TRANSVERSE BEAM PHASE-SPACE MEASUREMENT EXPERIENCE AT CTF3

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Abstract

One of the objective of The CLIC Test Facility (CTF3) at CERN is to demonstrate the CLIC Drive Beam Recombination concept. An accurate control of the transverse beam parameters is necessary in order to succeed in preserving the beam quality after the recombination. During the activity of the facility we improved our tools and technique for characterising the beam transverse phase space before and after recombination. The common quadrupole scan technique was improved by performing constant-beam-size measurement and it was enriched by a tomographic reconstruction of the phase-space. Moreover studies have been performed in order to estimate and subtract the impact of dispersion on such a measurements. An overview of these techniques will be presented with actual measurements performed over the last year of operation of the facility.

INTRODUCTION

The CLIC Test Facility (CTF3) [1] at CERN aims to demonstrate the feasibility of the key technologies of the Compact Linear Collider (CLIC) design [2]. One of the key aspects of CLIC is its Drive Beam recombination. At CTF3 an initially 1.2 µs long train of bunches at 1.5 GHz is recombined with itself in a complex of delay lines and rings in order to produce a 140 ns long train at 12 GHz. During the recombination process different parts of the initial train undertake different paths before being merged together. In order to preserve the projected emittance of the beam an optics that produces the same transverse (and longitudinal) phase-space distribution irrespectively of the path is necessary.

At CTF3 the quadrupole scan technique has been the primary tool to verify the optics and the orbit closure between the different paths, as well as to identify dispersion leakage and chromatic aberrations. In the following sections we introduce the basic mathematical concept used, and we discuss some actual measurements performed at CTF3.

QUADRUPOLE SCAN TECHNIQUE

The quadrupole scan technique is one of the main methods for measuring the transverse Twiss parameters of relativistic beams in transfer lines, and it is extensively documented in the literature (e.g. in [3]). Here we recall only the basic principles for the simplest case of a linear and uncoupled transfer line, where one can treat the horizontal and vertical phase-spaces independently using a 2D matrix formalism.

A quadrupole scan consists in reconstructing the transverse phase-space distribution at some location along a beam line by measuring the beam profile downstream. In linear optics the transfer matrix from a *reconstruction* (R) to a *measurement* (M) location can be written as:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_R \qquad (1)$$

where A, B, C, D are coefficients that depends on the layout of the beam line and on the strength of its quadrupoles. The beam variance (σ_M^2) at the measurement location can be expressed as a function of the Twiss parameters α_R, β_R and γ_R at the reconstruction location as:

$$\sigma_M^2 = \beta_M \epsilon = \begin{pmatrix} A^2 & -2AB & B^2 \end{pmatrix} \begin{pmatrix} \beta_R \\ \alpha_R \\ \gamma_R \end{pmatrix} \epsilon \qquad (2)$$

where ϵ is the beam *geometric* emittance[1]. One can measure the beam variance $\sigma_{M,i}^2$ (e.g. with an intercepting screen as normally done at CTF3) while varying the strength of the quadrupoles and therefore the coefficients of the transfer matrix, and so solve linear system of equations:

$$\begin{pmatrix} \sigma_{M,1}^2 \\ \sigma_{M,2}^2 \\ \vdots \\ \sigma_{M,n}^2 \end{pmatrix} = \begin{pmatrix} \beta_{M,1} \\ \beta_{M,2} \\ \vdots \\ \beta_{M,n} \end{pmatrix} \epsilon = \begin{pmatrix} A_1^2 & -2A_1B_1 & B_1^2 \\ A_2^2 & -2A_2B_2 & B_2^2 \\ \vdots & \vdots & \vdots \\ A_n^2 & -2A_nB_n & B_n^2 \end{pmatrix} \begin{pmatrix} \beta_R \\ \alpha_R \\ \gamma_R \end{pmatrix} \epsilon. \qquad (3)$$

Finally by knowing that the Twiss parameters must satisfy the relation $\beta\gamma - \alpha^2 = 1$, one can disentangle and obtain both the beam emittance and the Twiss parameters at the reconstruction location.

The necessary condition for obtaining a good fit is that the matrix in Eq. (3) is well conditioned. A standard way is to vary a single quadrupole strength such that the beam size at the measurement location goes through a minimum. However this is not necessary, and sometimes one might want to keep the beam size constant or within a certain range due to limitations of the measurement device (e.g. poor resolution or field of view of the screen used). This was already exploited at the former CLIC Test Facility 2 (CTF2) in [4] and used elsewhere, e.g. in [5]. Here one needs an initial estimate of the Twiss parameters that are going to be measured, and so build a well conditioned matrix of the coefficients in Eq. (3) according to the given constraint.

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[1] Often in this paper we will use the *normalised* emittance definition $\epsilon_N = \epsilon\, \gamma_{\rm rel}$ where $\gamma_{\rm rel}$ is the relativistic factor.

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Table 1: Twiss Parameters Fitted from the Measurements Shown in Fig. 1

	β_x [m]	α_x	ϵ_{Nx} [µm]
Standard	5.0 ± 0.2	-3.5 ± 0.1	155 ± 3
Constant size	4.0 ± 0.2	-2.6 ± 0.1	146 ± 3

At CTF3 we use the MATLAB *fsolve* solver [6] for setting up such a measurement. Figure 1 shows a comparison of two measurement performed on the Drive Beam at CTF3. The first one, in blue, has been obtained by varying only one quadrupole, while three quadrupoles have been used for the measurement in red. The second measurement, using the Twiss parameters measured from the first one, was set up trying to keep the beam size constant at the screen. The fitted Twiss parameters are reported in Table 1. Note that

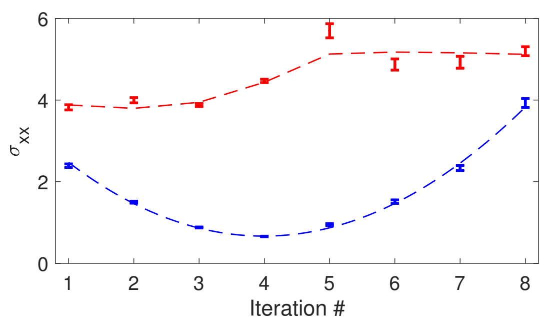


Figure 1: Horizontal beam variance σ_{xx} measured at the screen for different settings of quadrupole settings. Dashed are the expected values from the fitted Twiss parameters.

the fitted Twiss parameters, while being not too far, are not fully consistent. This is believed to be due to the Gaussian fit that is applied to the measured profiles, which sometimes can be far from being Gaussian as shown later in Fig. 5 (a).

TRANSVERSE MATCHING

In an ideal machine the beam is passing through the centre of the quadrupoles, therefore the beam centroid should not move while performing a quadrupole scan. In practice this is not always the case, but the movement of the beam centroid at the measurement location ($x_{M,i}$) can be used to fit the centroid phase-space coordinates (x_R, x'_R) by inverting the following system of equations:

$$\begin{pmatrix} x_{M,1} \\ x_{M,2} \\ \vdots \\ x_{M,n} \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \\ \vdots & \vdots \\ A_n & B_n \end{pmatrix} \begin{pmatrix} x_R \\ x'_R \end{pmatrix}. \tag{4}$$

Moreover, starting from the Twiss parameters and assuming Gaussian beams, one can represent the beam phase-space distribution as an ellipse of equation:

$$\epsilon_x = \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2. \tag{5}$$

At CTF3 the first recombination takes place into the Delay Loop (DL): half of the beam is *delayed*, while the other half *bypass* the DL. Both are then recombined in the following transfer line. Figure 2 shows the ellipse representation in phase-space of a typical well matched delayed beam, bypass beam and relative factor two combined beam measured at CTF3. The measured Twiss parameters and centroid coordinate are reported in Table 2.

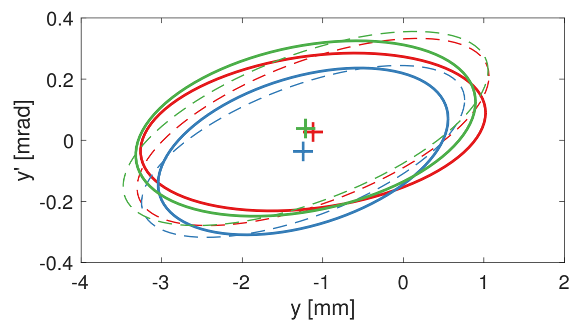


Figure 2: Phase-space representation of a bypass (red), delayed (blue) and combined (green) beam measured at CTF3. Dashed are the expected nominal ellipses.

DISPERSION EFFECT

One of the effects that can spoil the accuracy of a quadrupole scan is the presence of unwanted dispersion in conjunction with high beam energy spread. In the simplest case of a beam line without bending magnets and assuming only linear dispersion, Eq. (2) can be rewritten as:

$$\sigma_M^2 = \beta_M \epsilon + D_M^2 \sigma_p^2 \tag{6}$$

$$= \begin{pmatrix} A^2 & -2AB & B^2 \end{pmatrix} \begin{pmatrix} \beta_0 \epsilon + \sigma_p^2 D_R^2 \\ \alpha_0 \epsilon - \sigma_p^2 D_R D'_R \\ \gamma_0 \epsilon + \sigma_p^2 D'^2_R \end{pmatrix} \tag{7}$$

where D and D' are the dispersion coordinates, while σ_p is the beam r.m.s. energy spread. Note that from Eq. (7) there is no way with a simple quadrupole scan to disentangle the additional dispersion contribution from the “betatronic” one. On the other hand the presence of dispersion makes the beam centroid position change if the beam energy is varied. Each equation of the system in Eq. (4) can then be rewritten as:

$$x_M = \begin{bmatrix} A & B \end{bmatrix} \begin{pmatrix} x_R + D_{x,R} \, \Delta p/p_0 \\ x'_R + D'_{x,R} \, \Delta p/p_0 \end{pmatrix}_R \tag{8}$$

where the $D_{x,R}$ and $D'_{x,R}$ are the dispersion coordinates at the reconstruction location and $\Delta p/p_0$ the applied relative beam energy variation. During a quadrupole scan on can vary the beam energy as well, and collect all beam centroid positions. From this data one can fit the initial centroid coordinates and dispersion inverting Eq. (8), and use this information to subtract the dispersion contribution from Eq. (7).

Table 2: Beam Centroid and Twiss Parameters Related to Fig. 2

	y [mm]	y' [mrad]	β_y [m]	α_y	ϵ_{Ny} [μm]
Bypass beam	-1.12 ± 0.06	0.03 ± 0.01	8.56 ± 0.48	-0.25 ± 0.04	142 ± 4
Delayed beam	-1.24 ± 0.06	-0.04 ± 0.01	7.17 ± 0.53	-0.43 ± 0.05	119 ± 4
Combined beam	-1.21 ± 0.06	0.04 ± 0.01	7.71 ± 0.21	-0.32 ± 0.02	152 ± 2

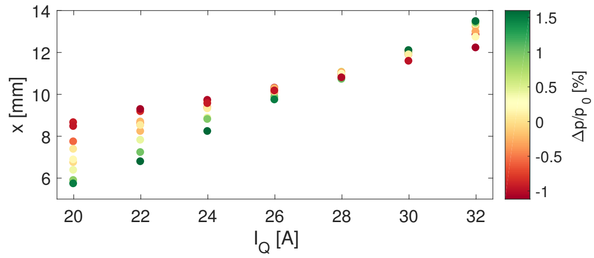
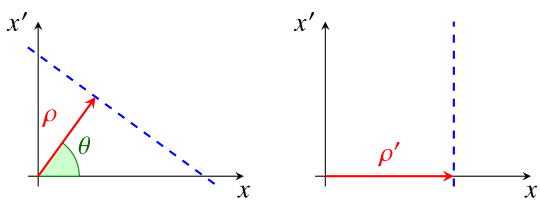


Figure 3: Horizontal beam position x measured at a screen as a function of quadrupole current I_Q. The colour code is the beam energy variation with respect to nominal.



(a) (b)

Figure 4: Correspondence between a phase-space profile integral line (dashed line) at the reconstruction location (a) and at the measurement location (b). The red arrows identify the distance of the integral line from the origin of the axes at the two locations.

Figure 3 shows an example of such a measurement performed at CTF3. From this data we could fit an incoming dispersion of $D_x = -141 \pm 16$ mm and $D'_x = -16 \pm 1$ mrad. On the quadrupole scan data, by including this contribution the measured beam emittance goes from $\epsilon_x = 171 \pm 5$ to 128 ± 17, i.e. a reduction of about 25%.

TRANSVERSE PHASE-SPACE TOMOGRAPHY

The idea of applying tomographic techniques for phase space reconstruction has been introduced in [7] and extensively studied in the literature, e.g. in [3, 8]. The concept is to use the whole beam profile measured, and not only the fitted beam variance. The mathematical background is well explained with the help of Fig. 4 from [8]. Each point of a measured beam profile can be seen as a line integral over the vertical-dashed line in Fig. 4 (b). This is equal to the integral along the dashed line in Fig. 4 (a) representing the phase space at the reconstruction location. Assuming the linear transformation in Eq. (1), the angle θ and the relation between the distances ρ and ρ' can be found to be:

$$\tan(\theta) = \frac{B}{A} \qquad \rho' = \rho\sqrt{A^2 + B^2}. \qquad (9)$$

In practice each measured profile is a compressed projected distribution of the beam phase space taken at different angles. By carefully decompressing the profiles one can therefore use them for reconstructing the actual beam phase-space distribution. At CTF3 this is done by using the inverse Radon transformation provided by MATLAB [9].

Figure 5 shows a typical measurement performed at CTF3 with an heavily non-Gaussian beam. Note the non-Gaussian

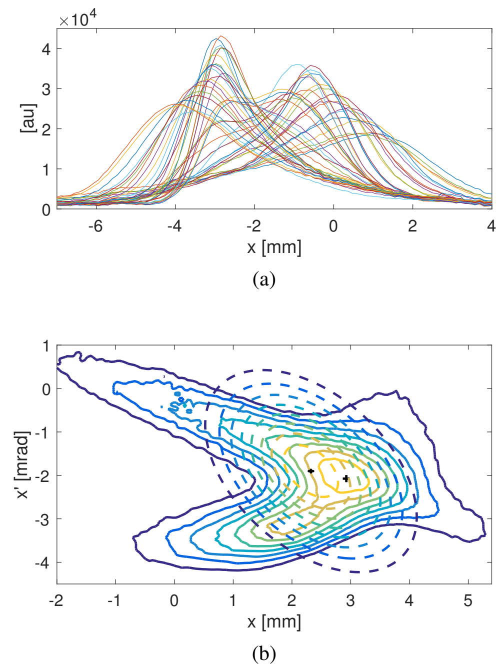


Figure 5: Measured horizontal beam profiles (a) used for the transverse phase-space tomographic reconstruction in (b). Dashed is the phase-space reconstruction using the simpler quadrupole scan technique.

profiles measured at the screen location in 5 (a), which are well exploited by the tomographic reconstruction but obviously ignored by the simple quadrupole scan reconstruction.

CONCLUSIONS

At CTF3 the standard quadrupole scan technique has been exploited in all its aspect and potentiality. The possibility of fitting the incoming beam centroid coordinate, verifying the presence or the absence of incoming dispersion, and performing tomographic reconstruction of the phase-space have been essential aids for verifying the optics matching between the different paths in the Drive Beam Recombination Complex, as well as to achieve many of the results presented in [10].

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