USE OF NONUNIFORM MAGNETS FOR EMITTANCE REDUCTION

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Abstract

We study a theoretical minimum emittance (TME) for a non-uniform bending magnet including a three-step bend (sandwich magnet), a dipole with linear ramp of the bending radius and the same but with a central segment of constant field. We derive expression for the minimum emittance and expand it into a power series with respect to the bending angle. A zero-order term naturally gives the uniform magnet TME while higher-order terms are responsible for the emittance reduction. Theoretical results are verified by numerical simulation.

INTRODUCTION

The equilibrium emittance in a relativistic electron storage ring is defined by the balance between radiation damping and quantum excitation and can be expressed as

$$\varepsilon_x = C_q \frac{\gamma^2}{J_x}\frac{I_5}{I_2},$$

where $C_q = 3.832\times10^{-13}$ m, γ is the relativistic factor, J_x is the horizontal damping partition number, and two synchrotron radiation integrals are [1]

$$I_2 = \oint_M \frac{ds}{\rho^2(s)}, \qquad I_5 = \oint_M \frac{H(s)ds}{|\rho(s)|^3},$$

Here $\rho(s)$ is the curvature radius and the dispersion action is given by

$$H(s) = \gamma_x\eta^2 + 2\alpha_x\eta\,\eta' + \beta_x\eta'^2,$$

with the Twiss parameters ($\alpha_x, \beta_x, \gamma_x$) and the dispersion function and its derivative (η, η'), respectively.

The minimum emittance for the isomagnetic lattice

$$\varepsilon_{xu\min} = C_q \frac{\gamma^2}{J_x}\frac{\theta^3}{12\sqrt{15}}. \tag{1}$$

is achievable in the TME cell consisting of a bending magnet with length L_u and bending angle θ, plus number of quadrupoles to adjust the optics [2].

To get over the constant field TME limit, A.Wrulich in 1992 proposed to use longitudinally non-uniform magnets [3] to compensate increase of $H(s)$ inside a magnet by enlarging the bending radius. As a result, additional minimization of I_5 is possible. The approach has been intensively elucidated in recent years both analytically and numerically [4-9]. The example of the detailed study can be found in trio papers [7-9]. The expressions in these papers are lengthy and cumbersome and the conclusions following from them are obscured by their complexity.

In our study we also derived closed-form formulas for the minimum emittance, initial beta and dispersion which are also entangled. But being expanded as a power series in θ [10] they produce simple and clear expressions with the isomagnetic TME emittance as a zero-order term while the next terms reduce the emittance due to the field variation.

MINIMUM EMITTANCE CALCULATION

Magnet with Linear Radius Ramp

First, we consider a dipole with linear rise (LR) of the curvature radius

$$\rho(s) = k\cdot s + \rho_c = \frac{2(\rho_s - \rho_c)}{L_{LR}}s + \rho_c,$$

corresponding to the hyperbolic field fall-off (Fig.1). Here L_{LR} is the total length of the magnet.

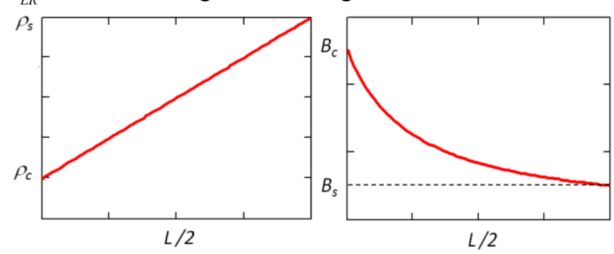

Figure 1: The linearly ramp radius (left plot) and the corresponding field profile for the half of the magnet.

In such field the total angle over the magnet is

$$\theta = \frac{2\cdot\ln(\rho_s/\rho_c)}{k} = \frac{2\cdot y}{k}.$$

The exact closed-form minimum emittance for such a magnet (here $J_x = 1$, $C_q\gamma^2$ is omitted) is given by [10]

$$\varepsilon_{LR\min} = \frac{\theta^3}{12\sqrt{15}}\cdot\frac{3\sqrt{15}\cdot e^{-y}}{y^3(e^y-1)}\cdot\left[\frac{A(y)B(y)}{e^{2y}-1}\right]^{1/2}, \tag{2}$$

where $y = \ln(\rho_s/\rho_c) = \ln(B_c/B_s)$, and two intermediary functions are

$$A = (4y-23)e^{4y} + 8(y+5)e^{3y} - (y^2+12y+10)e^{2y} - 8e^y + 1,$$
$$B = 2e^{2y} - y^2 - 2y - 2. \tag{3}$$

The explicit expressions (2-3) are complex and obscure but if one expands them into power series of θ, the minimum emittance takes simple form with isomagnetic minimum TME emittance as a zero-order term

$$\varepsilon_{LR\min} \approx \varepsilon_{xu\min}\cdot\left(1 - \frac{9k\cdot\theta}{32} + \frac{2337k^2\cdot\theta^2}{71680} - ...\right). \tag{4}$$

Figure 2 shows the emittance reduction with respect to the uniform TME as a function of $y = k\theta/2$ for exact formula (2) and decomposition (3) for the bending angle of $\theta = 0.152$.

It seems that the LR minimum emittance can be made arbitrary low for $B_s \to 0$ ($y \to \infty$), but for the fixed bending angle the length of such magnet

$$L_{LR} = L_u\cdot\sum_{n=0}^{\infty}\frac{y^n}{(n+1)!} \to \infty.$$

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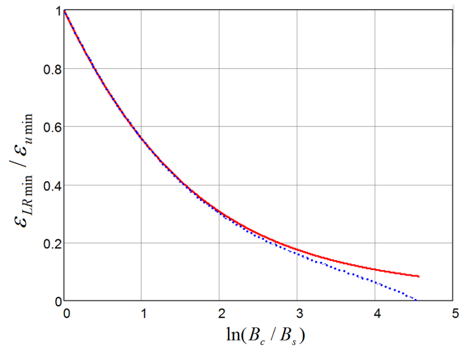

Figure 2: The LR magnet emittance reduction factor for $\theta = 0.152$. Solid line corresponds to the exact formula (2), dots – to the series (3).

Sandwich Dipole

The radius and the field profile of the three-step sandwich dipole (SD) are shown in Fig.3. Following the approach described in the previous section, we obtained the exact solution for the minimum emittance and then expanded it with respect to the bending angle.

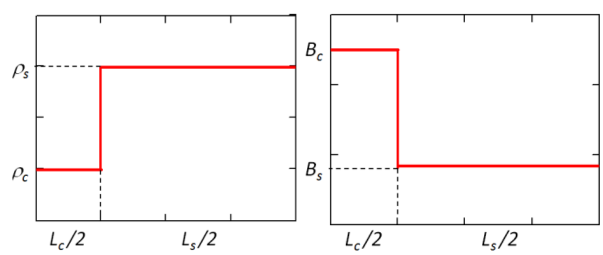

Figure 3: The radius (on the left) and the field profile for the sandwich dipole half.

For the SD, a natural expansion parameter is θ_s / θ, where θ and θ_s are the total and the total side magnets bending angles, respectively. Then the minimum emittance of the sandwich magnet has the form

$$\varepsilon_{SD\min}(\theta_s, n) \approx \varepsilon_{xu\min}\left(1 + a_1(n)\frac{\theta_s}{\theta} + a_2(n)\left(\frac{\theta_s}{\theta}\right)^2 + \ldots\right), \quad (5)$$

where $n = B_s / B_c = e^{-y}$ is the ratio of the side and the central fields, $\varepsilon_{xu\min}$ is the uniform TME magnet with the same total bend and maximum field and the polynomial coefficients

$$a_1(n) = (n-1)(4n+3), \; a_2(n) = -(n-1)(3n^3 + 7n^2 + 5n + 3).$$

Figure 4 shows the SD minimum emittance normalized to the isomagnetic TME for $\theta = 0.152$ as a function of the side bending angle θ_s. Further minimization of the second order approximation formula (5) with respect to θ_s and n gives

$$\theta_{s\min} = \frac{\theta}{2}\frac{4n+3}{3n^3 + 7n^2 + 5n + 3},$$

and

$$\varepsilon_{SD\min}(n) \approx 1 + \frac{1}{4}\cdot\frac{(n-1)\cdot(4n+3)^2}{3n^3 + 7n^2 + 5n + 3}.$$

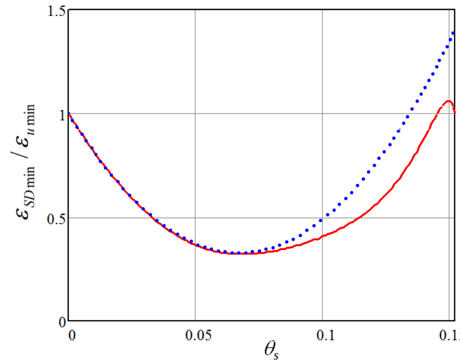

Figure 4: SD minimum emittance with respect to the constant field TME for $\theta = 0.152$ and $n = B_s / B_c = 0.2$. Solid line shows the exact formula while dots correspond to the series (5).

The minimum SD emittance $\varepsilon_{SD\min}(0) = \varepsilon_{xu\min} / 4$ relates to $n = B_s / B_c \to 0$ with the corresponding side magnet bending angle $\theta_{s\min} = \theta / 2$. The resulting three-step magnet has the central field $B_c \to \infty$, and the side field $B_s \to 0$ but the field integral for both segments is the constant providing $\theta_s = \theta_c = \theta / 2$.

The exact solution of the SD minimum emittance slightly differs from the second order approximation (5) and gives $\varepsilon_{SD\min} \approx 0.22557 \cdot \varepsilon_{xu\min}$. The interesting fact is that whereas the LR minimum emittance (2) can reach arbitrary low value with $y = \ln(B_c / B_s) \to \infty$ (in disregard of technological aspects), for the SD magnet there is the fundamental limitation mentioned above.

Magnet with Flat Top and Linear Radius Sides

Simulation shows that the magnet combining the features of the SD and LR magnets (Fig.5) is the most effective for emittance reduction.

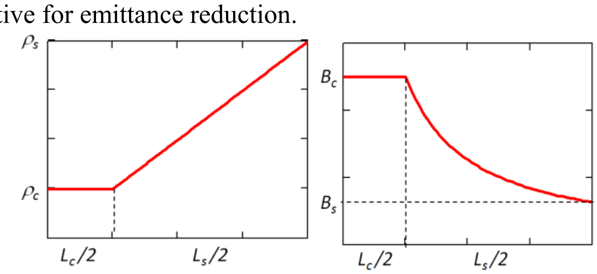

Figure 5: A half of the magnet with the flat top field and linearly ramped radius side segments (LRFT). The radius is on the left and the field is on the right.

The magnet has the attributes of the both models considered above. Combination of two models results in a very complex exact solution for the minimum emittance that we have obtained with Mathematica [11]. However, we have expanded it into series and derived the following expression

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$$\varepsilon_{LRFT\ \min} \approx \varepsilon_{xu\ \min}\left[1 + a_1(p)y + a_2(p)y^2 + a_3(p)y^3 + \ldots\right], \quad (6)$$

where $p = \theta_s/\theta$ and $y = k\cdot\theta_s/2$. The formula (6) is basically similar to (4) with replacement of the constant coefficients by the polynomials $a_i(p)$ with the number of terms growing rapidly with the index increase. Just for example here we show the first coefficient

$$a_1(p) = -\frac{1}{32}p\cdot\left(5p^4 - 30p^3 + 74p^2 - 96p + 56\right).$$

Three first coefficients are shown in Fig.6. The first one (red line), which is primarily responsible for the emittance reduction gives a hint that the best ratio of the side/total bending angle ratio is $\theta_s/\theta \approx 0.5$.

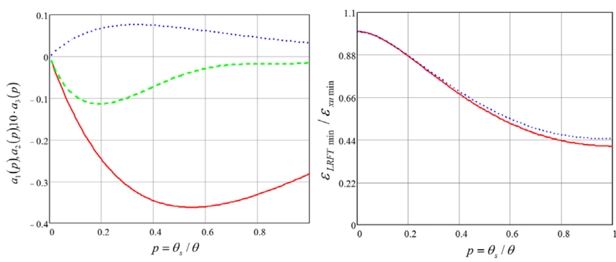

Figure 6: Three first coefficients of (6) are on the left plot. The emittance reduction below the isomagnetic TME is on the right plot ($\theta = 0.152$ and $k = 20$).

Figure 6 shows the LRFT magnet emittance reduction below the uniform TME obtained with the exact closed equation and the series expansion (6) as a function of *p*.

TEST LATTICE

For reference we use the isomagnetic TME magnet providing $\varepsilon_x = 1$ nm at $E = 3$ GeV with $\theta = 0.152$, $B_u = 10$ T and $L_u = 0.152$ m. For brevity, as an example, we apply only the LR dipole with the same bending angle and maximum field. In a compact lattice cell (which is our goal) it is difficult to satisfy optimal conditions, and the resulting emittance degrades due to the dipole optics mismatched with respect to the theoretical one. To cope with this problem we insert into the cell a small-angle anti-bend (–8 mrad) [12].

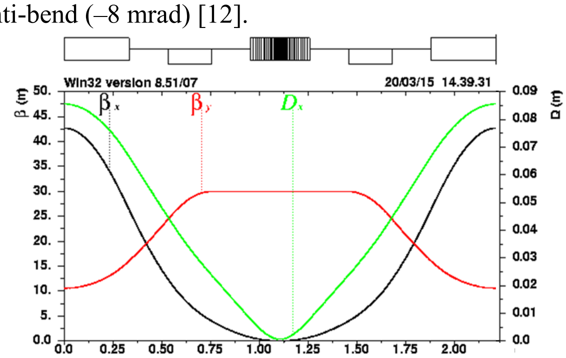

Figure 7: The lattice cell, optimized for the magnet with linear rise of the curvature radius.

The resulting lattice cell is shown in Fig.7. The parameters of the cell (theoretical prediction and simulation) together with the reference uniform field TME cell are listed in Table 1.

Insertion of the anti-bend allows us to tune the lattice very close to the ideal case including the lattice functions in the center of the bend (β_{x0}, η_{x0}) and the resulting emittance whose theoretical value (4) corresponds well to the simulated one. Note that for the cell with the anti-bend (both TME and LR), a momentum compaction α is negative.

Table 1: Main parameters of the example LR magnet lattice cell as compared with the uniform TME cell

Parameter	U-TME	LR (theo)	LR(simm)
L_{mag} [m]	0.152	0.304	0.304
L_{cell} [m]	2.1	-	2.2
ε_x [nm]	1	0.48	0.53
β_{x0} [mm]	19.6	30.5	30.7
η_{x0} [mm]	0.96	0.62	0.61
$\alpha\cdot 10^{-4}$	-0.013	-	-1.03

CONCLUSION

Simple expressions for minimum emittance of the three longitudinally non-uniform magnets are presented. For all cases the result is given as a power series depending on a single parameter: the ratio of the maximum and minimum field in the magnet. The compact TME lattice cell with the LR dipole producing twice less emittance than the constant field magnet is developed. To approach the theoretical minimum we inserted into the cell a small negative-angle anti-bend, which disentangles matching of the beta and dispersion functions. The numeric results correspond well to those predicted by the above theory.

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