

# MAGNETIC CENTER POSITION AND TILT ANGLE OF QUADRUPOLE BY VIBRATION WIRE METHOD

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## Abstract

Vibrating wire method and device are described to locate the magnetic center of a Quadrupole theoretically and experimentally. With rotating 180 degrees method, it is convenience to measure the position magnetic center from mechanical center. Tilt angle can also be measured because tilt of magnetic axis will cause the difference of measured magnetic center in different harmonic driving current frequency. Errors analysis shows that tilt of Quadrupole will cause the main error and improved device is described to adjust and measure the tilt angle to fix the errors caused by tilt.

## INTRODUCTION

There are three main methods to measure the magnetic center of the Quadrupole: wire vibration, rotating coil method, Hall probe method [1]. The rotating coil method will use coil rotating in the magnetic field to get magnetic center via the analysis of the Voltage change. Hall probe method use a hall probe rotating in the aperture of a Quadrupole to measure the distance between magnetic and mechanical axis. The single-stretched wire method is adopt to alignment of Quadrupole magnets [2-4].

By rotating 180 degrees measurement with vibrating wire method has been proposed for locating the magnetic center by the distance between magnetic center and mechanical axis which can be very accurate [5]. Errors analysis shows that tilt of Quadrupole will cause the main error and improved device is described to not only adjust the tilt angle to fix the errors caused by tilt, but measure the tilt angle.

## THEORY

### Rotate 180 Degrees to Locate

Locate magnet center by measuring its distance to the mechanical center. The method has already been described in a paper[5]: 1. Move the Quadrupole to get the wire and magnet center coincide; 2 rotate the Quadrupole by 180 degrees; 3 get wire and magnet center coincide again; 4 measure the displacement of position of Quadrupole from 1 to 3 and get dx and dy as it shows in Figure 1. we can get the position of magnet center from mechanical center (dx/2,dy/2)

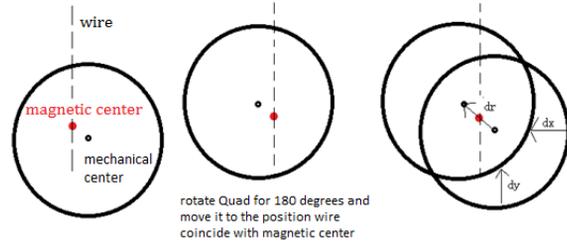


Figure 1: Rotating 180 degrees method for measurement.

### Physical analysis of tilt angle measurement

Set the equilibrium position of copper wire as the z-axis, one end is fixed with a plumb immersed in water where z=l, the upper end is fixed where z=0. Set u (z, t) as the string horizontal position. By is the horizontal magnetic field in y direction

Driving current, depends on time as  $I_0 e^{i\omega t}$

horizontal position in x direction u (z, t) will be

$$\mu \frac{\partial^2 u}{\partial t^2} = T \frac{\partial^2 u}{\partial z^2} - \gamma \frac{\partial u}{\partial t} + B_y(z) I_0 e^{i\omega t}$$

Expand U (z) and B (z) with the eigenfunction [2]

$$U(z) = \sum_{n=1}^{\infty} U_n \sin\left(\frac{\pi n}{l} z\right). \quad (2)$$

$$B_y(z) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{\pi n}{l} z\right). \quad (3)$$

$$U = \sum_{n=1}^{\infty} \frac{I_0}{-\mu(\omega^2 - \omega_n^2) + i\omega\gamma} B_n e^{i\omega t} \sin\left(\frac{\pi n}{l} z\right) \quad (4)$$

$$\omega_n = \frac{\pi n}{l} \sqrt{\frac{T}{\mu}} \text{ is } n \text{ times of natural frequency of wire}$$

When  $\omega \approx \omega_n$  it reach a state of corresponding harmonic

$$U \approx \frac{I_0}{-\mu(\omega^2 - \omega_n^2) + i\omega\gamma} B_n e^{i\omega t} \sin\left(\frac{\pi n}{l} z\right) \quad (5)$$

From the vertical direction of Quadrupole magnets, magnetic inclination angle of geometric axis Z is defined as the tilt angle. As it is shown In Figure 2, if the magnet tilts as Figure 2 left shows, the magnetic field along the wire will be like what the right side shows.

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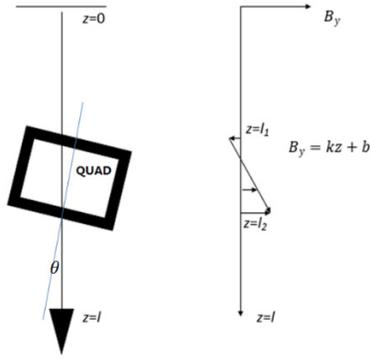


Figure 2: Rotating 180 degrees method for measurement.

The magnetic field along the wire can be described as

$$B_y = kz + b, z \in (l_1, l_2).$$

$l_1, l_2$  is the height coordinates of higher and lower surface of the magnet

$$k = G \frac{dx}{dz} = G \tan \theta. \quad (6)$$

$G$  is gradient of the magnetic field

$$\begin{aligned} B_n &= \frac{2}{l} \int_{l_1}^{l_2} (kz + b) \sin\left(\frac{n\pi}{l} z\right) dz \\ &= \frac{2}{l} \left\{ -k \frac{l}{n\pi} l_2 \cos\left(\frac{n\pi}{l} l_2\right) + k \frac{l}{n\pi} l_1 \cos\left(\frac{n\pi}{l} l_1\right) \right. \\ &\quad \left. + k \frac{l^2}{n^2 \pi^2} \left( \sin\left(\frac{n\pi}{l} l_2\right) - \sin\left(\frac{n\pi}{l} l_1\right) \right) - b \frac{l}{n\pi} \left( \cos\left(\frac{n\pi}{l} l_2\right) - \cos\left(\frac{n\pi}{l} l_1\right) \right) \right\} \end{aligned} \quad (7)$$

When  $w \approx w_n$  and the wire is coincide with magnetic center the  $U$  will be 0 then  $b$  will be

$$b = \frac{-kl_2 \cos\left(\frac{n\pi}{l} l_2\right) + kl_1 \cos\left(\frac{n\pi}{l} l_1\right) + k \frac{l}{n\pi} \left( \sin\left(\frac{n\pi}{l} l_2\right) - \sin\left(\frac{n\pi}{l} l_1\right) \right)}{\left( \cos\left(\frac{n\pi}{l} l_2\right) - \cos\left(\frac{n\pi}{l} l_1\right) \right)} \quad (8)$$

$B$  is relevant with  $n$ , If the location is operate on one harmonic, like  $n=1$ ; and then change the frequency to another harmonic like  $n=2$ , the position of the magnetic center measured will be different, the distance of two magnetic center of different harmonic  $x_s$  will be  $x_s = \Delta b / G$

And the value of  $\Delta b$  is proportional to  $k$ , Only when the tilt angle is 0,  $k$  will be 0,  $b_1=b_2=0$ .

By adjusting the tilt angle of the Quadrupole until it has same measured magnetic center for different harmonic. Then tilt angle can be obtained by measuring the tilt angle of table holding the Quadrupole.

## MEASUREMENT

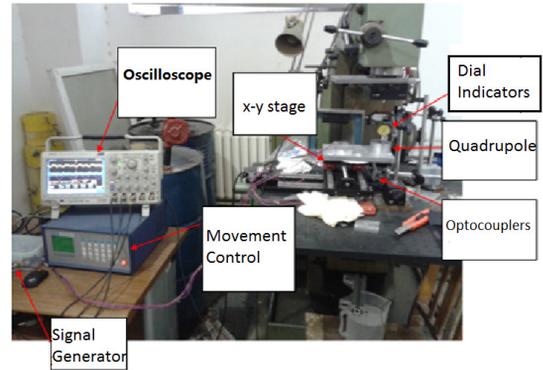


Figure 3: measuring device

As shown in Fig. 3, first, move the Quadrupole by x-y stage and observe the signal, when the amplitude gets down to the lowest or the positive amplitude change to negative with a step within  $2\mu\text{m}$ , it is assumed the wire coincide with magnetic center; get the position  $(x_1, y_1)$  by the dial indicators locate on x and y-axis edge of the Quadrupole. Relocate the magnetic center and get the position  $(x_2, y_2)$ . The magnetic center relative to mechanic center can be obtained  $\left( \frac{1}{2}(x_2 - x_1), \frac{1}{2}(y_2 - y_1) \right)$

To measure the tilt of the Quadrupole, a device is need below Quadrupole which can adjust x and y tilt degrees. First, locate the magnetic center at frequency with harmonic  $n=1$ , then relocate the magnetic center at frequency with harmonic  $n=2$ .

Second, adjust the tilt degrees according to the difference and repeat until the difference is within  $3\mu\text{m}$ . Third, measure the tilt degrees of the stage to get tilt degrees of Quadrupole. The measurement of tilt degrees is recommended to be done before measurement of magnetic center because the tilt of Quadrupole will cause errors.

## ERRORS OF MEASUREMENT

For the measuring devices in Tsinghua University, the parameters for (7) and (8) is  $G=47\text{T/m}$ ,  $l_1=0.255\text{m}$ ,  $l_2=0.30\text{m}$ ,  $l=0.8\text{m}$

If the tilt angle in x direction is 2 degrees, the magnetic center distance between  $n=1$  and  $n=2$ ,  $x_s=44\mu\text{m}$ . To verify the theory, an experiment of measuring the difference between  $n=1$  and  $n=2$  is made. The Quadrupole is tilt on x direction for different degrees. Experiment matches the theoretical estimation, the tilt with 1 degree can cause about  $20\mu\text{m}$  difference between different harmonic frequency.

The tilt can be caused by 3 reasons: tilt of the stage; tilt of the Quadrupole magnetic axis; tilt of the Quadrupole Mechanical shape.

The tilt of the Quadrupole Mechanical shape ont only cause the error of difference between different harmonic frequency, but error of using the dial indicator. When

measuring, dial indicator is at the outer surface of the magnet. After rotating 180 degrees, different height measurements to the corresponding values are different. If the Quadrupole itself is tilted, It will cause huge error compared to other errors. [5]

Table 1: Total errors

Reason of errors	Error amount( $\mu\text{m}$ )
Error caused by rotation	2 (can be reduced to 0.1 by using rotating stage)
Error of dial indicator	2.5
Error of optocouplers	3
Tilt of the Quadrupole Mechanical shape	0-80 (with 0.1 tilt degrees according to height of the dial indicator from 0 to 5cm)
Tilt of the magnetic axis	20 (with 1 tilt degree)

### DISCUSSION

If Quadrupole side surface is exactly perpendicular to the bottom surface and magnetic axis does not tilt, the error will be in order of  $\mu\text{m}$ . And experiments of measurement have been taken and it has been shown this method's stable repeatability[5].

To measure the Quadrupole with tilt angle, it is necessary to adjust tilt angle before magnetic center measurement. A rotating stage and tilt adjust stage are added to improve the setup as shown in Fig. 4.

Tilt measurement and adjustment are taken first, adjust the tilt angle according to the distance of different magnetic center at harmonic like  $n=1$  and  $n=2$  until the distance is less than  $3\mu\text{m}$ . Tilt angle can be obtained by measure the tilt of the adjust stage. The Rotating stage is below the adjust stage so that the magnetic axis will not tilt after rotating 180 degrees to measure the position. The mechanical shape of Quadrupole will tilt because this method will maintain the magnetic axis vertically but magnetic axis is tilt in the Quadrupole. So that the height of the Dial Indicator's need to be defined. It can be defined as same as the middle height of the Quadrupole for example. In this way, Quadrupole with tilt angle can be measured and error of tilt can be reduced.

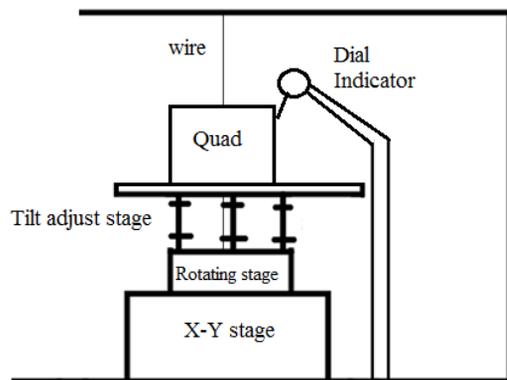


Figure 4: improved setup for Quadrupoles with large tilt angle

### CONCLUSION

With vibrating wire method and sensitive vibration monitors, precise distance between magnetic center and mechanic center can be measured by rotating 180 degrees method. Tilt will cause main error for measurement and error can be adjusted by the improved device.

Tilt angle measurement theory is described that for different harmonic frequency, different magnetic center will be measured because of the tilt. To measure tilt angle between magnet axis and mechanical, and to reduce the tilt error, an improved setup is proposed to measure the Quadrupole magnetic center position and tilt angle.

### REFERENCES

- [1] Henrichsen K N. Overview of magnet measurement methods[J]. 1998.
- [2] Melton J G, Burns M J, Honaberger D J. Pulsed taut-wire measurement of the magnetic alignment of the ITS induction cells[C]//Particle Accelerator Conference, 1993., Proceedings of the 1993. IEEE, 1993: 2944-2946.
- [3] Brueck H, Duda M, Bandelmann R, et al. First results of the magnetic measurements of the superconducting magnets for the European XFEL[J]. Applied Superconductivity, IEEE Transactions on, 2014, 24(3): 1-4.
- [4] A. Temnykh, Vibrating wire field-measuring technique, *Nucl. Instrum. Methods Phys. Res., A* 399(1999) 185 .
- [5] Wang B, Zheng S, Wu L, et al. Fiducialization of the small-aperture quadrupoles based on the vibrating wire method[J]. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 2016, 812: 37-42.