on the plane of moving particle is x and that perpendicular to x and s is y. The canonical momenta (p_x, p_y) are normalized by E_0/c. Electro-magnetic field at s is induced by the source particle at a different location, s', which is determined by the time relation of Eq.(4). The relation between s and s' is translated to

$$s - z = s' - z' + R(x, y, s - s'). \quad (5)$$

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ELECTRO-MAGNETIC FIELD NEAR SOURCE PARTICLE IN UNDULATOR

The motion of the particles in the undulator is represented as function of s' by:

$$\begin{aligned}
x'(s') &= \frac{K}{\bar{p}_s\gamma k_u}\sin k_u s', \qquad \bar{p}'_s \equiv \sqrt{1 - \frac{1+K^2/2}{2}}, \\
z'(s') &= -\frac{1+K^2/2}{2\gamma^2}s' - \frac{K^2}{8\gamma^2 k_u}\sin 2k_u s,' \\
\beta'_x(s') &= \frac{p'_s}{\bar{p}'_s}\frac{K}{\gamma}\cos k_u s', \\
\beta'_s(s') &= \frac{1}{1 + \frac{1+K^2/2}{2\gamma^2} + \frac{K^2}{4\gamma^2}\cos 2k_u s'} \equiv p'_s, \qquad (6) \\
\alpha'_x(s') &= -\frac{p'^2_s K k_u}{\bar{p}'_s\gamma}\sin k_u s + \frac{K^3 k_u}{2\bar{p}'_s\gamma^3}\sin 2k_u s' \cos k_u s', \\
\alpha'_s(s') &= \frac{p'_s\beta^2_s K^2 k_u}{2\gamma^2}\sin 2k_u s'.
\end{aligned}$$

All variables are expressed by s' explicitly in this expression. We first give x' and z' expressions as an approximation, and β and α are given by t' derivative of x', z', where $d/(cdt') = (1 - dz/ds)^{-1}d/ds'$. This expression is approximation, because $\beta_x^2 + \beta_s^2 \neq 1 - 1/\gamma^2$. We use these expressions to treat detailed positions of source and observer in the field calculation.

K and k_u characterize the magnetic field of the undulator.

$$B_y(s) = B_0 \sin k_u s \qquad K = \frac{B_0}{mck_u} \quad (7)$$

We calculate the electromagnetic field at (x, y, z) at a certain s. Lorentz force, which another charged particle located at $(x, y, z; s)$ experiences, is evaluated by the electromagnetic field. Lorentz force, which is convoluted by the beam distribution, is used to study a collective motion of the beam.

It is necessary to know the position s' where the source electron induces an electromagnetic field at $(x, y, z; s)$. Eq. (5) gives an implicit relation for s', thus root finding has to be done. Once s' is given, particle motion is determined by Eq.(6).

The relation between $z - s'$ for given x is shown in Fig. 1. By the way, this figure is given without solving s', but is plotted z for given x, s'. The parameters used are energy E = 8 to GeV, $\lambda_u = 2\pi/k_u = 1.8$ cm, $K = 1.5$. $s = 1$ m.

Fig.1 showed singular behavior at $s' \approx 1$ m, $-z = (1 + K^2/2)/(2\gamma^2) = 4.3$ mm. This is due to that Eq.(5) is treated with different approaches for $s > s'$ or $s < s'$. s' can be far smaller than s, since R and $(s - s')$ cancel for $s > s'$. $s \approx s'$ for $s < s'$. Oscillation amplitude of orbit in the undulator

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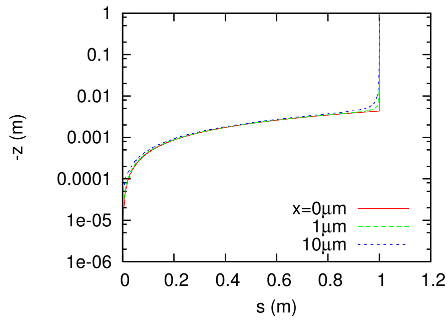

Figure 1: $z - s'$ relation at $s = 1$ m.

($|x'| = K/(k_u\gamma) = 0.27$ μm) is negligibly small compare with the beam size ($\sigma_x \approx 23$ μm). Ignoring the modulation in z, Eq.(5), which is reduced to a quadratic equation, has the following approximated solution,

$$s' = \frac{-z + \frac{s-z}{2\gamma_z^2} - \sqrt{\left(z + \frac{s}{2\gamma_z^2}\right)^2 + \left(\frac{1}{\gamma_z^2} + \frac{1}{4\gamma_z^4}\right)(x^2+y^2)}}{\frac{1}{\gamma_z^2} + \frac{1}{4\gamma_z^4}}. \tag{8}$$

This equation gives an approximated s' for given z. It can be used as an initial value for the root finding of Eq.(5).

When solving using the derivative in $s > s'$, the z modulation in the undulator period disturbs the convergence to solve Eq.(5) for an initial value of s' far from the solution. By using the approximate s', the solution is smoothly given.

$\kappa = (1 - \boldsymbol{n} \cdot \boldsymbol{\beta}')$ characterizes relation of the observation position (x, y, s) for moving direction of the source particle. Figure 2 shows κ. The behavior changes drastic at $z = -4.3$ nm similar as Fig.1. κ has frequency component of $(1 + K^2/2)\lambda_u/\gamma^2$for $z > -4.3$ nm. The frequency in κ becomes slower $z < -4.3$ nm for $x \neq 0$.

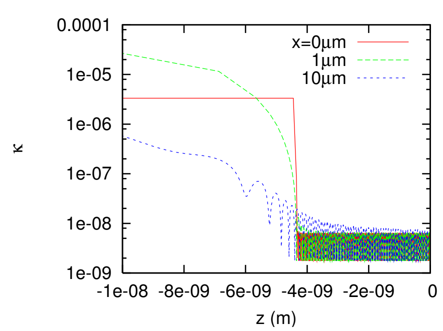

Figure 2: κ as function of z.

Now we are ready to calculate the electro-magnetic field and Lorentz force at position (x, y, z, s) for single electron. For given (x, y, z, s), s' is obtained by root finding. Particle motion in Eq.(6) is given as a function of s'. Electromagnetic field is evaluated by Eq.(1).

Figure 3 and 4 show Lorentz force F_z and F_x for longitudinal and horizontal directions. The force is calculated along z for $x = 0, 1, 10$ μm. Source electron is located at $z = -4.3$ nm ($s = 1$ m). Fig. 3(top) depicts F_z near the source electron. Lorentz force is singular near the electron. For $x = 0$, F_z is repulsive negative at downstream Periodic force, at upstream at $z > -4.3$ nm is seen in undulator. The period $\lambda_r = 0.15$ nm is consistent with the formula, $(1 + K^2/2)\lambda_u/\gamma^2$. The spiky force indicates to contain high frequency component. Fig. 3(bottom) depicts F_z at $z \sim -4$ nm. The force is periodic and spiky, but phase shift for x is seen.

Figure 4 shows the horizontal Lorentz force. The behavior is similar to that of F_s.

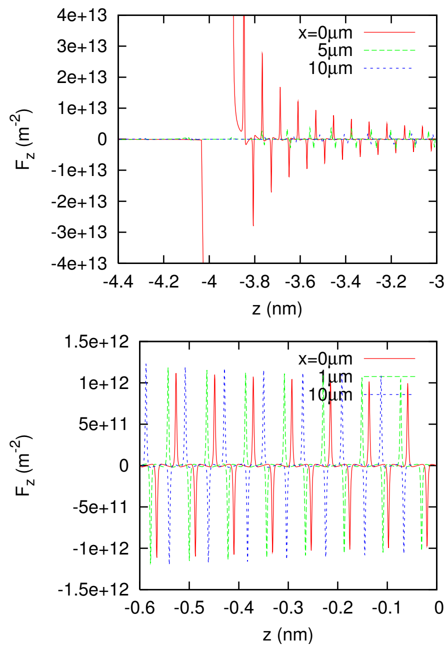

Figure 3: Lorentz force F_z near source particle. Top and bottom depict that near and upstream of the source particle.

INTEGRATED GREEN FUNCTION

To study beam motion under the self electro-magnetic field, high frequency component, which we are not interested in, is removed by the Integrated Green Function. We study the beam motion, whose the transverse size is 23 μm and the length is 1 μm. Mesh size is chosen $\Delta x = \Delta y = 4$ μm in transverse, and the covered area is $\pm 64 \times 64$ μm^2 with 32×16 meshes, where the vertical force is symmetric. Longitudinal mesh is chosen $\Delta z = 0.01$ nm, because the undulator radiation is essential for the beam motion. The integrated Lorentz force is integrated once more for s along electron trajectory with multiplying the velocity to calculate the energy loss/gain.

$$\int_0^{L_u} \beta_s(s)ds \int_{\Delta x \Delta y \Delta z} F_s(x, y, z, s)dxdydz$$

$$\int_0^{L_u} \beta_x(s)ds \int_{\Delta x \Delta y \Delta z} F_x(x, y, z, s)dxdydz \tag{9}$$

The integration step inside of a mesh in x, y, z is $dx = 0.1\lambda_c$ [1], where λ_c is characteristic wave length of the undulator

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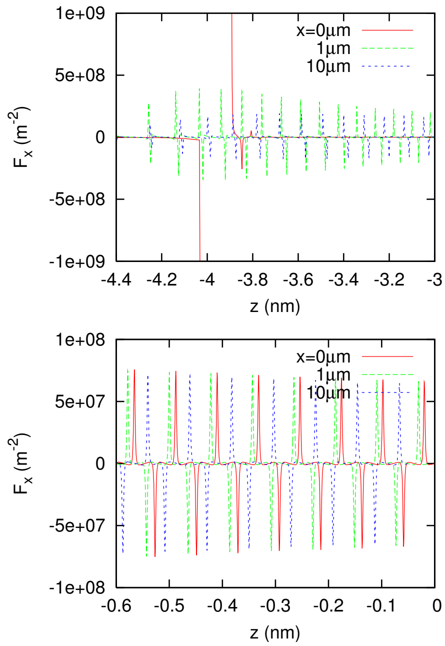

Figure 4: Lorentz force F_x near source particle. Top and bottom depict that near and upstream of the source particle.

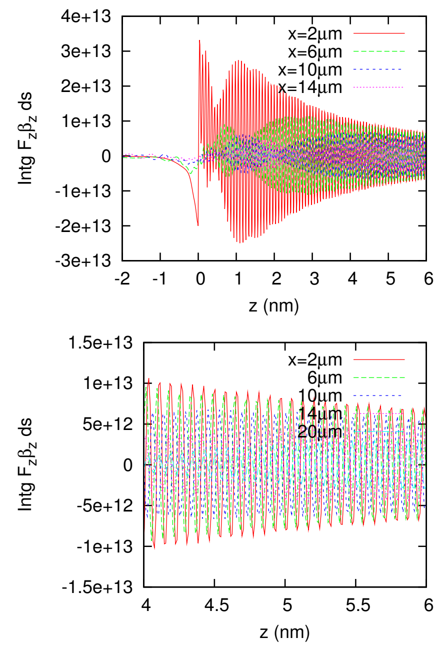

Figure 5: Integrated Lorentz force, F_z. The scale of z is different in top and bottom.

radiation, $\lambda_c = 4\pi E/(3\gamma^3 ecB_0)$. The step for transverse is $dx, dy = 2\gamma dz$. The spiky behavior and phse shift seen in Figs.3 and 4 are smeared and averaged by the integral. Figures 5 and 6 show integrated Lorentz force of F_z and F_x, respectively. $z = 0$ is re-coordinated as the position of the source electron. Oscillation for z corresponds to undulator radiation. The force basically decreases for a large z, because of radiation from early s. We can see a phase shift for x. For a large x, the force increases large z. The coherence for the undulator radiation is recovered at a large z.

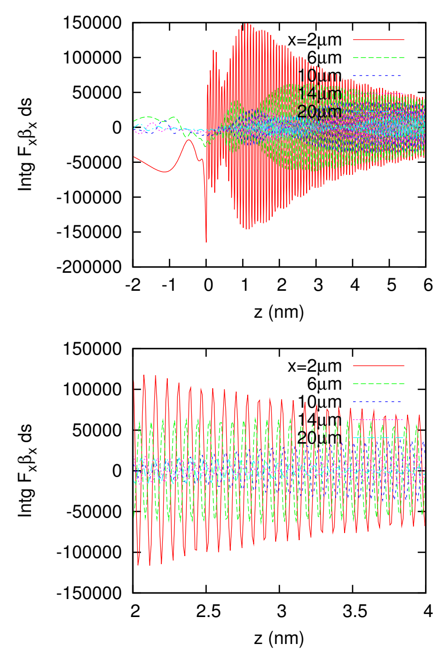

Figure 6: Integrated Lorentz force, F_z. The scale of z is different in top and bottom.

CONCLUSION

Electro-magnetic field was evaluated for an electron moving in an undulator using Lienard-Wiechert potential. Integrated Green Function was obtained from Lorentz force of the electro-magnetic field. Effects of three dimensional near field on beam in undulator can be studied using the Integrated Green Function.

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REFERENCES

[1] K. Ohmi, S. Chen, H. Tanaka, proceedings of IPAC16, WEPOY004, p. 2997, Busan, Korea, 2016.