

COHERENT SYNCHROTRON RADIATION SIMULATIONS FOR OFF-AXIS BEAMS USING THE BMAD TOOLKIT *

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Abstract

Except at the lowest beam energies, the one dimensional treatment of coherent synchrotron radiation (CSR) originally developed by Saldin [1] is an efficient and reasonably accurate way to simulate the effects of CSR on a particle beam. A possible problem with standard implementations of the 1D CSR formalism is that these implementations assume that the beam centroid is close to the reference trajectory that defines the lattice. In this paper, the one dimensional treatment is extended to take into account beams whose centroid is far from the reference trajectory and an example using the Cornell-BNL Fixed Field Alternating Gradient (FFAG) accelerator CBETA is given.

INTRODUCTION

Coherent synchrotron radiation (CSR) is a potential factor limiting the minimum transverse emittance achievable in accelerators [2] as well as possibly leading to micro bunching instabilities [3]. It is thus important to include CSR effects in simulations of present and future machines such as energy recovery linacs (ERL), free electron lasers, and low emittance light sources [4].

At very low energies, the space charge forces have been modeled successfully using particle-in-cell codes such as IMPACT-T [5] and OPAL [6]. While accurate, execution times for these codes can be long so, at higher energies, a 1-dimension CSR model has been developed based upon the work of Saldin [1]. This 1-dimensional model is computationally much faster and gives good results [7] as long as the beam shape obeys certain restrictions. For example, the transverse beam size σ_{\perp} must satisfy $\sigma_{\perp} \ll R(\sigma_z/R)^{2/3}$ where R is the bending radius and σ_z is the longitudinal beam size. This 1-dimensional model has been incorporated into a number of simulation packages including Elegant [8], IMPACT-T [5], and Bmad [9].

The 1-dimensional CSR model allows the beam particles to travel in 3D. However, when the CSR kick is calculated, the particles are projected onto a 1-dimensional reference trajectory. Typically, the reference trajectory used for the CSR calculation has been the reference orbit used to define the placement of the lattice elements. This choice is computationally convenient since the lattice reference orbit is typically made up of a series of straight lines and arcs of circles.

The drawback of using the lattice reference orbit for the CSR computation occurs when the beam centroid orbit is far from the lattice reference orbit. In this case, the CSR computation can be inaccurate due to differences between

the actual curvature of the beam orbit and the curvature of the lattice reference orbit. Situations where the beam centroid is far from the lattice reference orbit occur in Fixed Field Alternating Gradient (FFAG) accelerators where the beam can go through lattice elements several times on different trajectories. In such a case there may be no one lattice reference orbit that is simultaneously near all the different beam trajectories.

To get around this problem, the CSR calculation in the Bmad simulation toolkit [10] has been modified to use an orbit near the beam centroid orbit as the CSR reference orbit. As an added benefit, the new CSR calculation is less singular in nature and therefore easier to implement.

This paper outlines the beam centroid based 1-dimensional CSR calculation and its application to the Cornell-BNL ERL-FFAG Test Accelerator (CBETA).

ANALYSIS

The kick K a particle of a beam feels due to the field of another particle is divided into two pieces [9]

$$K = K_{CSR} + K_{SC} \tag{1}$$

where K_{SC} is the kick that would result if the particles were moving without acceleration along a straight line and K_{CSR} , the CSR kick, is essentially defined as $K - K_{SC}$. The analysis of K_{CSR} projects the particles of a bunch onto the CSR reference trajectory, the construction of which is described later. The analysis assumes that the CSR reference trajectory lies in a plane. Figure 1 shows the geometry. A “source” particle (red dots) is, at any given time, a distance z behind the “kicked” particle (blue dots). The radiation from the source particle at point P' and time t' interacts at some later time t with the kicked particle at point P . \mathbf{L} is the vector from P' to P and L_s is the distance along the CSR reference trajectory from P' to P .

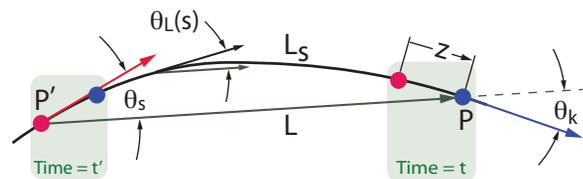


Figure 1: CSR calculation geometry. A “source” particle (red dots) is a distance z behind the “kicked” particle (blue dots). The radiation from the “source” particle at point P' and time t' interacts at some later time t with the kicked particle at point P .

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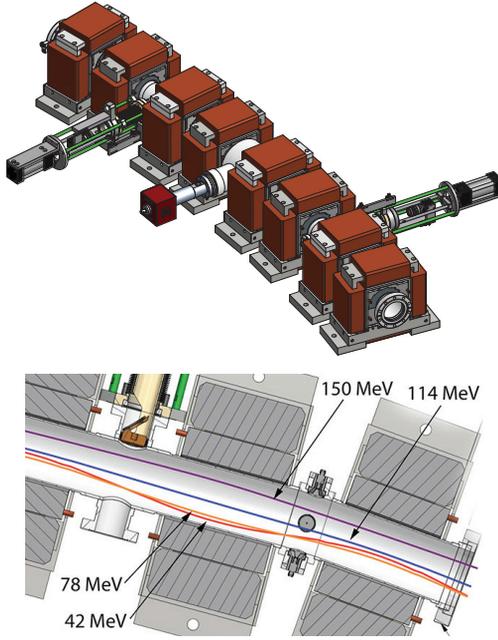


Figure 2: A) Section of the Cornell CBETA arc built out of Halbach magnets. B) Closeup of the four orbits in the CBETA arc for the four beams of differing energy.

The energy variation $d\mathcal{E}/ds$ due to the longitudinal component of the CSR kick is written as

$$\left(\frac{d\mathcal{E}}{ds}\right)_{\text{CSR}} = \int_{-\infty}^{\infty} ds' \frac{d\lambda(s')}{ds'} I_{\text{CSR}}(s-s') \quad (2)$$

where $\lambda(s)$ is the bunch line charge density and

$$I_{\text{CSR}}(s-s') = - \int_{-\infty}^{s'} ds'' K_{\text{CSR}}(s-s'') \quad (3)$$

Saldin et al. [11] gives a formula (Eq. (10)) for I_{CSR}

$$I_{\text{CSR}} = r_e m c^2 \left(\frac{1}{\gamma^2 z} - \frac{2}{L} \frac{1 + \gamma^2 \theta_s (\theta_s + \theta_k)}{1 + \gamma^2 \theta_s^2} \right) \quad (4)$$

where θ_s is the angle between the \mathbf{L} vector and the velocity vector of the source particle at position P' and θ_k is the angle between the \mathbf{L} vector and the velocity vector of the kicked particle at position P . Here r_e is the classical electron radius and mc^2 is its rest energy. The problem with the above equation is that even though I_{CSR} remains finite, for small z , and hence small L , the two terms in parentheses diverge to infinity as z approaches zero. Thus Eq. (4) is not suitable for arbitrary orbits where analytic equations cannot be derived.

To get around this, z is computed using

$$z = L_s - \beta L = \frac{L}{2\gamma^2} + \epsilon_L \quad (5)$$

where β is the normalized velocity of the particles which is assumed to be constant, the high energy approximation $1 - \beta = 1/2\gamma^2$ has been made, and

$$\epsilon_L \equiv L_s - L \quad (6)$$

Using this in Eq. (4) gives

$$I_{\text{CSR}} = -r_e m c^2 \left(\frac{2\epsilon_L}{zL} + \frac{2}{L} \frac{\gamma^2 \theta_s \theta_k}{1 + \gamma^2 \theta_s^2} \right) \quad (7)$$

The trick here is to *not* evaluate ϵ_L using Eq. (6) but rather to compute it using a small angle approximation

$$\epsilon_L \approx \frac{1}{2} \int_{s_{P'}}^{s_P} \theta_L^2(s) ds \quad (8)$$

where $\theta_L(s)$ is the angle between L and L_s as shown in Fig. 1. The error in ϵ_L is fourth order in the angles and so can be ignored. At small z , L , θ_L , θ_s , and θ_k scale linearly with z . Hence ϵ_L scales as z^3 so the two terms on the RHS of Eq. (7) are both well behaved as z approaches zero.

IMPLEMENTATION

The above algorithm for simulating the longitudinal CSR effect has been implemented as part of the Bmad [10] subroutine library for relativistic charged-particle and X-ray simulations in place of the old algorithm [9]. The transverse CSR calculation and the longitudinal space charge calculation as outlined in [9] remain unchanged in Bmad.

To establish a CSR reference trajectory that is near the beam centroid orbit, before any tracking of a beam is done, a single particle is tracked through the lattice with the particle's initial position at the start of the lattice matching the beam's centroid position there. From this, a piece wise linear reference line is defined by connecting, with straight line segments, all of the points where this particle's trajectory intersects an element boundary. This defines a coordinate system in which the CSR reference trajectory is constructed. The CSR reference trajectory in any given element is then defined by a cubic spline with respect to the piece wise linear reference line with coefficients of the spline determined by the tracked particle's trajectory at the edges of the element. For elements like wigglers and undulators, where a single spline fit may be inaccurate, the element can be split into sub-element sections.

When evaluating the integral for ϵ_L (Eq. (8)), The \mathbf{L} vector could be used as the s -axis of the integration. This would involve some computational overhead since the \mathbf{L} vector varies as z and P are varied. Instead, the piece wise linear reference line that is used to construct the CSR reference trajectory is also used as the integration axis.

The charge distribution is divided up into a number of bins and is smoothed by taking the charge distribution for each particle to be triangular with a finite width as described in [9]. For a given kick point P , and a given value of z , the source point P' is found using Eq. (5) using an implicit search as outlined in [9].

FFAG EXAMPLE

The Cornell CBETA machine, currently under construction [12], will be the first ever Energy Recovery Linac (ERL) based on a FFAG lattice. This non-scaling FFAG will have

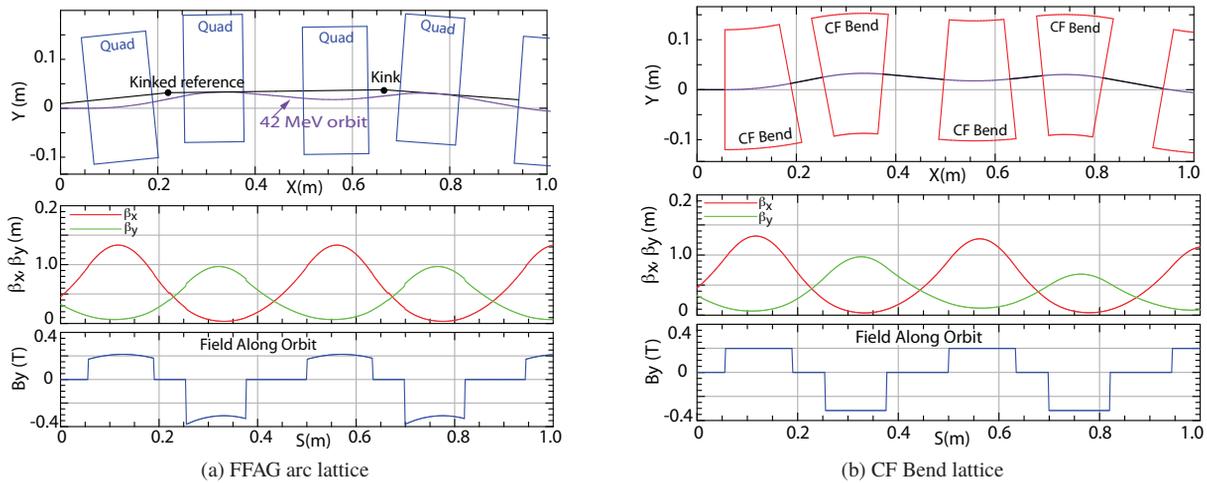


Figure 3: A) CBETA FFAG arc lattice using offset quadrupole magnets to bend the beam and patch elements to curve the lattice reference line. B) A close approximation to the FFAG arc lattice for the 42 MeV beam using combined-function (CF) bending magnets. Top plots: The physical layout and 42 MeV orbit. Middle plots: The beta functions. Bottom plots: The vertical magnetic field along the orbit.

four accelerating passes through the linac and 4 decelerating passes. The FFAG arc will have to handle beams at four different energies (42 MeV, 78 MeV, 114 MeV, and 150 MeV) traveling along four different paths as shown in Fig. 2.

Figure 3A shows the CBETA FFAG arc cell which uses Halbach magnets to bend the beam and “patch” elements which reorient the reference trajectory to follow the bend of the arc. These magnets can be reasonably modeled as shifted quadrupole magnets. The patch elements create a kinked reference orbit where the reference orbit tangent vector is discontinuous. Here the orbit is well-off the reference axis.

For use as a comparison, Fig. 3B shows an “equivalent” FFAG arc lattice but here combined-function (CF) bending magnets are used to get the lattice reference orbit close to the orbit of the 42 MeV beam which is the beam used in the simulation.

Figure 4 shows the longitudinal phase space after tracking a bunch through 10 cells of the FFAG arc using both the old

and new implementations of CSR in the Bmad library. The electron bunch has 77 pC of charge and an RMS duration of 3 ps at an average energy of 42 MeV. On the scale of this plot, the initial relative energy deviation δ is negligible. The blue curve shows the results of the new CSR implementation in the FFAG arc lattice (3A). The red curve shows the results of the new CSR implementation in the comparison CF bending magnet lattice. Because the bending felt by the beam is nearly the same in the two lattices, it is expected that there would be little difference in the results. And indeed, since the phase space distributions for tracking in both lattices are virtually identical, this shows that the new algorithm is able to handle off-axis beams.

Also shown in Fig. 4 is the final phase space distributions as calculated by the old CSR algorithm. For the CF bend lattice the old and new CSR calculations are in excellent agreement. On the other hand, the old CSR calculation with the FFAG arc lattice shows virtually no energy deviations because there are no bend magnets in this lattice. The old CSR calculation was not designed to handle a kinked reference orbit and hence ignores the kinks in the FFAG lattice. There is no way to remedy this for the old calculation because the CSR kick becomes infinite at a kink.

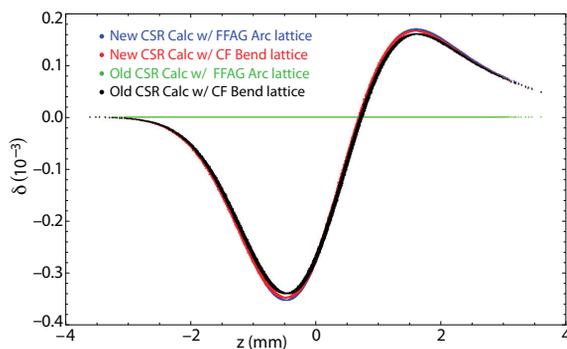


Figure 4: Longitudinal phase space after tracking a bunch through 10 cells of the two lattices using both the old and new CSR algorithm.

CONCLUSION

The new formulation for the kick integral I_{CSR} (Eq. (7)) has terms that are not singular in the limit as z approaches zero and thus Eq. (7) can be used with “arbitrary” plainer CSR reference orbits. This allows the use of a CSR reference orbit that is close to the beam centroid orbit enabling simulation of CSR in lattices where the beam centroid is far from the lattice reference orbit.

REFERENCES

- [1] E. L. Saldin, E. A. Schneidmiller and M. V. Yurkov, “On the coherent radiation of an electron bunch moving in an arc of a circle”, *Nuc. Instrum. Methods Phys. Res. A*, **398**, 373—394, (1997).
- [2] H. H. Braun, R. Corsini, L. Groening, F. Zhou, A. Kabel, T. O. Raubenheimer, R. Li, and T. Limberg, “Emittance growth and energy loss due to coherent synchrotron radiation in a bunch compressor”, *Phys. Rev. ST Accel. Beams* **3**, 124402 (2000).
- [3] S. Heifets, G. Stupakov, and S. Krinsky, “Coherent synchrotron radiation instability in a bunch compressor,” *Phys. Rev. ST Accel. Beams* **5**, 064401 (2002).
- [4] J. Qiang, J. Corlett, C. E. Mitchell, C. F. Papadopoulos, G. Penn, M. Placidi, M. Reinsch, R. D. Ryne, F. Sannibale, C. Sun, M. Venturini, P. Emma, and S. Reiche., “Start-to-end simulation of x-ray radiation of a next generation light source using the real number of electrons”, *Phys. Rev. ST Accel. & Beams* **9**, 044204 (2006).
- [5] J. Qiang, S. M. Lidia, R. D. Ryne, and C. Limborg-Deprey, “Three-dimensional quasi static model for high brightness beam dynamics simulation”, *Phys. Rev. ST Accel. Beams* **9**, 044204 (2006).
- [6] Y. J. Bi, A. Adelman, R. Dolling, M. Humbel, W. Joho, M. Seidel, and T. J. Zhang, “Towards Quantitative Simulations of High Power Proton Cyclotrons.”, *Phys. Rev. ST Accel. Beams*, 14:054402, (2011).
- [7] Chad E. Mitchell, Ji Qiang, Robert D. Ryne, “A fast method for computing 1-D wakefields due to coherent synchrotron radiation”, *Nuc. Instrum. and Methods in Phys. Research A* **715** 119—125 (2013).
- [8] M. Borland, “A Simple Method For Simulation Of Coherent Synchrotron Radiation In A Tracking Code” 6th International Comp. Accel. Phys. Conf., ICAP 2000, (2000).
- [9] D. C. Sagan, G. H. Hoffstaetter, C. E. Mayes, and U. Sae-Ueng, “Extended one-dimensional method for coherent synchrotron radiation including shielding”, *Phys. Rev. ST Accel. & Beams*, **12**, 040703, (2009).
- [10] D. Sagan, “Bmad: A relativistic charged particle simulation library,” *Nuc. Instrum. and Methods in Phys Research A* **558**, 356–359 (2006). <http://www.lepp.cornell.edu/~dcs/bmad>
- [11] E. L. Saldin, E. A. Schneidmiller and M. V. Yurkov, “Radiative interaction of electrons in a bunch moving in an undulator”, *Nuc. Instrum. Methods Phys. Res. A*, **417**, 158—168, (1998).
- [12] C. Mayes, *ed.*, “CBETA Design Report” (2017). To be published.
<https://www.classe.cornell.edu/Research/ERL/CBETA.html>