SIMULATION OF ION BEAM UNDER COHERENT ELECTRON COOLING*

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Abstract

The proof of coherent electron cooling (CeC) principle experiment is currently ongoing and it is essential to have the tools to predict the influences of cooling electrons on a circulating ion bunch. Recently, we have been developing a simulation code to track the evolution of an ion bunch under the influences of both CeC and intra-beam scattering (IBS). In this paper, we present the preliminary simulation results and show that they agree with numerical solutions of the Fokker-Planck equation.

INTRODUCTION

An accurate prediction of the ion bunch evolution in the presence of CeC is essential both for commissioning the on-going proof of the CeC principle experiment and for designing a possible CeC system for the future electron ion collider. Such a prediction is typically obtained through numerical simulations. The simulations of CeC consist of two aspects: the single pass simulations and the hadron cooling simulations. In the single pass simulation, an individual ion is injected into the electron beam at the entrance of the cooling section and the electrons are then tracked as they travelling through the cooling section. The single pass simulation provides the information about the responsive kicks from the cooling electrons to the injected ion. The hadron cooling simulation, on the other hand, tracks the ions as they circulate around the ring. During the hadron cooling simulation, the responsive kicks from the electrons, which are to be obtained from the single pass simulations, are applied to the ions for each turn, and the simulation predicts the ion bunch evolution as a result of cooling.

In this work, we concentrate on the hadron cooling simulations. The simulation code takes the energy kick from cooling electrons as input and tracks the circulating ions under the influence of the energy kicks. Section SIMULATION CODE provides a description of the simulation code. In section ENERGY KICKS FROM CEC, we derive the energy kick received by an ion, as expected from 1-D CeC model. As a first estimate, the energy kick derived in section ENERGY KICKS FROM CEC is applied to simulate ion beam evolution for the proof of CeC principle experiment. Section PRELIMINARY RESULTS consists of the preliminary results of the simulations, which is then benchmarked with results obtained from numerically solving the Fokker-Planck equation. We summarize in section DISCUSSION.

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SIMULATION CODE

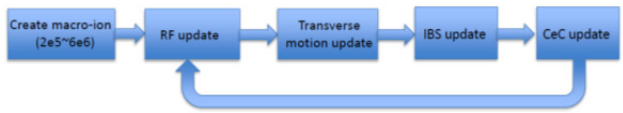

Figure 1: Flowchart illustration of the simulation process.

As shown in Fig. 1, typically 0.2~1 million macro-ions are generated when the simulation starts. The longitudinal coordinates of each macro-ion are then updated according to the rf voltage it sees and the phase slip factor of the lattice:

$$\bar{\varepsilon} = \varepsilon + \frac{q}{mc^2} V_{rf}(\tau) , \quad (1)$$

and

$$\bar{\tau} = \tau + \frac{T_0 \eta}{\beta^2 \gamma_0} \bar{\varepsilon} , \quad (2)$$

where q is the charge of the ion, m is the mass of the ion, c is speed of light, ε is the energy deviation of the macro-ion in unit of mc^2, τ is the arriving time of the macro-ion, $V_{rf}(\tau)$ is the rf voltage seen by an ion, $\gamma_0 mc^2$ is the energy of the reference ion, T_0 is the revolution period and η is the phase slip factor. The update of the transverse coordinate uses one turn linear transfer matrix:

$$\begin{pmatrix} \bar{x} \\ \bar{p}_x \\ \bar{y} \\ \bar{p}_y \end{pmatrix} = \begin{pmatrix} \cos\psi_x & \sin\psi_x & 0 & 0 \\ -\sin\psi_x & \cos\psi_x & 0 & 0 \\ 0 & 0 & \cos\psi_y & \sin\psi_y \\ 0 & 0 & -\sin\psi_y & \cos\psi_y \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{p}_x \\ \bar{y} \\ \bar{p}_y \end{pmatrix} , \quad (3)$$

where ψ_x and ψ_x are the one turn phase advances of horizontal and vertical betatron motion.

A random 3-D kick is applied to each macro-ion every turn to account for effects from intra-beam scattering. The R.M.S. amplitude of the kick is determined by the growth rate as calculated from the Piwinski's formula. Local ion line density is used in the IBS growth rate calculations.

To implement the one turn update due to CeC, we first estimate how the ions are mixed from turn to turn by synchrotron oscillation. The synchrotron period for the CeC experiment is about 4000 revolutions. With the RMS ion bunch length of 3.5 ns, the average longitudinal slippage of a typical ion in one revolution is

$$\langle \Delta\tau_{1\sigma} \rangle \sim \frac{4 \cdot 3.5ns \cdot c}{4000} = 1.05mm ,$$

which is ~80 times larger than the optical wavelength of the FEL amplifier (13 μm). Consequently, no phase

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information is preserved after one revolution and the incoherent kicks due to neighbour ions (and cooling electrons) can be implemented as a random kick. In the next section, we detail on how the coherent and incoherent kick are currently implemented in the code.

ENERGY KICKS FROM CEC

Using the 1-D FEL theory with high gain approximation [1], the electric field induced by a single ion at the entrance of the kicker section is

$$E_{1D}(z) = E_p e^{-\frac{z^2}{2\sigma_{z,rms}^2}} \sin\left(k_0 z - k_2^2 z^2 - \varphi_0\right), \quad (4)$$

where z is the longitudinal location with respect to the peak of the electron density wave-packet induced by the ion, $E_p = G \cdot E_0 = G \cdot Ze/(2\varepsilon_0 S)$ is the maximal electric field induced by the electron density wave-packet, S is the transverse area of the electron beam, $\sigma_{z,rms}$ is the RMS width of the wave-packet, φ_0 is a constant phase shift determined by the length of the FEL amplifier, and $k_2^2 z^2$ represents a slow phase variation along the wave-packet with $k_2^2 z^2 \sim (z/\sigma_{z,rms})^2 << k_0 z$. Thus the field observed by the j^{th} ion due to the wave-packet induced by the i^{th} ion is given by

$$E_{1D}(z_j - \varsigma_i) = E_p e^{-\frac{(z_j-\varsigma_i)^2}{2\sigma_{z,rms}^2}} \sin\left(k_0 (z_j - \varsigma_i) - k_2^2 (z_j - \varsigma_i)^2 - \varphi_0\right), \quad (5)$$

where z_j is the location of the j^{th} ion at the kicker section and ς_i is the location of the peak of the wave-packet induced by the i^{th} ion. By properly delaying the ions, the j^{th} ion can be placed at

$$z_j = D \cdot \delta_j + \Delta z_{sh} + \varsigma_j, \quad (6)$$

where δ is the relative energy deviation of the j^{th} ion, D is the longitudinal dispersion and $\Delta z_{sh} \in (-\pi/k_0, \pi/k_0)$ is a small delay of the electrons introduced by the phase shifter so that for an ion with zero energy deviation, the phase of the sinusoidal function in the first term of eq. (6) is $-\pi$, i.e.

$$\Delta z_{sh} \approx \left[\mathrm{mod}(\varphi_0, 2\pi) - \pi\right]/k_0. \quad (7)$$

Inserting eqs. (6) and (7) into eq. (5) and assuming the electric field do not change significantly inside the kicker of length l, we obtain the one turn energy kick received by the j^{th} ion in the CeC section:

$$\Delta E_j = \Delta E_{coh,j} + \Delta E_{inc,j}, \quad (8)$$

where

$$\Delta E_{coh,j} \equiv -Z_i e E_p l \sin\left(k_0 D \cdot \delta_j\right), \quad (9)$$

is the kick induced by the j^{th} ion itself, i.e. the coherent cooling kick, and the second term,

$$\Delta E_{inc,j} \equiv -Z_i e E_p l \sum_{i \neq j} e^{-\frac{(\varsigma_j-\varsigma_i)^2}{2\sigma_{z,rms}^2}} \sin\left(k_0 (D\delta_j + \varsigma_j - \varsigma_i) - k_2^2 (\varsigma_j - \varsigma_i)^2\right) \quad (10)$$

is the incoherent diffusive kick induced by all other ions. The variance of the incoherent kick is calculated as

$$\left\langle \Delta E_{inc,j}^2 \right\rangle = \frac{(Z_i e E_p l_1)^2}{2} \left\langle \sum_{i \neq j} e^{-\frac{(\varsigma_i-\varsigma_j)^2}{\sigma_{z,rms}^2}} \right\rangle, \quad (11)$$

where the angled bracket represents ensemble average. Assuming the ion density does not vary significantly over the width of the wave-packet, $\sigma_{z,rms}$, eq. (11) reduces to

$$\left\langle \Delta E_{inc,j}^2 \right\rangle = \frac{(Z_i e E_p l_1)^2}{2} \int_{-\infty}^{\infty} \rho_{ion}(\varsigma_i) e^{-\frac{(\varsigma_i-\varsigma_j)^2}{\sigma_{z,rms}^2}} d\varsigma_i, \quad (12)$$

$$\approx \frac{(Z_i e E_p l_1)^2}{2} \sqrt{\pi} \rho_{ion}(\varsigma_j) \sigma_{z,rms}$$

where $\rho_{ion}(\varsigma_j)$ is the local line number density of ions around the j^{th} ion. As mentioned in the previous section, the energy kick in eq. (8) can be approximated as the summation of the coherent kick as described in eq. (9) and a random kick with the R.M.S. amplitude described by eq. (12):

$$\Delta E_{j,N} \approx -Z_i e E_p l_1 \sin\left(k_0 D \cdot \delta_j\right) + \sqrt{\frac{\left\langle \Delta E_{inc,j}^2 \right\rangle}{\left\langle X^2 \right\rangle}} \cdot X_{j,N}, \quad (13)$$

where $X_{j,N}$ is a random number determining the incoherent kick acting on the j^{th} ion at the N^{th} turn and $\langle X^2 \rangle$ is the variance of $X_{j,N}$. For instance, if $X_{j,N}$ is an uniformly distributed random number from -1 to 1, its variance is

$$\left\langle X^2 \right\rangle = \frac{1}{2} \int_{-1}^{1} X^2 dX = \frac{1}{3}, \quad (14)$$

and eq. (13) becomes

$$\Delta E_{j,N} \approx -Z_i e E_p l \sin\left(k_0 D \cdot \delta_j\right) + Z_i e E_p l \sqrt{\frac{3}{2}\sqrt{\pi} \rho_{ion}(\varsigma_j) \sigma_{z,rms}} \cdot X_{j,N}. \quad (15)$$

Following a similar derivation, the incoherent kick due to cooling electrons' shot noise is derived as

$$\Delta E_{j,N}^e \approx e E_p l \sqrt{\frac{3}{2}\sqrt{\pi} \rho_e(\varsigma_j) \sigma_{z,rms}} \cdot X_{j,N}, \quad (16)$$

where $\rho_e(\varsigma_j)$ is the line number density of the electrons at location ς_j.

PRELIMINARY RESULTS

Assuming the ion bunch longitudinal profile is Gaussian, it can be derived from eq. (15) and (16), that the optimal field for a CeC system is given by [2]

$$E_{p,opt} = \frac{\langle \delta^2 \rangle \gamma_0 k_0 D}{N_{eff}} \frac{A_i m_u c^2}{Z_i e l}, \quad (17)$$

where $\sqrt{\langle \delta^2 \rangle}$ is the R.M.S. energy spread of the ion bunch, $A_i m_u$ is the mass of the ion,

$$N_{eff} \equiv \frac{\sqrt{\pi}}{2} \sigma_{z,rms} \left(\frac{N_i}{\sqrt{2\pi}\sigma_{ion}} + \frac{1}{Z_i^2} \frac{N_e}{l_e} \right) \quad (18)$$

is proportional to the numbers of particles in the wave-packet, N_i is the ion bunch intensity, σ_{ion} is the R.M.S. ion bunch length, N_e is the electron bunch intensity and l_e is the full electron bunch length.

For parameters listed in table 1 and table 2, the optimal electric field as calculated from eq. (17) is 130 V/m for $1.6 \cdot 10^8$ ion bunch intensity and 72 V/m for 10^9 ions per bunch. In the simulation, we assume $E_p = 46 V/m$, which corresponds to a field gain of 74. Figure 2(a) shows the simulation results for the ion bunch longitudinal profile after 40 minutes of cooling. The electron bunch sits at the center of the ion bunch and has a full bunch length of 10 ps (red) and 30 ps (green).

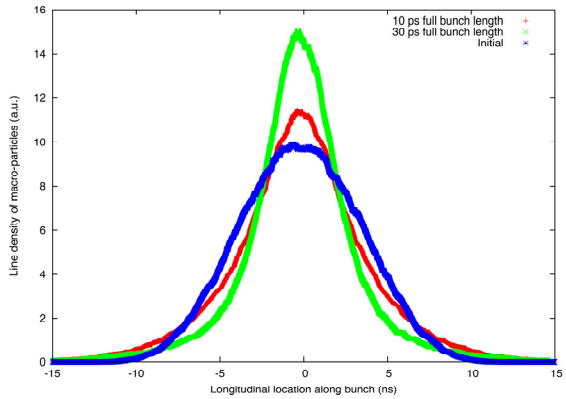
Table 1: Beam Parameters used in the Simulations

Ion bunch parameters, Au^{79+}		E bunch parameters	
Intensity	$1.6 \cdot 10^8$	Peak current	100 A
Bunch length, R.M.S.	3.5 ns	Normalized emittance	5 μm
Energy spread, R.M.S.	$3.8 \cdot 10^{-4}$	Energy spread, R.M.S.	$1 \cdot 10^{-3}$
Beam width at kicker, R.M.S.	600 μm	Beam width at kicker, R.M.S.	600 μm

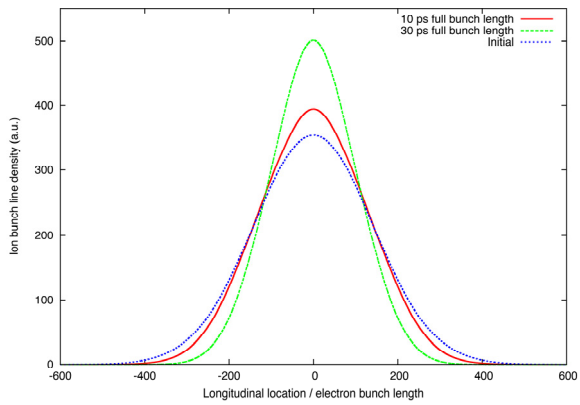
Table 2: CeC System Parameters used in the Simulation

Field gain, G	74	FEL wavelength	13 μm
Peak field, E_p	46 V/m	Dispersion, D	5.7 mm
Kicker length	3 m	Wave packet width, R.M.S.	276 μm

The simulation results are benchmarked with the numerical solution of the Fokker-Planck equation. As shown in Fig. 2(b) the numerical solutions of the Fokker-Planck equation qualitatively agree with the simulation results. There are some differences between Fig. 2(a) and Fig. 2(b) towards the tail of the distribution, which is likely due to the functional form of the cooling force. In the tracking code, the cooling force is proportional to $\sin(k_0 z)$ while in the Fokker-Planck equation, a linear cooling force is assumed.



(a)



(b)

Figure 2: The ion bunch longitudinal profiles after 40 minutes of cooling. (a) ion bunch profiles as obtained from macro-ion tracking; (b) ion bunch profiles as obtained from numerically solving Fokker-Planck equation.

DISCUSSION

It is noticed in the simulation that the ion bunch profile does not show any local blip during the cooling process, which is contrary to our previous expectations. The observation is confirmed with the numerical solutions of the Fokker-Planck equation [3]. For parameters listed in Table 1 and 2, both simulation and numerical solution of Fokker-Planck equation show that the blip starts to show up if the field gain is reduced by a factor of 100 and IBS is ignored from the simulation, which suggests that the fine structure in the distribution function (or high frequency component in the bunch spectrum) is susceptible to the random diffusive kicks. Since any stochastic cooling mechanism, including CeC, inevitably introduces diffusive kicks due to neighbour ions, it is unlikely to observe fine structures induced by local cooling.

We also noticed from the simulations that the cooling of 104 seconds (with 30 ps electron bunch length) results in 10% difference in the maximal ion bunch line density between the uncooled bunch and the cooled bunch. The relatively fast response of the ion bunch profile to cooling make it a valuable diagnostic tool in tuning the CeC system.

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