IMPROVEMENT OF THE ANALYTIC VLASOV SOLVER DELPHI

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Abstract

The simulation code DELPHI is an analytic Vlasov solver which allows to evaluate the beam transverse stability with respect to impedance effects. It allows to perform fast scans over parameters such as chromaticity, damper gain or beam intensity for a given impedance model and particle distribution.

In order to improve the simulation code, new longitudinal particle distributions have been implemented. The simulations results obtained with these distributions are compared to theoretical predictions. An additional post-processing of DELPHI's output has also been implemented, allowing to reconstruct the signal seen by head-tail stripline monitors, in particular in presence of bunch-by-bunch damper. The results are compared to theoretical models, to PyHEADTAIL simulations and to measurements performed in the LHC.

INTRODUCTION

Vlasov's Equation

Vlasov's equation describes the conservation of the local phase space with respect to time

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = 0 \tag{1}$$

where $\psi = \psi(s, J_z, \theta_z, \tau, \phi)$ is the phase space distribution density with $s = vt$ the longitudinal position along the accelerator orbit, (J_z, θ_z) the horizontal or vertical plane action-angle coordinates, (τ, ϕ) the polar coordinates for the longitudinal phase space [1].

Perturbation Formalism

To treat the stability problem, we assume that a small perturbation ψ_1 of the phase space density develops on top of the unperturbed distribution ψ_0. This mode develops along time at a complex frequency $\Omega = Q_c\omega_0$, with ω_0 the beam angular revolution frequency and Q_c the complex tune. The distribution ψ can be decomposed in transverse and longitudinal parts [1, 2]

$$\begin{aligned} \psi &= \psi_0 + \psi_1 \\ &= f_0 g_0 + f_1 g_1 \exp(j\Omega t) \end{aligned} \tag{2}$$

where f_0 and f_1 are functions of the transverse coordinates and g_0 and g_1 are functions of the longitudinal coordinates. DELPHI uses a decomposition over Laguerre polynomials of the functions g_0 and g_1 from Eq. (2). The treatment of Vlasov's equation leads to an eigensystem which once solved furnishes eigenvalues and eigenvectors. The eigenvalues give informations on the azimuthal and radial modes frequency shifts and their respective growth rates. The eigenvectors allow to reconstruct the longitudinal perturbation g_1.

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IMPLEMENTATION OF NEW LONGITUDINAL DISTRIBUTION

Principle and New Longitudinal Distributions Implemented

In DELPHI the unperturbed longitudinal particle distribution g_0 is written as a finite sum of Laguerre polynomials [2, 3]. This allows to implement multiple distributions to better fit the experimental beam profile or to compare simulations results with examples developed in the literature.

Only the Gaussian distribution was originally implemented in DELPHI. Three other distributions have been implemented: the parabolic line, parabolic amplitude, and an approximated uniform distribution. Their respective equations are given in [4]. The uniform distribution should be a step function $g_0(\tau) = \frac{4}{\pi\tau_b^2}$ for $\tau \in \left[0; \frac{\tau_b}{2}\right[$ and where τ_b is full bunch length in seconds. In DELPHI it has been approximated by a sigmoid shaped function

$$g_0(\tau) = \frac{4}{\pi\tau_b^2} \frac{1}{1 + \exp\left(\frac{25}{\tau_b}\left(\tau - \frac{\tau_b}{2}\right)\right)}, \tau \in [0; +\infty[\ . \tag{3}$$

This approximation is made to avoid the discontinuity of the uniform distribution that would lead to convergence issues when decomposing over Laguerre polynomials.

The resulting longitudinal distributions decomposed over Laguerre polynomials are showed in Fig. 1, alongside the initial longitudinal distribution.

Comparison of DELPHI's Results with Analytical Predictions

In order to check that the new distributions are correctly implemented, a comparison of DELPHI's results is made with analytical formulas. A broadband resonator impedance model (f_{res} = 1 GHz, R_s = 10 MΩ m^{-1} and $Q = 1$) is used and beam stability is computed in the horizontal plane. A scan in bunch intensity is performed for a fixed chromaticity of $Q' = -3$ [5].

The real part of the most unstable mode frequency shift is plotted in Fig. 2 where the dashed line shows the linear fits performed on the data and their respective equations.

These results are compared to analytical formulas from [6], which show that the tuneshift ΔQ_x caused by a general impedance at zero chromaticity and for a certain intensity is

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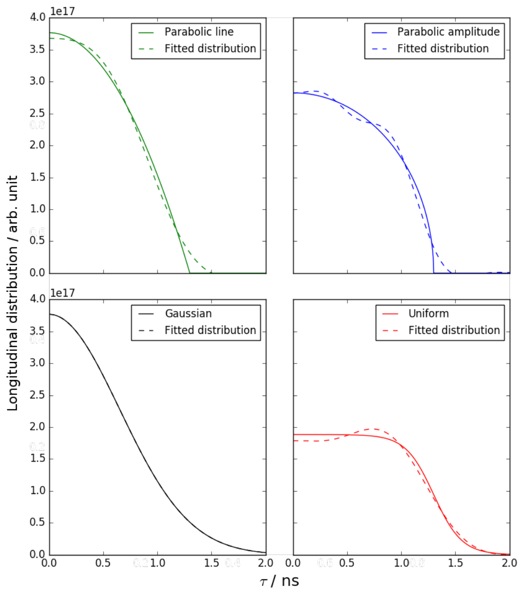

Figure 1: Longitudinal particle distributions as a function of length in ns. Four distributions are showed: parabolic line, parabolic amplitude, Gaussian and approximated uniform. The solid line shows the distribution from [4]. The dashed line show the distribution after the decomposition over Laguerre polynomials.

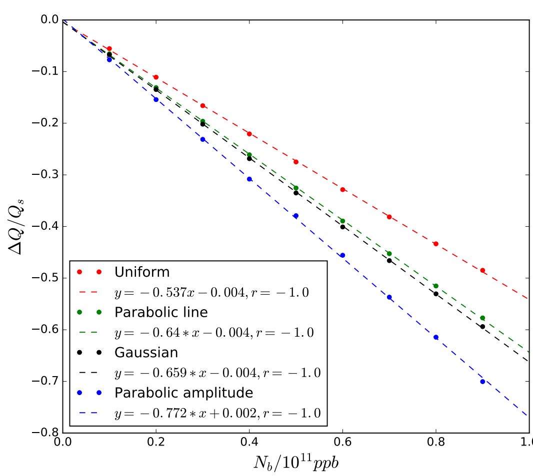

Figure 2: Real part of the most unstable mode frequency shift, normalised by the synchrotron tune, as a function of single bunch intensity (in protons per bunch).

proportional to

$$\Delta Q_x \propto \frac{\int_{-\infty}^{+\infty} g_0(\tau)^2 \, \mathrm{d}\tau}{\left(\int_{-\infty}^{+\infty} g_0(\tau) \, \mathrm{d}\tau\right)^2} \tag{4}$$

Equation 4 yields for the various distributions:

$$\Delta Q_x|_{\text{gaussian}} \propto \frac{4}{\pi^{\frac{1}{2}}\tau_b} \tag{5}$$

$$\Delta Q_x|_{\text{parabolic amplitude}} \propto \frac{12}{5\tau_b} \tag{6}$$

$$\Delta Q_x|_{\text{parabolic line}} \propto \frac{64}{3\pi^2\tau_b} \tag{7}$$

$$\Delta Q_x|_{\text{uniform}} \propto \frac{2}{\tau_b} \tag{8}$$

First, the three following ratios of the linear fits slopes reported in Fig. 2 are performed: Uniform/Gaussian, Parabolic amplitude/Gaussian and Parabolic line/Gaussian.

These three ratios are compared to the corresponding one obtained from analytical calculations i.e the ratios of equations (8)/(5), (6)/(5) and (7)/(5). The results are reported in Table 1. An agreement within 10% is reached between the simulations and the analytical predictions. Some differences could be expected as we used a non-zero chromaticity for the simulations to ensure that mode 0 is the most unstable at all intensities and an approximation of the longitudinal distributions by using a decomposition over Laguerre polynomials.

Table 1: Tune shifts ratios for different longitudinal distributions obtained with simulations and analytical calculations.

Ratio	**Simulations**	**Analytical calculations**
Uniform/Gauss.	0.816	0.886
Parab. amp./Gauss.	1.17	1.06
Parab. line/Gauss.	0.971	0.958

Check of DELPHI Results for the Uniform Longitudinal Distribution Case

A second verification on the implementation of the uniform longitudinal distribution in DELPHI was done. A scan in bunch intensity was performed and the real part of all the modes frequency shifts was compared to calculations made following Laclare formalism [4]. A first set of DELPHI simulations was done with a Gaussian longitudinal distribution and a second set of simulations was done with an uniform distribution. The impedance model used in these simulations is a broad-band resonator with $f_{res} = 1 \times 10^{15}$ Hz, $R_s = 10\,\mathrm{M\Omega\,m^{-1}}$ and $Q = 1$ in order to approximate a purely inductive impedance [7].

Figure 3 shows the resulting mode frequency shifts as a function of single bunch intensity. In these plots the modes frequency shifts are normalised to the synchrotron tune Q_s and the bunch intensity is normalised to the machine's parameters such as

$$\Delta Q_{coh} = N_b \frac{\beta e^2}{4\pi\gamma m_0 c Q_{x0} \tau_b \omega_s} Z_{eff} \tag{9}$$

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with e and m_0 the proton charge and mass, β the particle speed in units of c, γ the Lorentz factor, N_b the number of particles per bunch, Q_{x0} the unperturbed horizontal tune, τ_b the full bunch length (in seconds), ω_s the synchrtron angular revolution frequency and Z_{eff} the machine's effective impedance [7]. In both plots the black points represent the results obtained with Laclare formalism for an uniform distribution and the red points the results obtained with DELPHI.

Figure 3 shows that the uniform distribution (red points) is closer to the results obtained with Laclare formalism (black). The difference with the Gaussian distribution (green) is visible on modes 0, −1 and −2 where the shifts caused by a Gaussian distribution are slightly different from the ones obtained with an uniform distribution.

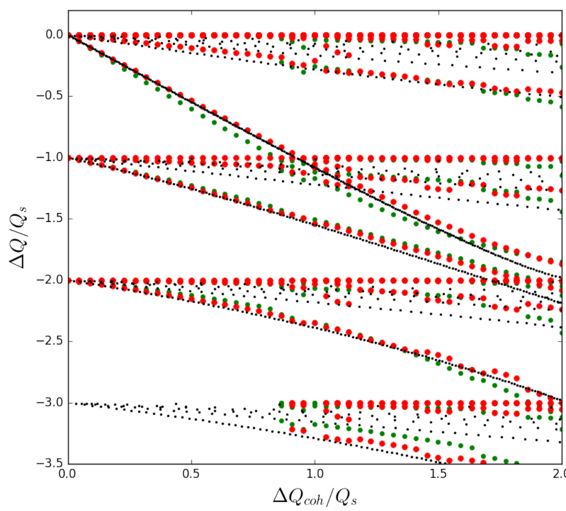

Figure 3: Real part of the modes frequency shifts obtained with DELPHI using a Gaussian longitudinal distribution (green), an uniform distribution (red), compared to Laclare formalism (uniform distribution, black) as a function of normalised beam intensity, for a constant inductive impedance. Results are normalised by the synchrotron tune.

TREATMENT OF THE EIGENVECTORS

In DELPHI only the eigenvalues were treated in the stability studies. The signal observed with stripline pickups can be obtained from the eigenvectors by reconstructing the transverse perturbation g_1 [1, 4, 5]. The reconstruction of the signal will allow to compare DELPHI results with the head-tail signals observed in the machines and to simulations from the tracking code PyHEADTAIL [8].

Simulations with the LHC Impedance Model and Comparison to PyHEADTAIL and Observations

Coherent instabilities are sometimes observed in the LHC, during machine development or physics time [9]. DELPHI simulations performed with the LHC impedance model [10] are compared to tracking simulations performed with

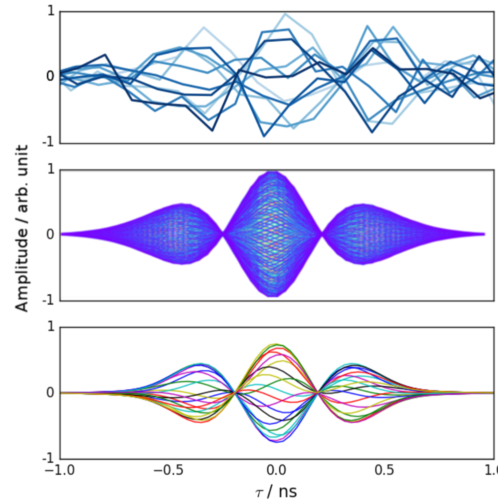

Figure 4: $Q' = 9$, measurements from the LHC head-tail monitor signal (top), PyHEADTAIL simulations (middle) and DELPHI simulations (bottom).

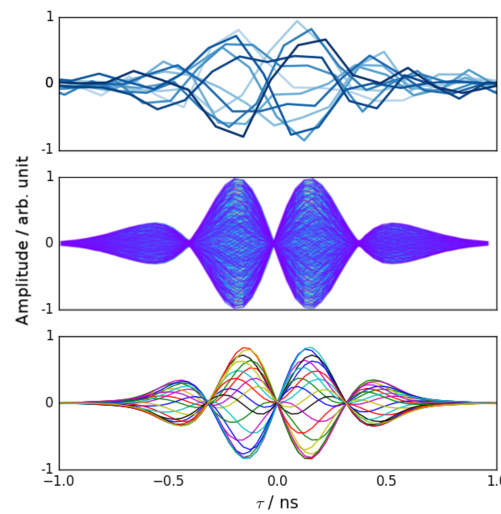

Figure 5: $Q' = 15$, measurements from the LHC head-tail monitor signal (top), PyHEADTAIL simulations (middle) and DELPHI simulations (bottom).

PyHEADTAIL and with head-tail monitor [11] observations from the machine. Two observations performed during machine development time are examined: Fig. 4 shows an instability with two nodes (head-tail mode 2) observed on the 16th of April, 2016 and Fig. 5 shows an instability with three nodes (head-tail mode 3) observed on the 7th of October, 2016. These instabilities were observed on beam 1 horizontal plane with a chromaticity of $Q' = 9$ and $Q' = 15$ respectively and with the transverse damper active (50 turns gain). Figures 4 and 5 show that DELPHI reconstruction of the head-tail monitor signal is coherent with PyHEADTAIL results and with observations.

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CONCLUSIONS

New longitudinal distributions have been implemented in DELPHI. Their implementation was checked with analytical formulas and simulations and will allow to better reproduce the observations made in the CERN accelerator complex.

The treatment of the eigenvectors output from DELPHI has also been completed and will allow to compare the measured LHC head-tail monitor signals to DELPHI simulations.

Further developments will take place on DELPHI to include new physics such as second order chromaticity or direct space charge. These improvements will allow to further improve the agreement between simulations and measurements in the LHC and its injectors.

REFERENCES

[1] A.W. Chao, *Physics of collective beam instabilities in high energy accelerators*. New York, NY, USA: Wiley, 1993.

[2] N. Mounet, "DELPHI: an Analytic Vlasov Solver for Impedance-Driven Modes", CERN, Geneva, Switzerland, Rep. CERN-ACC-SLIDES-2014-0066, May 2014.

[3] M. Abramowitz and I.A. Stegun, "Orthogonal Polynomials", in *Handbook of Mathematical Functions*, R Mars, Ed. New York, NY, USA: Dover Publications, 1970

[4] J.L. Laclare, "Bunched beam coherent instabilities", CERN Accelerator School, Rep. CERN-1987-003-V-1, Sep. 1985.

[5] D. Amorim, "Hadron collective effects studies for the LHC and FCC-hh project", MSc. thesis, Grenoble Institute of Technology, Grenoble, France, 2016.

[6] N. Biancacci, "Improved techniques of impedance calculation and localization in particle accelerators", Ph.D. thesis, Università degli Studi di Roma "La Sapienza", Italy, 2014.

[7] D. Amorim, "Shift of head-tail modes with an inductive impedance", presented at BE-ABP-HSC section meeting, CERN, Geneva, Switzerland, Sep. 2016, `http://cern.ch/go/SgN6`.

[8] PyHEADTAIL, `https://github.com/PyCOMPLETE/PyHEADTAIL`

[9] E. Métral *et al.*, "Measurement and interpretation of transverse beam instabilities in the CERN large hadron collider (LHC) and extrapolations to HL-LHC", CERN, Geneva, Switzerland, Rep. CERN-ACC-2016-009, Jul. 2016.

[10] N. Mounet, "The LHC Transverse Coupled-Bunch Instability", Ph.D. thesis, EPFL, Lausanne, Switzerland, 2012.

[11] T. Lefevre *et al.*, "Bunch by bunch beam instrumentation", in *Proc. 2nd Evian Workshop on LHC Beam Operation*, Evian-les-bains, France, Rep. CERN-ATS-2011-017, Dec. 2010, pp. 27–29.

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