

THE PHASE SLIP FACTOR OF THE ELECTROSTATIC CRYOGENIC STORAGE RING CSR

M. Grieser*, R. von Hahn, S. Vogel, A. Wolf
 Max-Planck Institut für Kernphysik, 69117 Heidelberg, Germany

Abstract

To determine the momentum spread of an ion beam from the measured revolution frequency distribution, the knowledge of the phase slip factor of the storage ring is necessary. The slip factor was measured for various working points of the cryogenic storage ring CSR at MPI for Nuclear Physics, Heidelberg and was compared with simulations. The predicted functional relationship of the slip factor and the horizontal tune depends on the different islands of stability, which has been experimentally verified. This behavior of the slip factor is in clear contrast to that of magnetic storage rings.

INTRODUCTION

The cryogenic storage ring CSR [1] shown in Fig. 1, is a fully electrostatic storage ring used to store atomic, molecular and cluster ion beams in the energy range of $q \cdot (20-300)$ keV, where q is the absolute value of the ion charge state. The entire storage ring can be cooled down to temperatures of only a few Kelvin. This very low temperature creates an extremely high vacuum. The observations from first cryogenic operation indicate the residual gas densities below 100 molecules/cm³. The cooling of all ion optics and the vacuum enclosure to 10 K temperature also provides the benefit of uniquely low level of blackbody radiation where the stored molecular ion beams reach their lowest quantum states. In March 2014 the functionality of CSR was demonstrated by storing a 50 keV ⁴⁰Ar⁺ beam under room temperature conditions. Later, in 2015, the storage ring was cooled down to an average temperature below 10 K. At this temperature the storage time for singly charged ions achieved up to 2500 s. The details about CSR are given elsewhere [1].



Figure 1: The cryogenic storage ring CSR with 35 m circumference.

* Manfred.Grieser@mpi-hd.mpg.de

THE PHASE SLIP AND MOMENTUM COMPACTION FACTOR

The momentum spread ($\Delta p/p$) of an stored ion beam can be determined by using the measured revolution frequency distribution and the phase slip factor η , which is defined by:

$$\eta = \frac{\Delta f/f_0}{\Delta p/p_0}, \quad (1)$$

where f_0 is the revolution frequency and p_0 the momentum of the central ion. The momentum and frequency deviation of an ion to the central particle is given by Δp and Δf , respectively. To calculate the slip factor of the storage ring we consider an ion moving with energy E on its closed orbit, where the revolution time ($T = 1/f$) related to the revolution frequency f is given by:

$$T = \int_0^{C(E)} \frac{1}{v(s')} ds'. \quad (2)$$

The length of the closed orbit of an ion with energy E is $C(E)$ and $v(s')$ denotes the velocity ion velocity, depending on its position s' . It is favorable to describe the ion motion in the coordinate system where the central particle is moving on its central orbit. In this coordinate system the differential ds' of the variable s' is transformed to ds [2] as follows:

$$ds' = \left(1 + \frac{x(s)}{\rho(s)}\right) ds, \quad (3)$$

where s is the position on the central orbit, $x(s)$ denotes the horizontal deviation of the ion orbit to the central orbit and $\rho(s)$ is the radius of the central orbit.

In the coordinate system of the central particle the revolution time T of an ion on its closed orbit is given by:

$$T = \int_0^{C_0} \frac{1 + \frac{x(s)}{\rho(s)}}{v(s)} ds. \quad (4)$$

An electrostatic storage ring can be used only for non relativistic ion velocities $v(s)$ and hence $v(s)$ at the potential $\phi(s)$ is given by:

$$v(s) = \sqrt{\frac{2}{m} (E - Q\phi(s))}. \quad (5)$$

Since the CSR consists of electrostatic cylindrical deflectors and electrostatic quadrupoles, the potential $\phi(s)$ can be expressed by:

$$\phi(s) = V_0(s) \ln \left(\frac{\rho(s) + x(s)}{\rho(s)} \right) + K(s)(x(s)^2 - y(s)^2), \quad (6)$$

where $V_0(s) = \frac{2E_0}{Q}$ for cylindrical deflectors. E_0 denotes the energy and Q the charge of the central ion. Using the Taylor series expansion, Eq. (6) can be expanded in $x(s)$ as:

$$\phi(s) = V_0(s) \frac{x(s)}{\rho(s)} - V_0(s) \frac{x(s)^2}{2\rho(s)^2} + K(s)(x(s)^2 - y(s)^2) + \dots \quad (7)$$

Neglecting second order and higher terms in Eq. (7) results in:

$$\phi(s) = V_0(s) \frac{x(s)}{\rho(s)}. \quad (8)$$

Using Eq. (8) in Eq. (5) gives with Eq. (4):

$$T = \int_0^{C_0} \frac{1 + \frac{x(s)}{\rho(s)}}{\sqrt{\frac{2}{m}(E - 2E_0 \frac{x(s)}{\rho(s)})}} ds. \quad (9)$$

Eq. (9) can be expanded in $x(s)$. Considering only first order terms of $x(s)$ yields in:

$$T = \sqrt{\frac{m}{2E}} C_0 + \frac{E_0 + E}{E} \sqrt{\frac{m}{2E}} \int_0^{C_0} \frac{x(s)}{\rho(s)} ds. \quad (10)$$

The closed orbit deviation $x(s)$ in Eq. (10) is given by the dispersion ($D_p(s)$) of the storage ring and the energy deviation ($\Delta E = E - E_0$) of an ion to the central particle having an energy E_0 :

$$x(s) = D_p(s) \frac{1}{2} \frac{\Delta E}{E_0}. \quad (11)$$

In Eq. (11) there is a factor $\frac{1}{2}$ because the dispersion function is defined via the ion momentum. The integral in Eq. (10) can be expressed via the momentum compaction factor (α_p) of the storage ring [3]. α_p describes the relation between circumference (C_0) of the closed orbit and the ion momentum (p):

$$\alpha_p = \frac{\Delta C/C_0}{\Delta p/p_0} = \frac{1}{C_0} \int_0^{C_0} \frac{D_p(s)}{\rho(s)} ds. \quad (12)$$

With the introduction of the momentum compaction factor α_p , Eq. (10) can be rewritten as:

$$T = \sqrt{\frac{m}{2E}} C_0 + \frac{E_0 + E}{E} \sqrt{\frac{m}{2E}} C_0 \frac{\alpha_p}{2} \frac{E - E_0}{E_0}. \quad (13)$$

Finally, with Eq. (13) we can calculate the slip factor of the storage ring, defined by Eq. (1):

$$\eta = -2 \frac{E_0}{T_0} \frac{\partial T}{\partial E} \Big|_{E=E_0} = 1 - 2\alpha_p. \quad (14)$$

Whereas for magnetic storage rings, slip factor η_m in the non relativistic approach is given by [4]:

$$\eta_m = 1 - \alpha_p. \quad (15)$$

The difference of the slip factor of an electrostatic storage ring to a magnetic storage ring can be explained by the acceleration and deceleration processes taking place in the electrostatic deflectors of the storage ring depending on the horizontal ion position $x(s)$. The measurement of the slip factor η and the momentum compaction factor α_p for CSR is explained in the next section.

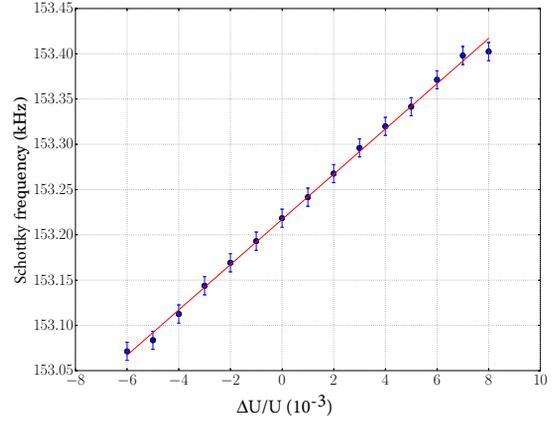


Figure 2: Schottky frequency measured at 10^{th} harmonic of the revolution frequency as a function of the variation of the voltages of all CSR deflectors and quadrupoles expressed by $\Delta U/U_0$.

MEASUREMENT OF α_p AND η

As derived in the previous section the phase slip factor and the compaction factor α_p of an electrostatic storage ring are related to each other. The momentum compaction factor can be measured experimentally by measuring the change in the revolution frequencies (Δf) due to variation of the voltages by $\Delta U/U_0$ on all ion optical elements [6]:

$$\frac{\Delta f}{f_0} = \alpha_p \frac{\Delta U}{U_0}. \quad (16)$$

This method has been used to measure the momentum compaction factor α_p where the change of the revolution frequency was measured with Schottky noise analysis. Fig. 2 shows the measured Schottky frequency f as a function of the variation of the voltages of all electrostatic elements. The frequencies shown in Fig. 2 correspond to the 10^{th} harmonic of the revolution frequency of a $^{40}\text{Ar}^+$ beam with 60 keV energy. The frequencies were measured with the Schottky pick-up connected to a spectrum analyzer which is triggered few ms after injection. The sweep time was set to 0.5 s (corresponding to the first 7500 revolutions of the ion beam). The errors in the determination of the Schottky frequencies are estimated to be ± 10 Hz. A linear fit to the data is shown as a red straight line. From its slope using Eq. (16) the momentum compaction factor is calculated to be $\alpha_p = 0.163$. Using Eq. (14) a slip factor of $\eta = 0.674$ can be calculated. With MAD8 [5], where each electrostatic element is described by transfer matrices, gives for the same quadrupole settings and working point a phase slip factor of $\eta_{mad} = 0.679$, which is in good agreement with the experimentally determined value.

THE SLIP FACTOR η AS A FUNCTION OF THE HORIZONTAL TUNE Q_x

In magnetic storage rings the slip factor η is related to the horizontal tune Q_x . For example at the TSR storage

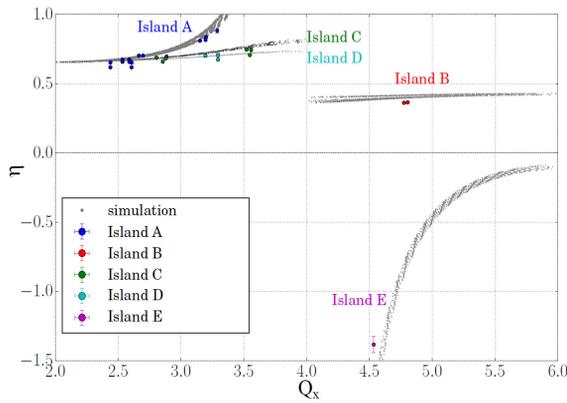


Figure 3: The measured (markers with error bars) and simulated (gray markers) phase slip factors for the cryogenic storage ring CSR as a function of the measured and simulated horizontal tune Q_x , respectively.

ring in Heidelberg following dependency between η and the horizontal tune in the non relativistic approach was found [7]:

$$\eta \approx 1 - 1/Q_x^2. \quad (17)$$

In order to study the functional relationship between η and Q_x , simulations were carried out using the MAD8 [5] program. The results of these calculations are shown in Fig. 3. Simulations (gray markers) predict a functional relationship $\eta = f(Q_x)$, where the functions depend on the quadrupole focusing strengths. The quadrupole focusing strengths for the stable ion orbits are used to calculate $\eta = f(Q_x)$ (Fig. 4). The stable working points are arranged in five islands, labeled with A-E, where island A and B are splitted into two sub islands. Each of these islands can be assigned to a relation $\eta = f(Q_x)$ shown in Fig. 3, which are different for the various islands. This behavior of the slip factor is different to any magnetic storage ring.

To proof the simulated relationship $\eta = f(Q_x)$, measurements were carried out for η and Q_x for the different stability islands. The experimentally measured values are shown as colored markers in Fig. 3. The denoted statistical errors in Fig. 3 are propagated from the fitting errors, whereas the errors in Q_x are too small to be visible. Although there are a few deviations between the experimental data and the simulations, the overall agreement is good and the functional relation $\eta = f(Q_x)$ predicted from the simulations is experimentally verified.

CONCLUSION

In this paper a relationship between the slip factor η and the momentum compaction α_p for an electrostatic storage ring is established. The difference of the η value from electrostatic to magnetic storage rings can be explained by acceleration and deceleration processes in the electrostatic deflectors, depending on the ion position. To measure the momentum compaction factor α_p and the phase slip factor

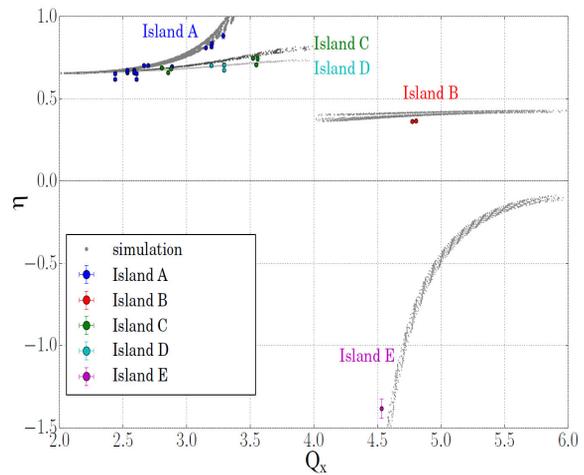


Figure 4: Stability diagramm of the CSR simulated with the MAD8 program, k_1 and k_2 are the quadrupole strengths of the two quadrupole families.

η , a new method was introduced where the change of the revolution frequency is measured as a function of the scaled voltages of all ion optics elements. In the usual measurements of η the revolution frequencies have to be measured for different momentums of the injected ion beam. This common method is very time consuming due to the necessary energy changes of the ion source and the changes in the settings of the transfer line, consisting of many magnetic and electrostatic elements. But in this new method of measuring η , the energy of the injected ions remains constant and therefore the determination of α_p and η could be done quickly. In contrast to magnetic storage rings the phase slip factor of the electrostatic storage ring CSR does not follow the approximate dependency on the horizontal tune η (compare Eq. (17)). It has different functional dependencies which vary for the different islands of stability.

REFERENCES

- [1] R. von Hahn et al. "The cryogenic storage ring CSR", Review of Scientific Instruments 87,063115, 2016.
- [2] P. Schmüser, "Longitudinal beam dynamic in circular accelerators", CERN accelerator school, Aarhus, 1986, CERN 87-10, p. 131.
- [3] A. Wu Chao, M. Tigner, "Handbook of Accelerator Physics and Engineering", Word Scientific, 2002, second Printing, p. 52.
- [4] J. Le Duff, "Basic course on accelerator optics", CERN accelerator school, Paris, 1984, CERN 85-19, p. 33.
- [5] H. Grote and F. C. Iselin, The MAD Program, Version 8.13/8, CERN/SL/90-13 (AP), 1990.
- [6] Stephen Vogel, "Developments at an Electrostatic Cryogenic Storage Ring for Electron Cooled keV Energy Ion Beams", Ph.D. thesis, University of Heidelberg, 2016.
- [7] M. Grieser et al., Workshop on beam cooling and related topics, Montreux, Switzerland, 4-8 October 1993, CERN 94-03, p. 257.