TRANSVERSE FEEDBACK PARAMETER EXTRACTION FROM EXCITATION DATA

G. Kotzian*, CERN, Geneva, Switzerland

Abstract

In this paper we present a simple and fast approach to extract essential parameters of a transverse feedback system such as phase advances between pick-ups and kickers, fractional tune, kicker delay, or per-bunch transverse activity from discrete-time samples of position signals. In this approach the beam is excited and subsequent beam oscillations are recorded. Given that any number of pick-ups can be evaluated at once with only a marginal increase of transverse beam size this method is suitable for regular health checks of a transverse feedback system, e. g., for every injection. The fundamental idea relies on the reconstruction of the transverse phase space by means of digital filters. We sketch a simple mathematical model to illustrate the underlying method. Examples are given together with a set of filter kernels for the fractional tunes of the LHC transverse feedback system.

INTRODUCTION

For the operation of our transverse feedback system (TFB) in CERN's Large Hadron Collider (LHC) we aim on defining a suitable method to extract vital feedback parameters in a simple, fast, and non-destructive manner, which can be carried out routinely to ensure that the active feedbacks for the two beams are correctly configured and at their stable working point.

TRANSVERSE PHASE SPACE

For our analysis we introduce an analytic signal, $x[n]$, which describes the evolution of a particle in normalized transverse phase space coordinates, (y, y'), obtained from the equations of motion, (Y, Y'), by applying the linear map, Λ_s,

$$\begin{pmatrix} y \\ y' \end{pmatrix} = \Lambda_s \begin{pmatrix} \mathrm{Y} \\ \mathrm{Y'} \end{pmatrix}, \quad (1)$$

with Twiss parameters, α and β, being a function of the observer's longitudinal coordinate, s,

$$\Lambda_s = \frac{1}{\sqrt{\beta(s)}} \begin{pmatrix} 1 & 0 \\ \alpha(s) & \beta(s) \end{pmatrix}. \quad (2)$$

Normalized coordinates are satisfying the relation $(y)^2 + (y')^2 = \text{const}$.

A discrete-time complex-valued sequence, $x[n]$, is then formed as the sum of the normalized transverse position, $y[n]$, with the corresponding normalized slope, $y'[n]$, multiplied by the imaginary unit $j = \sqrt{-1}$, such that

$$x[n] = \begin{pmatrix} 1 & j \end{pmatrix} \cdot \begin{pmatrix} y \\ y' \end{pmatrix}. \quad (3)$$

* gerd.kotzian@cern.ch

For simplicity we assume that the particle motion is dominated by active damping (see also Ref. [1]), thus reducing the analysis to linear optic effects of the magnetic guidance field and exponential amplitude decay.

Thanks to the trigonometric nature of the normalized coordinates and the complex notation of $x[n]$ we reformulate Eq. (3) by applying Euler's formula,

$$x[n] = A_0\, e^{-j\phi_0} \left(\alpha \cdot e^{-j\omega_0}\right)^n. \quad (4)$$

Equation (4) describes a damped harmonic oscillation at turn index n, with angular frequency ω_0 and a decay factor, α, and with initial amplitude and phase denoted as A_0 resp. ϕ_0. Note that the negative exponents preserve the direction of rotation in normalized phase coordinates (positive phase $\mapsto$ clockwise; downstream).

FEEDBACK PARAMETERS

In order to extract essential parameters of a transverse feedback system from bunch-by-bunch beam data we shall consider the case of a beam in a steady state — any transients have settled — which has been excited transversely by the TFB for less than one turn (illustrated in Fig. 1).

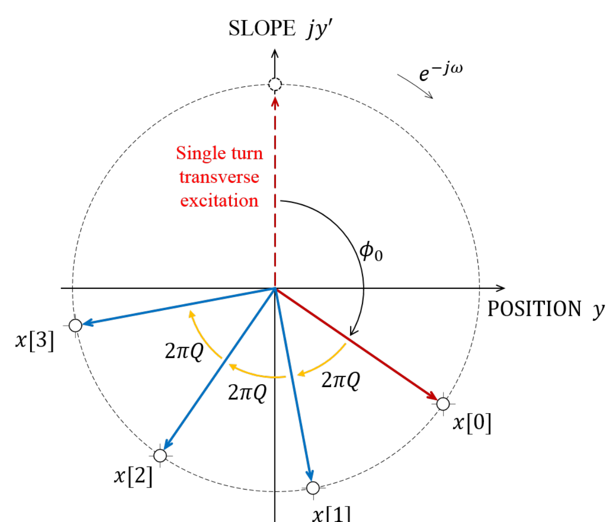


Figure 1: Transverse phase space plot (normalized) at the position of a pick-up. A transverse deflection commutes from the location of the kicker to the coordinates of the pick-up by a fixed phase angle (denoted as ϕ_0), with subsequent turns advancing in phase by the fractional tune ($\Delta\varphi = 2\pi Q$).

Fractional Tune

By rewriting Eq. (4) as recurrence formula we obtain the coordinates for consecutive turns by evaluating

$$x[n] = x[n-1] \cdot \alpha e^{-j\omega_0}. \quad (5)$$

As can be seen, after a turn the sequence has advanced in phase by $\Delta\varphi = \omega_0 \equiv 2\pi Q$. Therefore, by taking the ratio over two consecutive turns we can express the per-turn or instantaneous fractional tune, $Q[n]$, as

$$Q[n] = \frac{1}{2\pi} \arg\left\{\frac{x[n-1]}{x[n]}\right\} . \tag{6}$$

It is worth noting that Eq. (6) allows for correctly characterizing the fractional tune to be below or above the half-integer resonance.

Phase Advance

As shown in Fig. 1, in the very same turn when the kick ($\pi/2$ or $+j$) has been applied, i.e. $n = 0$, the betatron phase advance between kicker and pick-up effectively transforms the transverse deflection in normalized coordinates, thus leading for the initial condition of Eq. (4),

$$\phi_0 = \frac{\pi}{2} - \arg\{x[0]\} . \tag{7}$$

More generally, according to Eq. (5) we notice that any subsequent beam oscillations recorded by the pick-up will advance by the fractional tune. We can therefore determine also for later turns an initial phase, $\psi[n]$, from the argument of the analytic signal by including a linear phase term,

$$\psi[n] \doteq \frac{\pi}{2} - \arg\left\{x[n] \cdot e^{j2\pi Q\cdot n}\right\} . \tag{8}$$

From this we instantly obtain the phase advance between the kicker and the pick-up by averaging over M consecutive turns,

$$\zeta = \frac{1}{M}\sum_{k=0}^{M-1} \psi[k] . \tag{9}$$

Transverse Activity

We define the transverse activity, $A[n]$, as the magnitude of the vector $x[n]$,

$$A[n] = \mathrm{abs}\{x[n]\} , \tag{10}$$

which is a measure of the instantaneous oscillation amplitude in the normalized transverse phase space.

From Eq. (4) it follows that,

$$A[n] = A_0\alpha^n , \tag{11}$$

with the magnitude A_0 defined by the initial excitation amplitude of the transverse deflection at turn $n = 0$.

Decay Time

By noting that the change in amplitude per turn in Eq. (5) is constant and defined by the decay factor, $\alpha \equiv e^{-1/\tau_d}$, we obtain the decay time, τ_d, from the transverse activity, described in Eq. (11), at two time instances, n_1 and n_2, as

$$\tau_d = (n_2 - n_1)\left(\log\frac{A[n_1]}{A[n_2]}\right)^{-1} . \tag{12}$$

Kicker Delay

For the stability of a TFB it is essential that kick signals are well aligned with the time of arrival of the bunches. In the following we derive a method which aims on quantifying the kicker delay offset.

We now consider the case of a kick signal which is modulated in amplitude over one turn. Thereby, a sinusoidal kick waveform with M periods per machine turn is sampled by a bunch with index k depending on the time of arrival at the location of the kicker. The resulting bunch oscillation magnitude, described as,

$$A_k = A_0 \cdot \cos\left(\frac{2\pi M}{h}\cdot k + 2\pi M\cdot\eta\right) , \tag{13}$$

is then recorded as betatron oscillation decay at a downstream pick-up. Here, the harmonic number h represents the maximum number of buckets per turn, and a delay offset factor, $\eta = \Delta T/T_{\mathrm{Rev}}$, defined as the ratio between the kicker delay offset, ΔT, and the revolution period, T_{Rev}.

If this kick exercise is repeated with two phase-shifted versions of the modulation signal in quadrature, denoted as A_I and A_Q, we can reconstruct an IQ-footprint of the traversing bunches at the kicker as,

$$\chi[k,n] = A_I[k,n] + jA_Q[k,n] . \tag{14}$$

As can be easily verified, bunches are equally distributed around a circle with constant radius. Therefore, unwinding the phase response of Eq. (14) by taking into account a linear position-dependent phase term,

$$\rho[n] = \arg\left\{\chi[k,n]\cdot e^{-j2\pi Mk/h}\right\} , \tag{15}$$

and averaging over populated bunches and N turns results in,

$$\theta = \frac{1}{N}\sum_{m=0}^{N-1}\rho[m] \doteq 2\pi M\cdot\eta . \tag{16}$$

Since $\eta = \Delta T/T_{\mathrm{Rev}}$ it follows for the delay offset:

$$\Delta T = \frac{\theta}{2\pi M}\cdot T_{\mathrm{Rev}} . \tag{17}$$

It is worth noting that the offset factor in Eq. (13) is weighted by M, thus increasing the sensitivity to delay offsets. Ultimately, if $M = h$ then the resulting phase in Eq. (15) depends solely on the delay offset factor.

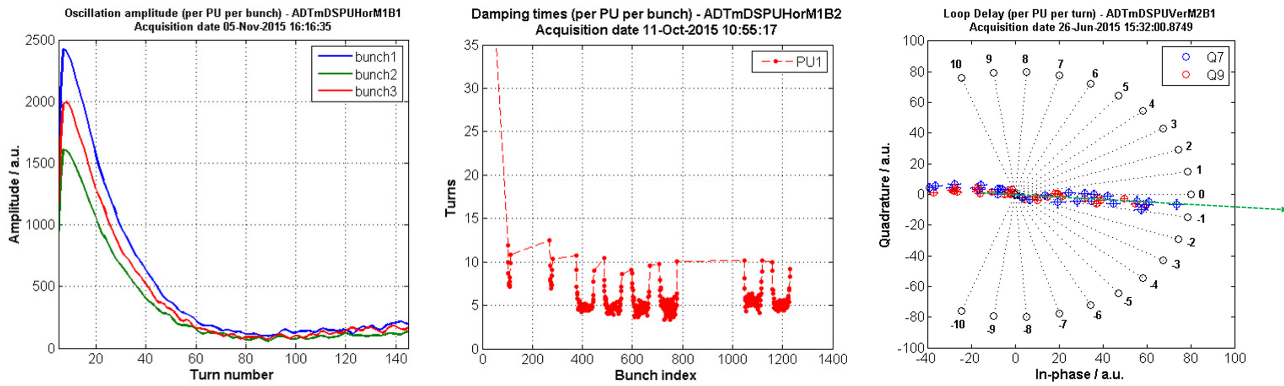
PHASE SPACE RECONSTRUCTION

In the following we assume that the pick-up signal processing is providing a stream of normalized discrete-time samples, $y[n]$, with position values recorded bunch-by-bunch and turn-by-turn, as is the case for the LHC TFB.

In order to reconstruct the analytic signal in Eq. (3) we are looking for a solution that allows to transform a sequence of position samples, $y[n]$, into a sequence of corresponding slope samples, i.e.

$$y'[n] = L\{y[n]\} . \tag{18}$$

ISBN 978-3-95450-182-3



(a) Transverse activity per bunch per turn. (b) Damping times per bunch. (c) Delay offset in nanoseconds.

Figure 2: Numerical results obtained from measurements with the LHC transverse feedback system.

Thereby, the operation denoted by $L\{\cdot\}$ in Eq. (18) is commonly known as *Hilbert transform* and is explained for example in Ref. [2].

A more practical approach can be found by noticing that $y'[n]$ is the 90° phase-rotated version of $y[n]$. This phase shift can be generated by simple means of digital filtering — as it is already been done in the feedback phase controller [3].

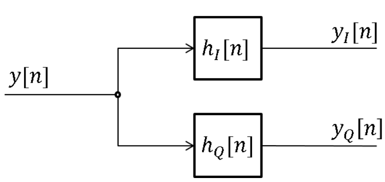


Figure 3: Phase space reconstruction using two digital filters.

Figure 3 shows how the phase space is reconstructed by means of two individual digital filters. The two branches with filter kernels h_I[n] for the in-phase component, and $h_Q[n]$ for the quadrature component generate two quadrature output signals, named $y_I[n]$ and $y_Q[n]$, which can be combined to a final analytic signal, representing a reconstruction of the transverse phase space,

$$c[n] = y_I[n] + j y_Q[n] = y[n] * (h_I + j h_Q) . \qquad (19)$$

Elaborate filter kernels including DC suppression can be defined, tuned for the fractional tunes of the particular plane and attenuate out-of-band signals. Examples of filter kernels for fractional tunes used in the LHC transverse feedback systems are listed in Table 1. With only five coefficients these filters are usable with possible damping times of 10 turns or less.

RESULTS

Figure 2(a) shows the transverse activity of three individual bunches with active feedback, as recorded by the LHC TFB. Measurements with several circulating batches showed that damping times of ≈ 5 turns are achievable at the center of batches with 72 bunches (see Fig. 2(b)).

Table 1: Filter Kernels LHC

(a) Horizontal, $Q_f = 0.275$

$h_I[n]$	$h_Q[n]$
-0.1837	+0.0447
-0.1224	-0.4922
+0.6122	+0.0000
-0.1224	+0.4922
-0.1837	-0.0447

(b) Vertical, $Q_f = 0.31$

$h_I[n]$	$h_Q[n]$
-0.1322	+0.1136
-0.1983	-0.4542
+0.6612	+0.0000
-0.1983	+0.4542
-0.1322	-0.1136

Figure 2(c) details the delay offset for one kicker module of the LHC TFB, measured by two independent pick-ups (red and blue), both confirming the fine-delay adjustment of better than 0.5 ns.

SUMMARY AND CONCLUSION

We describe an alternative method for feedback parameter extraction, based on transverse excitations generated by the kicker of a TFB or at every injection, and a transverse phase space reconstruction with digital filters for each individual pick-up. The analysis are carried out solely in time domain (no conversion to frequency domain).

Tests can be carried out as routine health checks at any time, also during the acceleration, allowing evaluation of closed loop feedback parameters (gain, phase, delay) or beam parameters (tune, damping/decoherence time).

With this technique novel ideas for alternative feedback controllers exploiting new control inputs for instantaneous tune, oscillation amplitude, or feedback phase can be envisaged.

ACKNOWLEDGEMENT

Particular credit is given to Daniel Valuch, for continuous encouragement and enduring support of the underlying work.

REFERENCES

[1] G. Kotzian, W. Höfle, E. Vogel, "LHC transverse feedback damping efficiency", in *Proc. EPAC'08*, Genoa, Italy, June 2008, paper THPP114, pp. 3632-3634.

[2] A.V. Oppenheim, R. W. Schafer, and J. R. Buck, "Discrete Hilbert Transforms", in *Discrete-Time Signal Processing – SE.* New Jersey, USA: Prentice Hall, 1999, pp. 775–801.

[3] G. Kotzian, "Possibilities for transverse feedback phase adjustment by means of digital filters", presented at IPAC'17, Copenhagen, Denmark, May 2017, paper TUPIK095, this conference.

ISBN 978-3-95450-182-3