OBSERVING SUPPRESSION OF SYNCROTRON OSCILLATION AMPLITUDES

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Abstract

We proposed a method to reduce loosing particles in acceleration stage of synchrotrons. A slowly varying horizontal electrostatic field may be useful to de-excite synchrotron oscillations. Then we have to somehow observe the damping of amplitudes of synchrotron oscillations to confirm the effect. We assume that the synchrotron component of rationalized Hamiltonian in acceleration stage is kept constant. Our experimental results did not contradict with this assumption. Taking advantage of this assumption, we can easily confirm the damping of synchrotron oscillation amplitudes experimentally through the increase of synchrotron frequencies.

INTRODUCTION

The author reported the energy exchange between the synchrotron and the betatron oscillations through the synchro-betatron difference resonant coupling [1]. Using an external power supply and inside situated electrodes to couple the synchrotron and the betatron oscillations, we may difference resonate the synchrotron oscillation with the betatron oscillation. The synchrotron oscillation damps out through the betatron oscillation. Then particles near the separatrix orbit might be pushed back deep inside the bucket to reduce the number of losing particles [2]. Here we discuss a mechanism to observe de-excitement of the synchrotron oscillation through observing increase of its frequency.

We consider a reference particle (RP) of mass m. The particle is revolving on the reference closed orbit (RCO) of an average radius R_0 with a revolution frequency ω_0, a velocity v_0, a total energy E_0 and a momentum p_0. In the right-handed curvilinear coordinate system (x,s,z), $p_{x,s,z}$ is the canonical momentum and s is the orbit length. We have $\vec{p}_0=(p_x,p_s,p_z)$, $p_0^{\,2}=p_x^2+p_s^2+p_z^2$ and $p_0\approx p_s$. $\overline{x}$ and $\overline{p}_x$ are the off-momentum horizontal coordinate and momentum. We neglect the vertical motion and put $z=0$ and $p_z=0$.

From Eq. (21) of the reference 1, we have the rationalized Hamiltonian H, which is composed of coasting, synchrotron and betatron motions, for an orbiting particle as follows.

$$H=-\frac{p_s}{p_0}$$

$$=-(1+\delta_C+\delta_S)+\frac{1}{2}\left(\frac{\bar{p}_x}{p_0}\right)^2+\frac{1}{2}K_x\overline{x}^2+\frac{1}{2}(-\eta)(\delta_C+\delta_S)^2 \quad (1)$$

$$-\frac{hqV}{2\pi\beta_0^2E_0}\{\cos(\phi+\phi_D)-\cos(\phi_S+\phi_D)+(\phi-\phi_S)\sin(\phi_S+\phi_D)\}$$

where $\delta_C=\frac{\Delta E}{\beta_0^2E_0}$, $v_0=R_0\omega_0$, $\frac{ds}{dt}=v_0$, $\beta_0=\frac{v_0}{c}$, $p_0=m\gamma\beta_0c$, $E_0=m\gamma c^2$, $\gamma=\frac{1}{\sqrt{1-\beta_0^2}}$, $K_x=\frac{1}{\rho^2}-\frac{1}{B_0\rho}\frac{\partial B_z}{\partial x}$, $\psi_S=\phi_S+\phi_D$ and $\phi_D=-\frac{D}{R_0}\left(\frac{\bar{p}_x}{p_0}\right)+\frac{D'}{R_0}\overline{x}$.

D is the dispersion function, δ_C is the energy deviation of the coasting motion, δ_S the energy deviation of the synchrotron oscillation, ρ is the curvature of the dipole B-field, h is the harmonic number, η is the phase slip factor and ψ_S is the phase angle for the synchronous particle. V is the effective rf cavity voltage seen by particles per passage [3]. We divide H into three components: H_C for the coasting particle, H_β for the betatron oscillation and H_S for the synchrotron oscillation.

$$H_C=-(1+\delta_C+\delta_S)\ . \quad (2)$$

$$H_\beta=\frac{1}{2}\left(\frac{\bar{p}_x}{p_0}\right)^2+\frac{1}{2}K_x\overline{x}^2\ . \quad (3)$$

In fact, $\phi_D\ll 1$ and $\psi_S\sim\phi_S\to\pi$, we have

$$H_S=\frac{(-\eta)}{2}(\delta_C+\delta_S)^2-\left(\frac{\omega_S}{\omega_0}\right)^2\frac{(1+\cos\phi)}{|\eta|}\ . \quad (4)$$

$$\frac{\omega_S^2}{\omega_0^2}=\frac{hqV\,|\eta\cos\ (\phi_S+\phi_D)|}{2\pi\beta_0^2E_0}=\frac{hqV\,|\eta|}{2\pi\beta_0^2E_0}\ . \quad (5)$$

where ω_S is the synchrotron frequency and $H=H_C+H_\beta+H_S$. In Eq. (4), we call the first term the dispersion part and the second term the potential part.

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HAMILTONIAN CHANGE IN ACCELELATION STAGE

We assume accelerations affect only δ_C but not δ_S.

a) Original circulating RP is on RCO. The rationalized Hamiltonian is consists of H_C, H_β and H_S.

From Eq. (1),

$$-\frac{p_s}{p_0} = -(1+\delta_C+\delta_S) + H_\beta + H_S. \quad (6)$$

b) When the on-momentum particle is accelerated, δE is the amount of increase in kinetic energy, δp is the increase of momentum and δR is the increase of average radius. Defining $\Delta = \frac{\delta E}{\beta_0^2 E_0}$, we put $\delta_C \to \delta_C + \Delta$. The energy deviation ΔE and the orbital momentum are increased in acceleration stage $\Delta E \to \Delta E + \delta E$ and $p_s \to p_s + \delta p$. In Eq. (2), (3), (4) and (5), we define as: $H_C, H_S, H_\beta \to H'_C, H'_S, H'_\beta$ and $K_x, \eta, \omega_s \to K'_x, \eta', \omega'_s$

We have

$$H'_C = -(1+\delta_C+\Delta+\delta_S). \quad (7)$$

$$H'_\beta = \frac{1}{2}\left(\frac{\bar{p}_x}{p_0}\right)^2 + \frac{1}{2}K'_x\bar{x}^2. \quad (8)$$

$$H'_S = \frac{(-\eta')}{2}(\delta_C+\Delta+\delta_S)^2 - \left(\frac{\omega'_S}{\omega_0}\right)^2 \frac{(\cos\phi+1)}{|\eta'|}. \quad (9)$$

$$-\frac{p_s+\delta p}{p_0} = -(1+\delta_C+\Delta+\delta_S) + H'_\beta + H'_S. \quad (10)$$

Some parameter changes are shown with "→".

v_0, E_0, $\Delta E \to \Delta E + \delta E$, p_0, $p_s \to p_s + \delta p$, $R_0 \to R_0 + \delta R$, ω_0

c) The dipole magnetic field has to be increased so that the particle goes back to the original RCO under the velocity $v_0 + \delta v$. The particle is revolving with an average radius R_0, a revolution frequency $\omega_0 + \delta\omega$, a total energy $E_0 + \delta E$ and momentum $p_0 + \delta p$. However, the energy deviation is reduced back to the original values $\Delta E + \delta E \to \Delta E$ ($\delta E, \Delta \Rightarrow 0$), but $E_0 \to E_0 + \delta E$. Notice $p_s + \delta p$ stays the same value. In Eq.(7), we put $\delta_C + \Delta \to \delta''_C$. In Eq.(7), (8) and (9), we define as:

$H'_C, H'_\beta, H'_S \to H''_C, H''_\beta, H''_S$ and $K'_x, \eta', \omega'_s \to K''_x, \eta'', \omega''_s$.

We have

$$H''_C = -(1+\delta''_C+\delta_S). \quad (11)$$

$$H''_\beta = \frac{1}{2}\left(\frac{\bar{p}_x}{p_0+\delta p}\right)^2 + \frac{1}{2}K''_x\bar{x}^2. \quad (12)$$

$$H''_S = \frac{(-\eta'')}{2}(\delta''_C+\delta_S)^2 - \left(\frac{\omega''_S}{\omega_0}\right)^2 \frac{(\cos\phi+1)}{|\eta''|}. \quad (13)$$

Some parameter changes are shown with "→".

$v_0 \to v_0 + \delta v$, $E_0 \to E_0 + \delta E$, $\Delta E + \delta E \to \Delta E$,

$p_0 \to p_0 + \delta p$, $p_s + \delta p$, $R_0 + \delta R \to R_0$, $\omega_0 \to \omega_0 + \delta\omega$

Since $\delta_C = \frac{\Delta E}{\beta_0^2 E_0}$ and $\beta_0 = \frac{v_0}{c}$,

$$\delta''_C = \frac{\Delta E}{(\beta_0+\delta\beta)^2(E_0+\delta E)} \approx \frac{\Delta E}{(\beta_0)^2 E_0}\left(1-\frac{\delta\beta}{\beta_0}\right)^2\left(1-\frac{\delta E}{E_0}\right)$$

$$\approx \frac{\Delta E}{(\beta_0)^2 E_0}\left(1-\frac{\delta E}{E_0}-\frac{2\delta\beta}{\beta_0}\right) \approx \frac{\Delta E}{(\beta_0)^2 E_0}.$$

In the acceleration stage δ_C has the same value: $\delta''_C \approx \delta_C$ ($\delta_C + \Delta \to \delta''_C \to \delta_C$) and $H''_C \to H_C$.

Since $-\frac{p_s+\delta p}{p_0+\delta p} \approx -\frac{p_s}{p_0}\left(1+\frac{\delta p}{p_s}\right)\left(1-\frac{\delta p}{p_0}\right) \approx -\frac{p_s}{p_0}$.

LHS and the 1st term in RHS (coasting part) of Eq. (10) return to the value before the acceleration. When the dipole magnetic field is increased after the acceleration,

$$-\frac{p_s}{p_0} = -(1+\delta_C+\delta_S) + H''_\beta + H''_S. \quad (14)$$

Subtracting Eq. (14) from Eq. (6), we have

$$H_\beta + H_S = H''_\beta + H''_S. \quad (15)$$

The synchrotron plus betatron Hamiltonian is kept in the same value in the acceleration stage. Unless there is a special mechanism to exchange energy between the synchrotron and the betatron oscillations, each Hamiltonian keeps the same value independently. Finally, we obtain

$$H_C = H''_C, \quad H_\beta = H''_\beta \quad \text{and} \quad H_S = H''_S. \quad (16)$$

Each component of the rationalized Hamiltonian H is kept constant in the acceleration stage.

EXPERIMENTAL RESULTS

Figure 1 and 2 are images of the spectrum analyzer (Tektronix RSA 3303B) taken for $^{12}C^{6+}$ beam of a heavy ion synchrotron at Gunma University Heavy Ion Medical Center (GHMC) [4]. Two symmetrical sidebands of synchrotron oscillations $\pm\omega_s$ around the revolution frequenc ω_0 of two different energy were clearly recorded. The image was taken by the electrostatic beam positioning monitor, of which plate was used as an antenna and $V_{\rm rf}$, which is proportional to V, is the peak voltage of the rf cavity. Fig. 1 is the spectrum image of 400 MeV/u beam ($V_{\rm rf} = 174\rm V$). Two sidebands $\pm\omega_s$ are a little less than 200Hz. Their signal strengths are longer than that of the revolution frequency ω_0 (3.37809MHz). Fig. 2 is the spectrum image of 290 MeV/u beam ($V_{\rm rf} = 280\rm V$). Two sidebands $\pm\omega_s$ are a little more than 300Hz. Its signal strength is equivalent to that of the revolution frequency ω_0 (3.05760MHz). Apparently synchrotron frequency ω_s is larger than the case of 400 MeV/u beam since its energy 290 MeV/u is less and its peak voltage is larger (See Eq. (5)). On the other hand, its signal strength of the synchrotron oscillation (Fig. 2) is less than that of 400MeV/u beam (Fig. 1).

DISCUSSION

We assume that the phase slip factor has the same value since their RCO are the same. Since the synchrotron Hamiltonian H_S, which is constant in the acceleration stage, consists of the dispersion and the potential parts, the potential part increases when the dispersion part decreases and vice versa.

As we discussed in the previous section, there is an apparent difference between Fig.1 and Fig.2. In Fig.2 (290 MeV/u beam), the potential part in Eq.(15) is expected to be larger than that of Fig.1(400 MeV/u beam) because its value of $\frac{\omega_s}{\omega_0}$ is larger. Then its dispersion part should be smaller. We observe shorter signal strength of the synchrotron oscillation in Fig.2 compared the signal strength in Figs.1. Although measurements of the rationalized dispersion δ_S are necessary for precise argument, signal strengths may proportional to values of the dispersion part, which is expected to be smaller in Fig.2 than that of Fig.1. Our argument that H_S is constant looks promising.

Taking advantage of this assumption, we might observe the variation of δ_S when a slowly varying perpendicular electrostatic field couples the synchrotron and the betatron oscillations through the difference resonance coupling. If the synchrotron oscillation amplitude is decreased, the synchrotron frequency will increase and the synchrotron oscillation damping is easily observable. We plan more precise experiments.

The synchrotron oscillation frequency $\pm\omega_s$ (peaks in both sides) are a little less than 200Hz. Their signal strengths are longer than that of the revolution frequency ω_0 (centre peak 3.3780955MHz).

The synchrotron oscillation frequency $\pm\omega_s$ (peaks in both sides) are a little larger than 300Hz. Their signal strengths are equivalent to that of the revolution frequency ω_0 (centre peak 3.0576025MHz).

Apparently ω_s is larger than the case of 400 MeV/u beam (Fig.1) since its energy 290 MeV/u is less and its peak voltage of the rf cavity is larger. On the other hand, their synchrotron signal strengths are less than that of 400 MeV/u beam.

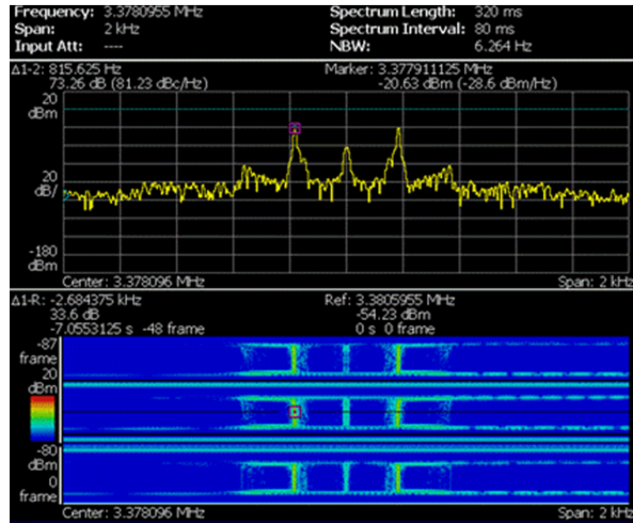

Figure 1: the spectrum image of 400 MeV/u beam ($V_{\rm rf} = 174\rm V$).

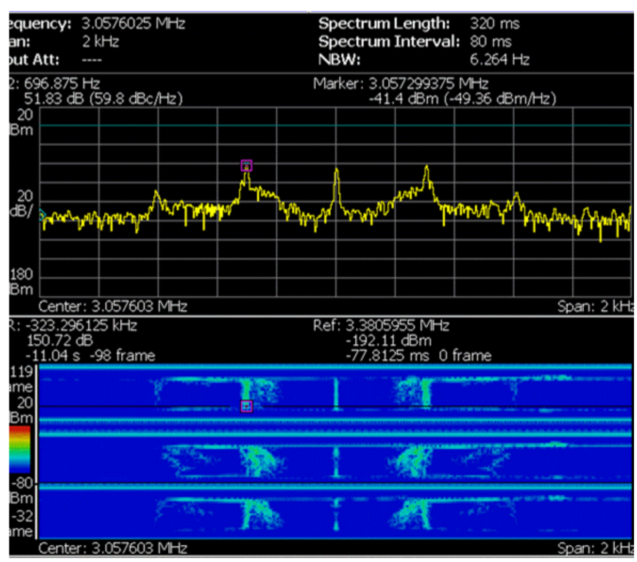

Figure 2: the spectrum image of 290 MeV/u beam ($V_{\rm rf} = 280\rm V$).

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