CSR-DRIVEN LONGITUDINAL SINGLE BUNCH INSTABILITY WITH NEGATIVE MOMENTUM COMPACTION FACTOR*

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Abstract

Acceptable agreement is found between experimental results obtained at the Metrology Light Source (MLS) operated with negative momentum compaction factor, α, and theoretical estimates of the CSR-driven threshold currents. Theoretical instability thresholds are estimated by numerically solving the Vlasov-Fokker-Planck equation and/or by multi particle tracking and taking into account the shielded CSR-interaction. Some of the issues with the calculations, the determination of the theoretical thresholds as well as the derivation of a general scaling law will be presented.

INTRODUCTION

The first observations of the stable emission of coherent synchrotron radiation at BESSY II in 2001 [1] and the increasing interest in shorter light pulses delivered by storage ring based light sources has triggered a large amount of experimental and theoretical work dealing with the stability of short bunches circulating in rings. The Stanford group was the first to present theoretical results based on the model of shielded CSR [2]. This model describes the interaction of electrons by radiation moving midway between two infinite conducting parallel plates [3], $2h$ apart. For the usual case of a positive momentum compaction factor and because of the scaling properties for wake and impedance the predicted threshold is a quite simple function of the shielding factor, Π: $S_{csr}^{thr} = 0.5 + 0.12 \cdot \Pi$. With the dimensionless strength parameter, S_{csr}^{thr}, given by: $S_{csr} = \frac{N r_e}{2\pi \nu_s \gamma \sigma_\varepsilon} \cdot \rho^{1/3} (c\sigma_0)^{-4/3}$ with N (number of particles in the bunch), r_e (classical radius of the electron), ν_s (synchrotron tune), γ (Lorentz factor), σ_ε (natural energy spread), ρ (bending radius), σ_0 (zero current bunch length), and c (speed of light). The shielding parameter, $\Pi = \rho^{1/2} c\sigma_0 / h^{3/2}$, is related to the normalized resonance frequency, $2\pi \cdot F_{res} \cdot \sigma_0$. F_{res} is the local maximum of the shielded CSR-impedance [4] and can be calculated by: $F_{res} = c\sqrt{\pi/24}\rho^{1/2} h^{-3/2}$. With this model the instability is weak for $2\pi \cdot F_{res} \cdot \sigma_0 < 1$ [2, 5]. In this regime the threshold current depends on the longitudinal damping time in relation to ν_s.

Despite the simplicity of the model good agreement was found between calculated and measured thresholds especially in smaller rings [6]. In addition there are other theoretically predicted features of the instability which were observed in experiments: The frequency of the first unstable mode [7], some of the intensity dependent features of these modes [8], and for certain parameters and with increasing number of particles, N, a once again stable intensity region before a second, higher threshold of instability is reached [9]. The question arose, whether the model would work similarly well for negative momentum compaction factors and this motivated the studies presented in this paper.

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DETERMINATION OF THRESHOLDS

Generally, the numerical solution of the Vlasov-Fokker-Planck equation including the wake from the shielded CSR-wake is considered to be a quite fast and robust approach to determine the instability thresholds [2]. With the numerical approach described in [10] it turned out, that with α<0 and below a certain bunch length, $2\pi \cdot F_{res} \cdot \sigma_0 < 0.8$, the bunch would turn unstable only at extremely high intensities and only due to numerical issues. Therefore, thresholds were estimated with multi particle tracking. The induced voltage due to the short range part of the wake was calculated based on binning the particles into up to 40 bins per σ_0. Integration of this stepwise constant distribution with the sharply peaked unshielded part of the wake, $w_0(s)$, was simplified by using causality and the fact that $\int_{-\infty}^{+\infty} w_0(s-s')ds' = 0$. Thus the integration over one bin of width Δs, and the spike of the wake multiplied with a constant density, $\lambda(s')$, can be simplified:

$$\int_0^{\Delta s} \lambda(s') w_0(s-s')ds' = -\lambda(s') \int_{\Delta s}^{+\infty} w_0(s-s')ds'.$$

Markus Ries has published a set of threshold current measurements from the MLS with negative momentum compaction factor and for different bunch lengths in his PhD thesis [11]. Thus the parameters of the MLS were used in the calculations. They are collected in Table 1.

Table 1: MLS Parameters

Parameter	Value
Energy	629 MeV
Bending radius	1.528 m
Cavity Voltage/kV	20, 50, 100, 500
Accelerating Frequency	2π·500 MHz
Revolution Time	160 ns
Natural Energy Spread	4.36 10^{-4}
Longitudinal Damping Time	11.1 ms
Height of Dipole Chamber	4.2 cm

In Fig. 1 the results of particle tracking are shown. The bunch length is 2 ps and in this case the VFP-solver does not yield useful results and the calculated normalized energy spread remains constant=1. With multi particle tracking the energy spread grows slowly with increasing strength parameter and the growth is larger for smaller numbers of tracked particles. The higher the number and correspondingly, the smoother the distribution function,

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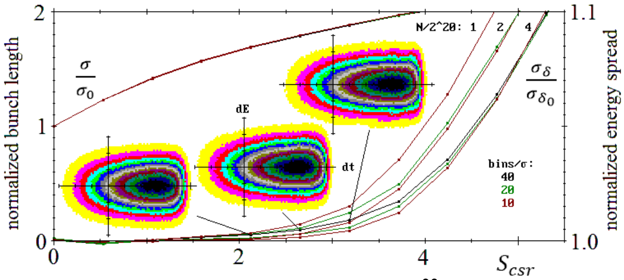

Figure 1: Results of tracking up to 2^{22} particles with a bunch length of 2 ps. The number of bins per σ_0 has only a small effect.

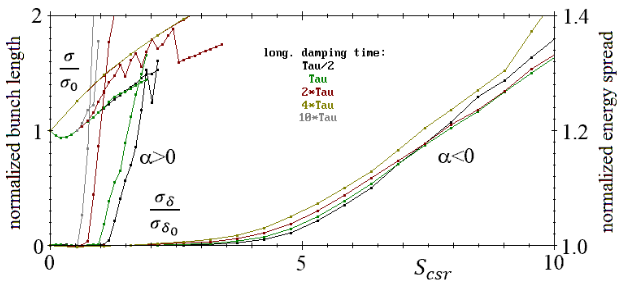

Figure 2: Comparison of multi particle tracking results for positive and negative compaction factors without shielding. In the calculation 2^{22} particles and 20 bins/σ_0 were used.

the larger the strength has to be in order to create a noticeable increase of the energy spread. Therefore it is conceivable that for a really smooth distribution function, like in the VFP-solver, the beam might not get unstable at all. Even though the spread in the tracking results has increased only slightly the distributions show the onset of the instability at the tails and where the particle density is highest. It is the wavy form of the colour interfaces of the density which signals instability. In comparison to the clear influence of the number of particles on the resulting curves, $\sigma_\delta/\sigma_{\delta 0}(S_{csr})$, the impact of the resolution, that is the number of bins per σ_0, is very small. Note that 1 million electrons in the bunch correspond to 1 µA at the MLS. This is not too far below the observed threshold currents.

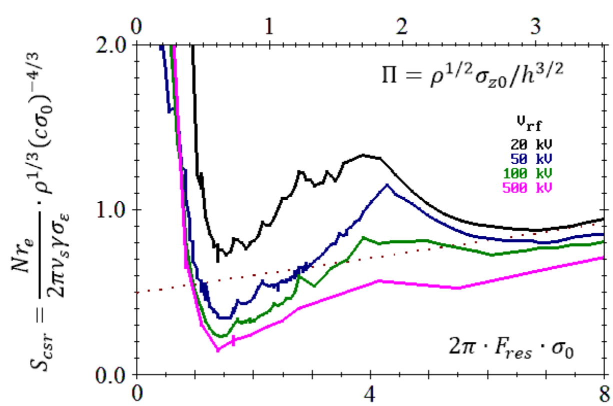

Figure 3: Thresholds for negative momentum compaction factor. The red dotted line is the simple scaling law found for α>0.

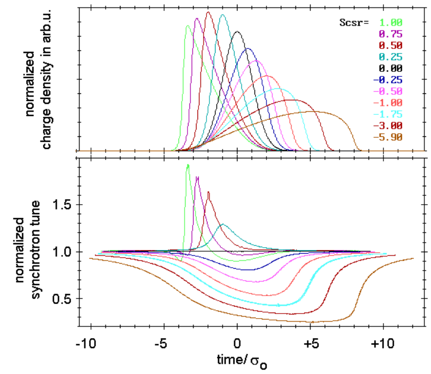

Figure 4: Result for very short bunches – without shielding of the CSR-interaction. Solutions of the Haissinski equation for the charge density and the normalized synchrotron tune derived from this distribution.

A clear determination of the threshold is difficult for α<0 and the initially shorter bunches. This is quite different from the case with α>0. In Fig. 2 the unshielded situation, or $\sigma_0 \to 0$, for both signs of alpha are shown for different longitudinal damping times and a cavity voltage of 50 kV. The weak instability for α>0 is apparent and the thresholds are clearly visible due to the steep increase of the energy spread above them. For α<0 the instability still appears to be mildly weak. The thresholds determination lacks accuracy because of the small growth of the energy spread as the number of particles increases.

THRESHOLDS FOR ALPHA<0

Taken these uncertainties into account and based on the two different numerical approaches which do agree in an intermediate regime of bunch lengths the final result for the threshold strength, S_{csr}, is presented in Fig. 3. The calculations have been performed for different cavity voltages and show the characteristics of a weak instability for ~0.5 < Π < 2.5. This result is a bit disappointing as the dependence of the thresholds with α<0 vs. the bunch length is more complicated than in case of α>0. In the following some additional theoretical observations will be presented to better understand the more complicated behaviour.

Thresholds of Very Short Bunches

For very short bunches the shielding effect of the plates is vanishingly small and Π→0. Figure 4 the shows the charge distribution from the solutions of the Haissinski equation [12] for both signs of α which is reflected in the sign of S_{csr}. From the distribution functions the tune ν_s/ν_{s0} has been determined [13]. With negative α bunches are lengthened with increasing N (Fig. 2). Longer bunches will have higher thresholds in comparison to the still short bunches with positive α. Based on the results presented in Fig. 2 the threshold is: S_{csr}~4. The strong lengthening can be attributed to the “inductive” part of the CSR-interaction. With reversed sign of the momentum compaction factor, α, this leads to bunch shortening at first and the dominance of the resistive part of the CSR-impedance shifting the bunches to the left in order to compensate for the increasing energy loss – see Fig. 2 and 4.

Threshold of Very Long Bunches

With the known particle distribution at the threshold one can calculate the synchrotron tune and fold it with the distribution function to obtain the average tune and the relative tune spread of the ensemble of particles. The tune spread produces Landau damping and shifts the thresholds to higher values. In Fig. 5 these parameters are presented for positive and negative α. The local minimum of the tune spread for α>0 at around Π~0.6 and the loss of Landau damping was blamed to be one of the reasons for the weak instability [Bane]. Similarly one could argue that for α<0 the smallest thresholds should occur around Π~1 and followed for longer bunches by a local maximum at Π~3 which is only roughly true. Here we rather look at the very long bunches, with the large shielding factors where the average synchrotron tune approaches 1 and the tune spread goes 0 for both signs of α. Independent of the sign of α and close to the threshold the particles approaches a Gaussian distribution and they move in a quadratic potential with an oscillation frequency independent of the amplitude. For the onset of the instability it does not matter whether the electrons are circulating clock- or anti clockwise in this potential and therefore the long bunches should have a threshold independent of the sign of α.

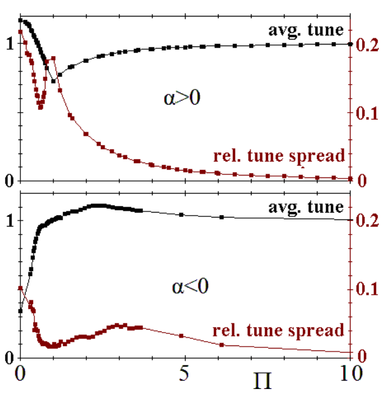

Figure 5: Average synchrotron tune and relative synchrotron tune spread for both signs of α at the onset of the instability. For long bunches and large shielding factors both parameters are identical and the threshold should not depend on the sign of α.

Scaling Law

With these observations one can write the threshold strength for example as a function of the shielding parameter, Π:

$S_{csr}^{thr} \sim \begin{cases} 0.5 + 0.12 \cdot \Pi \text{ for } \Pi > 3 \\ 4 \text{ for } \Pi < 0.5 \end{cases}$. In between the instability is weak and one needs to know the actual parameters in order to estimate the threshold. Figure 6 presents the theoretical results together with these two approximations shown as red lines. Here the results are differently scaled and the graph applies to the MLS only. In a double log-plot the agreement appears to be satisfactory.

Comparison with Experimental Results

Figure 6 also shows the experimental results obtained at the MLS. In the experiments the momentum compaction factor is kept fixed and the bunch length is varied by changing the RF cavity voltage. The experimental results are shown as large squares which are colour coded according to their actual accelerating voltage. In the simulations only the 4 cavity voltages from Table 1 were used. Nevertheless a comparison is possible.

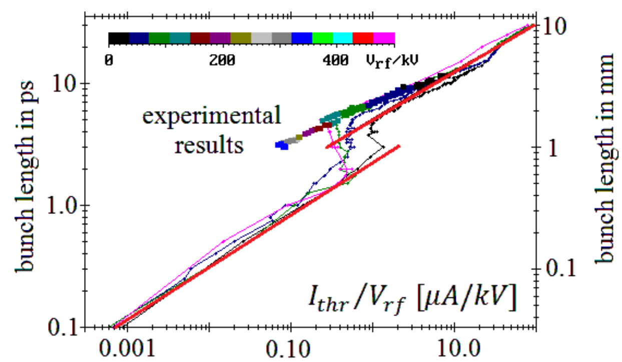

Figure 6: Comparison of experimental results measured by M. Ries at the MLS with the theoretical predictions. Approximations of the two solution branches are shown as red lines. The experimental results are shown as coloured squares with colours corresponding to the RF-voltage during the measurement.

The agreement is acceptable for bunches longer than 5 ps. For shorter bunches the discrepancy increases. One explanation is the missing inductive impedance of the vacuum chamber in the calculations. If this contribution (|Z/n| ~ 0.05–0.1 Ω) is included in the calculations the resulting focusing effect with α<0 will shorten the bunches and reduce the threshold current significantly which improves the agreement between measurement and simulation. The inductive chamber impedance will even remove some of the issues with finding the thresholds.

CONCLUSIONS

Thresholds of the longitudinal single bunch instability driven by the shielded CSR-wake have been simulated for negative momentum compaction factor with multi particle tracking and a VFP-solver. Two different scaling laws were found for long and short bunches. The agreement between theoretical predictions and available experimental results is satisfactory. It occurred that the VFP-solver might be too robust to yield instability thresholds which are due to the noise of the particle distribution which in reality consists out of a finite number of electrons. This needs further investigations.

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