ESTIMATING THE TRANSVERSE IMPEDANCE IN THE FERMILAB RECYCLER *

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Abstract

Impedance could represent a limitation of running high intensity bunches in the Fermilab recycler. With high intensity upgrades foreseen, it is important to quantify the impedance. To do this, studies have been performed measuring the tune shift as a function of bunch intensity allowing the transverse impedance to be derived.

INTRODUCTION

For high intensity operations in the Recycler, high chromaticity is used to help stabilise the beam against the resistive wall instability. Thus, it is important to understand the instability itself and the associated impedances.

The transverse impedance can be found by measuring the tune shift as a function of intensity [1, 2]. The transverse vertical tune shift $\Delta Q_{x,y}$ for a single bunch and flat vacuum chamber is given by

$$\Delta Q_{x,y} = -\frac{q^2 N_b R}{8\pi^{3/2} Q^0_{x,y} \beta^2 E_o \sigma_b} Im\left(Z^{eff}_{x,y}\right)\frac{3}{2} \quad (1)$$

where q is the proton charge, N_b is the bunch population, $Q^0_{x,y}$ is the unperturbed tune, $R = 528.28$ m is the radius of the recycle, β is the relativistic factor, E_o is the proton energy and σ_b is the rms bunch length. The factor of 3/2 comes from including the contribution from both the dipole wake and the quadrupole wake [3]. For the horizontal plane, the contributions cancel giving an impedance of zero. Thus, the impedance can be found using

$$Im\left(Z^{eff}_{x,y}\right) = -\frac{\Delta Q_{x,y}}{N_b}\frac{8\pi^{3/2} Q^0_{x,y}\beta^2 E_o \sigma_b}{q^2 R}\frac{2}{3} \quad (2)$$

where $\Delta Q_{x,y}/N_b$ is gradient found by performing a fit of tune against intensity.

TUNE MEASUREMENT

Turn by turn data from the bpms was taken at different intensities with 10 bunches in there recycler and then repeated with 20 and 30 bunches. Data was taken without pinging as errors in the machine provided a strong tune measurement. For the measurements, the horizontal chromaticity was set to -6 and vertical chromaticity set to -1. For each measurement, 2048 turns were recorded.

The intensity was changed by varying the number of turns in the booster starting at 2 turns and increasing to 12 turns in steps of 2. For each intensity, 3 measurements are recorded.

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A DCCT was used to record the intensity. To calculate the protons per bunch, the intensity was recorded and divided by the number of bunches.

With turn by turn bpm data recorded, the tune can then be determined in the frequency domain by using a Fourier transform [4] or in the time domain by using a sinusoidal fit.

FFT Method

The tune can be found by performing a Fourier transform on the turn by turn data and is identified as the frequency with the highest amplitude.

$$x(n) = \sum_{j=1}^{N} \psi(Q_j)\exp(2\pi i Q_j) \quad (3)$$

where Q refers to the fractional part of the tune and $\psi(Q_j)$ is the amplitude of the spectrum at index j. The frequency array is created by generating an array from 0 to 1 in steps of $1/N$ where N is the number of turns. The error due to the discreteness of the frequency step is equal to

$$|\delta Q| \leq \frac{1}{2N} \quad (4)$$

Interpolated FFT

Assuming that the shape of the Fourier spectrum is know and is equal to that of a pure sinusoidal oscillation with tune Q_{Fint}.

$$|\psi(Q_j)| = \left|\frac{\sin N\pi(Q_{Fint} - Q_j)}{N\sin\pi(Q_{Fint} - Q_j)}\right| \quad (5)$$

The interpolated tune Q_{Fint} is given by

$$Q_{Fint} = \frac{k}{N} + \frac{1}{\pi}\arctan\left(\frac{|\psi(Q_{k+1})|\sin(\pi/N)}{|\psi(Q_k)| + |\psi(Q_{k+1})|\cos(\pi/N)}\right) \quad (6)$$

where $|\psi(Q_k)|$ is the peak of the fourier transform and $|\psi(Q_{k+1})|$ is the highest neighbor. Thus, instead of using only the peak value of the FFT, an interpolation between the highest to points is used. In the case of large N, the error is given by

$$|\delta Q| \leq \frac{C_{Fint}}{N^2} \quad (7)$$

where C_{Fint} is a numerical constant.

Fitting Method

The tune can measured using a time domain technique in which the following function is fitted to the turn by turn data

$$Ae^{-Bx}\sin(\phi - 2\pi Q x) \quad (8)$$

where A is the amplitude, B is the decay constant, ϕ is the phase and Q is the tune. In order to provide the best estimate, the the turn by turn data from multiple bpms are fit simultaneously forcing them to have a common tune value.

VERTICAL TRANSVERSE IMPEDANCE

Data from the vertical bpms was analyzed by performing an FFT on the turn by turn data for each bpm and summing the resulting spectrum. The tune was then found by interpolating the FFT peak as described in the previous section. Figure 1 shows the tune measured tune using both the time domain and frequency domain method. It can be seen that the two methods agree very well for 10 bunches. A linear fit of the frequency domain results is also shown.

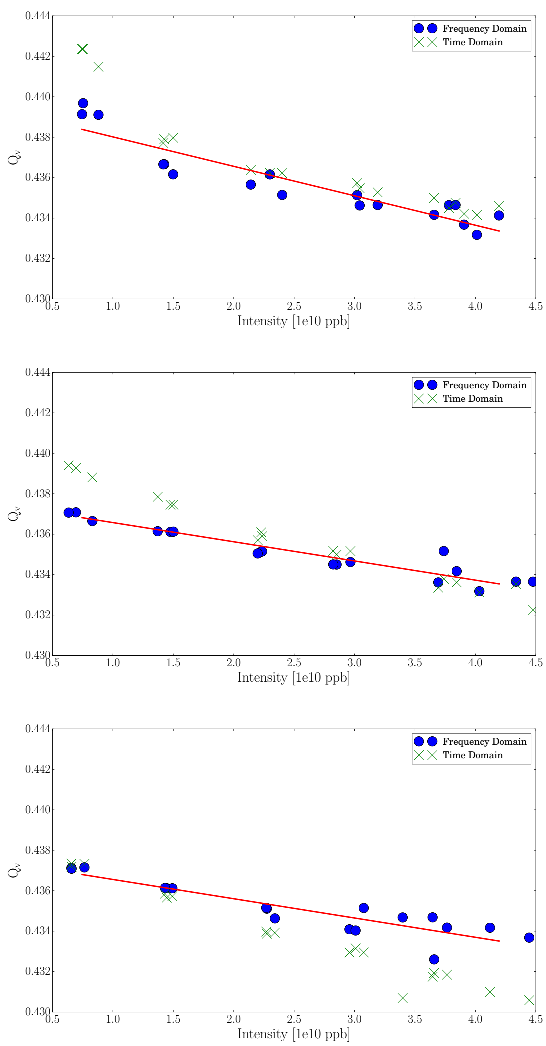

Figure 1: Vertical coherent tune shift as function of intensity for 10 (top), 20 (middle) and 30 (bottom) bunches using both the time domain and frequency domain methods. A linear fit of the frequency domain fit is also shown.

HORIZONTAL TRANSVERSE IMPEDANCE

The measured horizontal tune shift was found to be much smaller. When performing the time domain fit, in some cases, the resulting tune came out very far from the initial guess. In such cases, any tune values that differed by more than 0.02 from the initial guess was removed as this also removed cases when the vertical tune was found instead. The results for 10 bunches are shown in Figure 2.

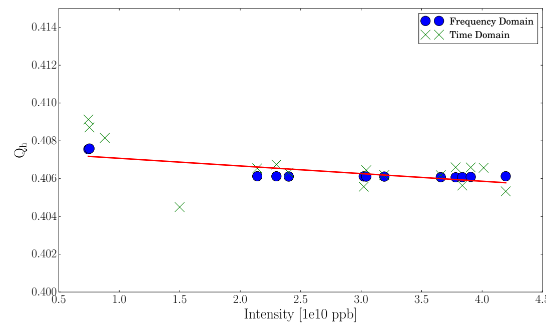

Figure 2: Horizontal coherent tune shift as function of intensity for 10 bunches using both the time domain and frequency domain methods. A linear fit of the frequency domain fit is also shown..

IMPEDANCE CALCULATION

With the tune shifts now measured, the transverse impedance can be estimated. The horizontal tune shift was measured to be very small in all cases. For elliptical beam pipe, the transverse impedance is expected to be close to zero as the dipole wake cancels the quadrupole wake. This agrees well with the small tune shifts measured.

The vertical transverse impedance was then estimated using the measured tune shifts. The tunes obtained using the time domain method appeared less linear and so data was fit to the frequency domain data. The estimated impedances are shown in Table 1. No real dependence on the number of bunches is observed using this method.

Table 1: Calculated Vertical Transverse Impedances for 30, 20 and 10 Bunches using the Frequency Domain Method (FDM)

$Im\left(Z_y^{eff}\right)$ [$M\Omega$/m]			
10 bunches	20 bunches	30 bunches	$\sigma_b[ns]$
12.3	11.1	11.3	1.8

TRANSVERSE IMPEDANCE ESTIMATION FROM LONGITUDINAL IMPEDANCE

The transverse impedance can also be found from the longitudinal impedance.

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$$Im\left(Z_{x,y}^{eff}\right) = \frac{2R}{b^2}\frac{\left(Z_l^{eff}\right)}{n}\frac{\pi^2}{12} \tag{9}$$

The longitudinal impedance Z_l/n has been estimated previously to be 12Ω [5]. Assuming the recycler is mostly made from elliptical beam pipe for which the vertical radius is 2.2225 cm, the vertical transverse impedance is estimated to be 21 MΩ/m.

COUPLED BUNCH CONTRIBUTION

The impedance derivation is for a single bunch however, the measurements shown here involved bunch trains of at least 10 bunches. Some approximations are performed to estimate the size of the contribution from coupled bunches. First, consider the single bunch contribution as

$$\Delta Q_1 \sim \frac{N}{\sigma_b}Z(\frac{1}{\sigma_b}) \sim \frac{N}{\sqrt{\sigma_b}} \tag{10}$$

For this case we have assumed the impedance $Z(\omega) \sim \frac{1}{\sqrt{\omega}}$. Now consider a train of bunches. As an approximation, the train is considered to be one long bunch where the bunch length RMS is given by $M\lambda_0/4$ where M is the number of bunches, λ_0 is the bunch spacing. The factor of 4 is if we assume the total bunch length to be 4σ.

$$\Delta Q_M \sim \frac{4NM}{M\lambda_0}Z(\frac{4}{M\lambda_0}) \sim \frac{2N}{\lambda_0}\sqrt{M\lambda_0} \tag{11}$$

Finally, to estimate the contribution compared to the single bunch case we look at the raio

$$\frac{\Delta Q_m}{\Delta Q_1} \sim 2\sqrt{\frac{M\sigma_b}{\lambda_0}} \tag{12}$$

For the recycler, the bunch length is about 2 ns and the bunch spacing about 20 ns. For 10 bunches the ratio is approximately 2. It can be seen that the multi bunch contribution is not negligible and this could be the reason why as the number of bunches increases, the tunes obtained using the frequency and time domain methods begin to differ.

SUMMARY

The transverse impedance was measured in the Recycler by measuring the tune shift as a function of intensity. In the vertical plane, it was estimated to be $\sim 11 - 12$ MΩ/m. Taking into account the quadrupole term, we expect the impedance to be smaller and then the estimated value here can be used as an upper limit for the vertical impedance.

This paper describes a first approach to this measurements in order to obtain approximations. Further work is planned taking a more rigorous approach. So far measurements have also be made looking at the growth rate of the resistive wall instability i.e. measuring the real part of the impedance. Analysis is currently ongoing.

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