WAKE FIELD AND HEAD-TAIL INSTABILITY IN BEAM-BEAM COLLISION WITH A LARGE CROSSING ANGLE

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Abstract

Collision scheme with a large crossing angle is being very popular in design of future colliders in combination with the crab waist scheme. We discuss that a strong wake field with correlation between turns is induced by the beam-beam interaction. Recently strong-strong beam-beam simulations have shown a strong coherent instability in head-tail mode in collision with a large crossing angle. The wake field explains the mechanism of the coherent head-tail instability. Study of this instability is essential for collider designs based on a large crossing angle and crab waist scheme.

INTRODUCTION

A coherent head-tail instability has been seen in collision with a large crossing angle in strong-strong beam-beam simulation. We try to explain this beam-beam instability using wake field induced by the beam-beam interaction. When a positron bunch with a dipole moment $\rho^{(+)}(z')$ collide with an electron bunch, parts of the electron bunch (z_-) experiences a momentum kick as a function of z_- and $\rho^{(+)}(z')$. In collision with a large crossing angle, the kick depends on z and z'. We present two kinds of wake field in the beam-beam collision.

One is single beam approach, and second is two beams approach. In the single beam approach, a beam interacting with another beam regarded with a particle cloud. A wake field is obtained by the similar way with the electron cloud. The beam particle cloud interacts with the beam in many turns. The wake field contains a turn-by-turn correlation.

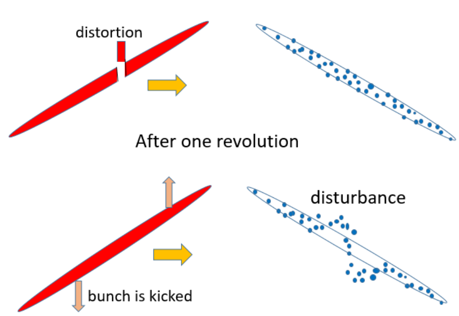

Figure 1: Sketch for evaluation of wake field induced by beam-beam interaction in single beam approach.

In second approach, a wake field describes correlation between two beams. The wake field is evaluated by calculating kick of positron/electron beam induced by a delta function like dipole moment of electron/positron beam. Figures 1 and 2 shows schematic views of the wake field models.

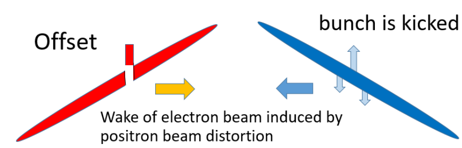

Figure 2: Sketch for evaluation of wake field induced by beam-beam interaction in two beam approach.

WAKE FIELD CALCULATION IN SINGLE BEAM APPROACH

Effective wake field induced by the beam-beam collision is evaluated by a simulation based on weak-strong model. The basic algorithm is the same as that done to study the electron cloud effect [2]. The so-called strong beam is expressed by a series of micro-bunches (n_{mb}) with a transverse rigid Gaussian distribution. While the so-called weak beam is represented by macro-particles (n_e). The numbers n_{mb} and n_e are chosen to take into account of Piwinski angle ($\sigma_z\theta_c/\sigma_x$) and statistical noise of the wake field, respectively, where σ_z is the bunch length, θ_c the half crossing angle, and σ_x the horizontal beam size at IP. Typically $n_{mb} = 100$ and $n_e =$ 100k or 1M are used.

The momentum change of micro-bunch in the strong beam is related to the wake field as follows

$$\Delta p_x(z,t) = -\sum_{t'=0}^{t}\int_{-\ell}^{\ell} W_x(z-z',t-t')\rho_x(z',t')dz' \quad (1)$$

where the integration is carried out $\ell \approx 3-5\sigma_z$. The distortion is applied by a displacement Δx of one (at $z'=0$) of macro-bunches n_{mb}, that is, $\rho_x(z',0) = \Delta x\delta(z')\delta_{n'1}/n_{nm}$. Wake field is given by the momentum change as follows,

$$W_x(z,t) = -n_{mb}\frac{\Delta p_x(z,t)}{\Delta x}. \quad (2)$$

The wake field/momentum kick is calculated for FCC-ee-Z [3]; where the parameters are emittance, $\varepsilon_{x/y}$ = 0.2 nm/1 pm, $\beta^*_{x/y}$ = 50/0.1 cm, the bunch population $N_\pm = 10^{11}$, half crossing angle θ_c = 15 mrad, bunch length σ_z = 6.7 mm.. Figure 3 shows momentum kick of micro-bunches, for displacement ($\Delta x = \sigma_x = 10^{-5}$ m) of a micro-bunch at $z' = 0, t = 0$. Wake field is given by $W_x(z,t)[\mathrm{m}^{-1}] = -10^7\Delta p_x(z,t)$. Picture (a) depicts momentum kick at $t = 2-6$-th turn. The wake field damp

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about in 5 turns. For $t = 0$, $\Delta p_x(0,0) = -5.93 \times 10^{-6}$ and $\Delta p_x(z < 0, 0) \sim \pm 7 \times 10^{-9}$ correspond to the tune shift and short range wake in a bunch, respectively. The short range wake is 2 order smaller than that for $t \geq 1$. The momentum kick has a peak near $z = 0$ and oscillate turn by turn. Picture (b) depicts the peak momentum kick as function of turn. The frequency and quality factor are estimated to be $\nu = 0.61$and $Q = 5.7$. The frequency is reasonable with considering the horizontal tune ($\nu_x = 0.54$), the synchrotron tune ($\nu_s = 0.018$) and beam-beam tune shift ($\xi_x = 0.024$).

Linearity and translational invariance of the wake field is checked as shown in Figure 4. Wake field for the displacement 1, 2 and $3\sigma_x$ is plotted in Picture (a) Linearity for the displacement is satisfied well, though it is not perfect. Translational invariance, which guarantees the function form $W(z - z')$, is also satisfied well: that is, the wake field shift for changing $z' = 0, \pm 2.4, \pm 4.8$ mm.

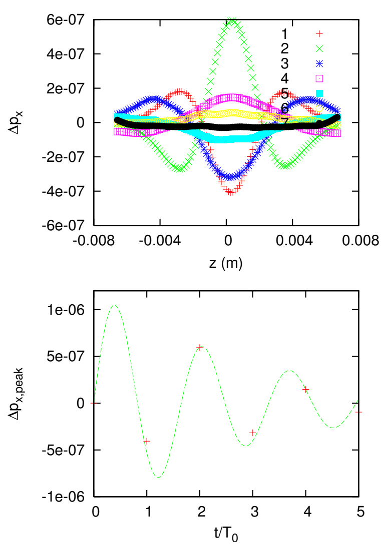

Figure 3: (a) Momentum kick of micro-bunches at (z, t) for displacement ($\Delta x = \sigma_x = 10^{-5}$ m) of a micro-bunch at $z' = 0, t = 0$, where $n_{mb} = 100$. Wake field is given by $W_x(z, t)[\mathrm{m}^{-1}] = -10^7 \Delta p_x(z, t)$. (b) peak momentum kick as function of turn. (t).

Simulation for beam instability is performed using the wake field. Particles ($\sim$ 10k) are generated with Gaussian distribution for the design emittance and beta in the 6 dimensional phase space. The kick induced by the wake field is calculated turn by turn using Eq.(1). where the beam dipole moments $\rho_x(z', n')$ are recorded for the past several turns. After the kick (effective collision), coordinate of particles are multiplied by revolution matrix. Figure 5 (a) shows evolution of $\langle x^2 \rangle$ for various β_x^*. Exponential growth in $\langle x^2 \rangle$ and $\langle xz \rangle$ is seen. Note that this wake field model is linear for betatron amplitude. Actually since the beam-beam force is nonlinear and is saturates at several σ_x.

Figure 5(b) shows particle distribution in $z - \delta p/p - x$ phase space. Complex head-tail motion is seen clearly. The amplitude is huge, since linear wake model is used.

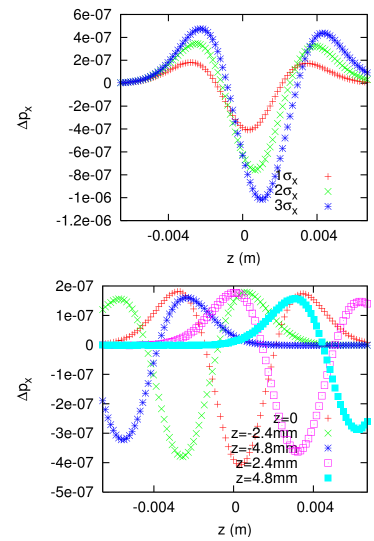

Figure 4: (a) Wake field for the displacement 1, 2 and $3\sigma_x$, (b) Wake field $W(z - z')$ for $z' = 0, \pm 2.4, \pm 4.8$ mm.

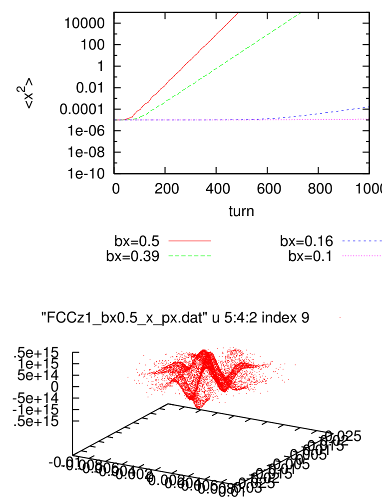

Figure 5: (a) evolution of $\langle x^2 \rangle$ for various β_x^* after 1000 turn ($\beta_x^* = 0.5$ m), and (b) Particle distribution in $z - \delta p/p - x$ phase space.

WAKE FIELD BETWEEN TWO COLLIDING BEAMS

A wake field representing a correlation between two colliding bunches is discussed. We apply a horizontal displacement in a macro-bunch of electron (positron) bunch. Momentum kick which positron (electron) bunch experiences is expressed by

$$\Delta p_{x,\pm}(z_\pm) = -\int_l^l W_x(z_\pm - z'_\mp)\rho_x(z'_\mp)dz'_\mp. \quad (3)$$

We consider that a part of positron bunch $\rho_0(z_+)\delta(z'_+ - z_+)$ deviates Δx,

$$\Delta p_x^{(-)} = -W_x(z_- - z_+)\rho_0(z_+)\Delta x. \quad (4)$$

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Effect of the deviation in the momentum kick is given by the beam-beam force,

$$\Delta p_x^{(-)} = \frac{N_+\rho_0(z_+)r_e}{\gamma}(F(x_- - x_+ - \Delta x) - F_x(x_- - x_+)). \quad (5)$$

For a transverse Gaussian beam, F is represented by complex error function as follows,

$$F(x,y) = F_y + iF_x = \frac{2\sqrt{\pi}}{\Sigma}\left[w\left(\frac{x+iy}{\Sigma}\right) - \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) w\left(\frac{\sigma_y x/\sigma_x + i\sigma_x y/\sigma_y}{\Sigma}\right)\right], \quad (6)$$

where $\Sigma = \sqrt{2(\sigma_x^2 - \sigma_y^2)}$. The beam sizes are convoluted ones of two beams $\sigma_{x(y)} = \sqrt{\sigma_{x(y),-}^2 + \sigma_{x(y),+}^2}$. $x_\pm \approx z_\pm \theta_c$ for collision with the half crossing angle θ_c.

$$F_x((z_- - z_+)\theta_c - \Delta x, 0) - F_x((z_- - z_+)\theta_c, 0) = -\left.\frac{\partial F_x(x,0)}{\partial x}\right|_{x=(z_- - z_+)\theta_c} \Delta x \quad (7)$$

The wake force is expressed by derivative of the beam-beam force,

$$W_x(z_- - z_+) = \frac{N_+ r_e}{\gamma} \left.\frac{\partial F_x(x,0)}{\partial x}\right|_{x=(z_- - z_+)\theta_c} \quad (8)$$

For $z_+ = z_-$, $W(z)$ is the minimum value,

$$W_x(0) = \frac{N_+ r_e}{\gamma} \frac{2}{\sigma_x(\sigma_x + \sigma_y)} \quad (9)$$

$W(z) = 0$ at $z \approx \pm 1.3\theta_c/\sigma_x$, and W is the maximum $\approx 0.28|W_x(0)|$ at $z \approx \pm 2.2\sigma_x/\theta_c$. Figure 6 shows the wake field. The wake field is also calculated by a numerical method. The wake linearly depends on Δx around $\Delta x \leq 3\sigma_{x,+}$.

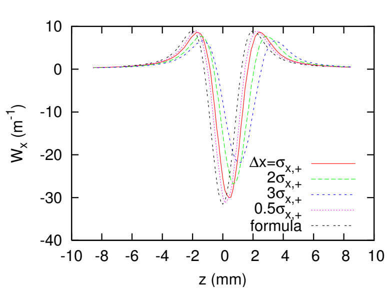

Figure 6: Wake field given by a numerical method and formula in Eq.(8).

Particle tracking simulation using the wake in Fig.6 was carried out. Figure 7 shows evolution of the horizontal bunch size and $\langle xz \rangle$ correlation. The growth of the beam size is very fast ($\sim$ 20 turns) and the head-tail phase of two bunches was the same. This behavior is consistent with a strong-strong simulation.

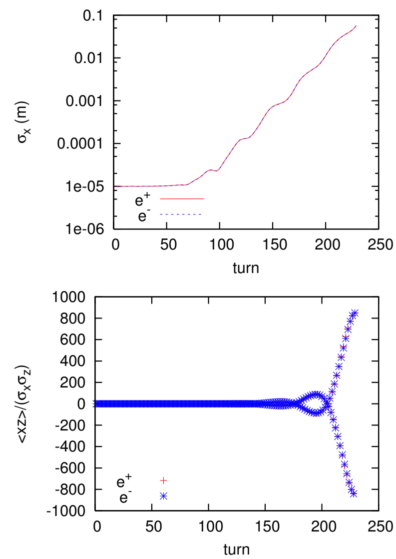

Figure 7: Evolution of beam size and $\langle xz \rangle$.

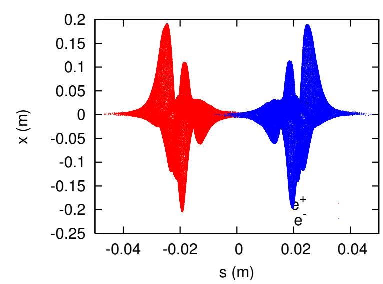

Figure 8: Beam distribution at the collision.

Figure 8 shows distribution of electron/positron bunches after 230 revolutions. The distributions of two bunches are mostly identical.

CONCLUSION

Wake fields induced by beam-beam collision with a large crossing angle were evaluated. A head-tail instability is caused by the wake fields. The instability explains the strong-strong simulation results.

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