TRANSVERSE EMITTANCE EXCHANGE FOR IMPROVED INJECTION EFFICIENCY *

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Abstract

In most cases beam is injected into electron storage rings in the horizontal plane and off-axis. The larger the horizontal emittance of the injected beam the larger the acceptance of the ring has to be. The injected beam is usually delivered by a synchrotron. In case the vertical acceptance of the ring is sufficiently large one can take advantage of the small vertical emittance reached in well aligned and tuned synchrotrons since the transverse emittances can be exchanged with the help of skew quadrupole magnets. A few possible processes will be discussed: emittance exchange with static magnets in the transfer line between synchrotron and ring or emittance exchange in the synchrotron shortly before extraction with time dependent magnets. This could be a suddenly switched-on normal or skew quadrupole magnet or skew quadrupole fields oscillating at a frequency fulfilling the resonance condition. Estimates for these magnets and their design will be given.

INTRODUCTION

BESSY II, a third generation synchrotron light source based on a 1.7 GeV electron storage ring, has recently introduced top-up operation under very stringent conditions for the injection efficiency. In the course of preparing the facility and in addition to many necessary hardware modifications it became clear that these requirements would be reached more easily with a smaller emittance of the injected beam. This would make the injection process less dependent on the actual acceptance which might be reduced by insertion devices. The potential improvement is visualized in Fig. 1 showing the injection process in the x,x'-phase space. The horizontal aperture requirement, shown as a red circle, is dominated by the large emittance of the injected beam, σ_{inj} , in comparison to the low emittance of the already stored beam, σ_{sto} :

acceptance > $6 \cdot \sigma_{ini} + 6 \cdot \sigma_{sto} + eff$. septum thick.



Figure 1: Impact of the injected beam emittance on the aperture requirement.

In nearly all light sources synchrotrons deliver the injected beam with a large horizontal and a small vertical emittance or at least could be tuned in such a way. Without any tuning at all the emittance ratio of the BESSY synchrotron is >3. For the BESSY II storage ring we verified that the acceptance of the ring is sufficiently large to capture a beam after fully exchanging the emittance. This was done by vertically miss steering the injected beam in angle and position at the injection point and observing the injection efficiency. It turned out that the vertical acceptance for injection is large enough to go for a full emittance exchange. The resulting reduction in the required aperture is obvious – see Figure 1.

In the following, ways will be discussed to take advantage of the large emittance ratio obtained in properly aligned and tuned injector synchrotrons. A moderate reduction is obtained from emittance sharing. Full benefits can be reached by emittance exchange through resonance crossing, by applying a so called π -pulse, or by an appropriate skew quadrupole arrangements in the transfer line.

EMITTANCE SHARING

It is well known [1], that, with equal damping rates in both transverse planes, the natural emittance, ε_0 , will be shared equally between the planes: $\varepsilon_x = \varepsilon_y = \varepsilon_0/2$ on the linear, difference, coupling resonance – here written including a skew gradient which oscillates at frequency F_{sq} : $Q_x - Q_y = n \mp F_{sq}/F_0$, where n is an integer and F_0 is the revolution frequency. Even if operation on the coupling resonance is not desired emittance sharing can be achieved by bridging the tune difference with an appropriate oscillating skew quadrupole field [2]. In order to



Figure 2: Vertical beam size close to the coupling resonance for decreasing coupling strengths. On resonance the emittance reaches a value independent of coupling above a certain coupling strength.

reach full coupling the coupling strength for the resonance [3] has to be sufficiently large compared to the natural diffusion of the transverse beam motions. In Fig. 2 the result of a recent decoupling experiment at the BES-SY ring is shown. The smaller the coupling the more the width of the resonance is reduced. On resonance emittance sharing is reached until the coupling is too small to fully combat the diffusion and the vertical emittance is

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now smaller than $\varepsilon_0/2$. The widening of the resonance curves as the coupling is increased could be called power broadening, a term which is used in other but very similarly behaving systems.

EMITTANCE EXCHANGE

So far we were dealing with the "long term steady state" solution of the coupled beam motion. Now we are entering the field of time dependent coupling effects which will allow for full emittance exchange. Compared to sharing the emittance, this will produce the ultimate improvement of the horizontal aperture. It can be applied in cases where the vertical acceptance for injecting beam is sufficiently large and the ring accepts the full horizontal emittance in the vertical plane. Three techniques to exchange the emittance will be presented. Two of them require operating the synchrotron close to the coupling resonance and demand a good compensation of the coupling strength of this particular resonance.

Emittance Exchange by Resonance Crossing

The easiest way to achieve the exchange is by crossing the coupling resonance in the synchrotron shortly before beam is extracted [4]. Carli, et al., the authors of this paper have developed the technique and the corresponding theory for proton machines where damping does not play a role. In Fig. 3 results from multi particle tracking studies including damping and excitation are shown which demonstrate that given a fast enough crossing the emittance could at least be partly exchanged. The transverse damping times in the BESSY II synchrotron are 9.82 ms at 1.7 GeV.



Figure 3: Variation of the normalized emittances as the tunes are swept across the linear coupling resonance with increasing speed. The coupling strength is constant and for very slow sweeps the dotted, theoretical curve would be obtained, similar to Fig. 2. The horizontal axis is also the time axis with the indicated full sweep time. Time increases from left to right.

Estimates for the technical specifications of a single pulsed normal quadrupole magnet are simple and depend on the distance to the coupling resonance, $\Delta Q_{x,y}$, which has to be covered. The maximum integrated K-value needed is given by: $K \cdot L = 4\pi \cdot \Delta Q_{x,y}/\beta_{x,y}$, and the smaller the closer the synchrotron is operated to the resonance. The magnet has to be pulsed with duration of 1-2

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ms. In case the resonance is not excited at all then a small tilt of this magnet could simultaneously produce the required skew component which is needed by this approach. That is the way we would do it at BESSY where the magnets of the synchrotron are part of White-circuits and are cycled at 10 Hz. With more slowly ramped magnets the already existing hardware will probably immediately allow to drive the beam across the resonance shortly before extraction.

The emittance exchange is limited by the resonance crossing speed and at best a ratio of 9 to 1 seems achievable. The next technique is based on a very fast pulse and promises to reach really full exchange of the transverse emittances.

Emittance Exchange by Applying a π *-Pulse*

Like in many two-level systems, NMR, atomic transitions resonantly driven by a strong laser field, spin=1/2and other systems, the inversion of states is achieved with a π -pulse. In our case and in order to be efficient the tune should lay as close to the coupling resonance as possible. For example, in the results of multi particle tracking presented in Fig. 4 it was assumed that the resonance was perfectly compensated for and the working point was on the resonance. The pulse duration of the skew quadrupole magnet is 6 turns and the strength follows a half sine wave which is a simple approximation to what can be achieved in reality. As a matter of fact, the shape is not important. A π -pulse is rather characterized by its time integrated value. From numerical calculations follows that the peak skew gradient is approximately given by:

 $\frac{\partial B_x}{\partial x} \cdot n \cong \frac{4.94 \cdot B\rho}{\sqrt{\beta_x \beta_y \cdot L_{skew}}}, \text{ where n is the pulse duration in}$

number of turns, and L_{skew} the length of the pulsed magnet. The other parameters have their usual meanings. The longer the pulse the smaller the peak field can be. A shorter pulse and its higher peak field is however desirable in order to reach the largest exchange ratio if the working point is only near and not on the coupling resonance. The power broadening helps to force all particles to behave equally and thus to maximize the exchange. The technique works only for tunes very close to the resonance. This requires an extremely careful and stable compensation of the resonance in order to reach the small vertical emittance before the π -pulse is applied.



Figure 4: Impact of a skew quadrupole π -pulse on the emittance in the synchrotron.

Taking the above example from the BESSY II synchrotron the required peak gradient of a pulsed skew quadrupole magnet located in one of the straight sections would be: $\frac{\partial B_x}{\partial x} \approx \frac{4.94 \cdot B\rho/n}{\sqrt{\beta_x \beta_y \cdot L_{skew}}} \approx \frac{4.94}{6 \cdot 0.3 m} \cdot \frac{5.7 Tm}{\sqrt{4.7 m \cdot 5.0 m}} \sim 3.2 T/m.$

Such gradients can be obtained with a symmetric 4 wire arrangement as shown in Figure 5. With the distance between the wires, a, and the current through neighbouring wires, I, flowing in alternating directions the skew gradient will be: $\frac{\partial B_x}{\partial x} \left[\frac{T}{m}\right] = \frac{4 \cdot \mu \cdot I}{\pi \cdot a^2} = \frac{1.6 \cdot 10^{-6} \cdot I[A]}{a^2[m^2]}$. A gradient of 4T/m can be reached with a=2 cm and I=1kA. The pulse duration, in our case of 1.92 µs, and the overall design of such a magnet are very similar to the parameters of our non-linear injection kicker magnet developed, built, and installed in the BESSY II storage ring [5].



Figure 5: left – wire arrangement for producing pulsed skew gradient fields, right – sketch of our non-linear injection kicker magnet where the currents in all wires flows in the same direction and return wires further out are needed.

Emittance Exchange in the Transfer Line

Finally, exchanging the emittance after the beam is extracted from the synchrotron by an appropriate arrangement of skew quadrupole magnets in the transfer line is another option which would not require any special operating conditions of the synchrotron except for a low intrinsic vertical emittance in order to be useful. Different magnet structures have been studied and here more details are given for a system based on 5 symmetrically placed skew quadrupole magnets as shown in Fig. 6. The 4x4-transfer matrixes of these exchange structures have the following form [6]: $R = \begin{pmatrix} 0 & D \\ D & 0 \end{pmatrix}$ with $D = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$. In the example of Fig. 5 the total length of the insert is equal to *l* and the structure acts like a simple drift.



Figure 6: Emittance exchange with 5 skew quadrupole magnets.

As can be seen in the optics plot shown in Fig. 7 the structure would fit into the available space of our transfer line, however, the horizontal dispersion created by the dipole magnets following the exchange insertion and the vertical dispersion after the exchange cannot be corrected for. They are so large that any final improvement of the **ISBN 978-3-95450-147-2**

horizontal emittance would be outweighed by the increased contribution from the energy spread, $\sigma_E=5.6 \cdot 10^{-4}$, delivered by the synchrotron. Also the vertical dispersion would nearly triple the vertical beam size and make efficient injections impossible. Very likely no transfer line would be capable of dealing with these issues of dispersion. Even if we would start with zero horizontal dispersion, transfer lines have to contain bending magnets and even if they could be set up in an achromatic arrangement the space needed for the exchange insertion are excessive and do not allow for installation at the end of the transfer line where the dispersion should approach zero and where their exchange would no longer cause much trouble.



Figure 7: BESSY II transfer line optics with emittance exchange by 5 skew quadrupole magnets. The creation of vertical dispersion and the horizontal dispersion produced further downstream turn this scheme useless. The Twiss parameters at the end of the transfer line are shown at the top.

CONCLUSIONS

Clearly, a smaller horizontal emittance would generally improve horizontal off-axis injections. As demonstrated, emittance sharing and the halving of the horizontal emittance on the natural or an artificial coupling resonance would already be a desirable step in this direction. Emittance exchange, in order to take full advantage of the large emittance ratio in modern synchrotrons, requires first of all that the vertical acceptance of the storage ring is sufficient for the vertically enlarged injected beam. If that is the case, then the exchange can be performed by quickly crossing the coupling resonance or by applying a skew gradient π -pulse. Emittance exchange of the already extracted beam seems to be difficult because of the issues with uncontrollable horizontal and vertical dispersion.

Probably the easiest way to realize emittance exchange in existing facilities is the crossing of the coupling resonance. Take advantage of an emittance reduction by a factor approaching 10 and sacrifice the additional factor of 5 which could be expected from the π -pulse technique.

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REFERENCES

 A.W. Chao and M.J. Lee, "Particle distribution parameters in an electron storage ring", Journal of Applied Physics 47, 4453 (1976).

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- [2] P. Kuske and R. Görgen, "Vertical Emittance Control at BESSY", EPAC 2002, WEPLE015.
- [3] G. Guignard, CERN-ISR-MA/75-25, 1975 (unpublished).
- [4] C. Carli et al., "Emittance Exchange by Crossing a Coupling Resonance", EPAC 2002, WEPRI04.
- [5] T. Atkinson et al., "Development of a Non-Linear Kicker System to Facilitate a New Injection Scheme for the BESSY II Storage Ring", IPAC 2011, THPO024.
- [6] M. Aiba et al., "Round Beam Operation in Electron Storage Rings and Generalisation of Möbius Accelerator", IPAC 2015, TUPJE045.