Beamline map computation for paraxial optics†

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Free Electron Oscillator

$$E_f = T_{Optics} T_{FEL}(E_i)$$

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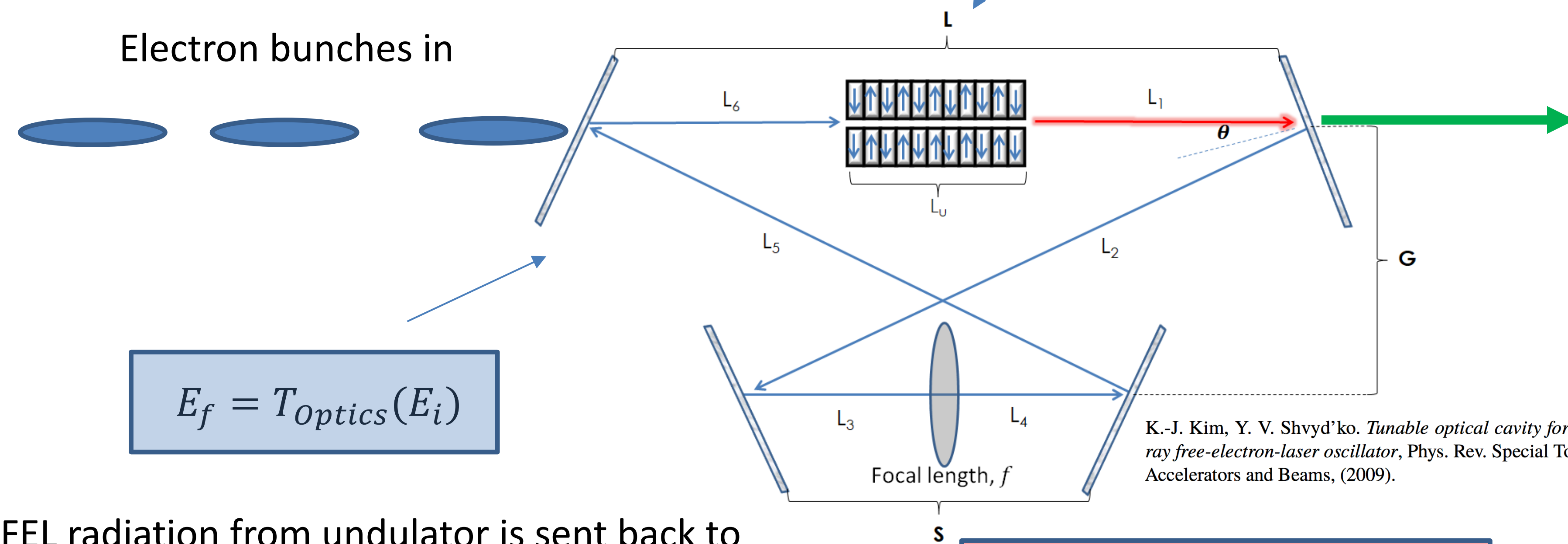
Electron bunches in



$$E_f = T_{Optics}(E_i)$$

K.-J. Kim, Y. V. Shvyd'ko. *Tunable optical cavity for an x-ray free-electron-laser oscillator*, Phys. Rev. Special Topics - Accelerators and Beams, (2009).

FEL radiation from undulator is sent back to interact with fresh electron bunch.
Over many such interactions,
Longitudinal coherence of radiation is dramatically improved.

One would simulate with Genesis+SRW
Many passes needed.
How to speed up modelling?

How does one propagate radiation wavefronts?

For simplicity, we consider separable solutions:

$$E(x, y; z) = E_0 E_x(x; z) E_y(y; z)$$

Free space propagation in paraxial approximation:

$$\nabla_\perp^2 E + 2ik\frac{\partial E}{\partial z} = 0 \qquad \frac{\partial^2 \bar{E}}{\partial z^2} \ll 2ik\frac{\partial \bar{E}}{\partial z}$$

(paraxial Helmholtz eqn)

$$E(x; z+l) = -\frac{i}{\lambda l}\int_{-\infty}^{\infty} E(x'; z) e^{\frac{2\pi i}{\lambda}\left(\frac{(x-x')^2}{2l}+l\right)} dx'$$

Fresnel propagator

$$E(\theta; z+l) = E(\theta; z) e^{\frac{2\pi i l}{\lambda}\left(1-\frac{\theta^2}{2}\right)}$$

SRW

Angular representation:

$$E(\theta) = \frac{1}{\sqrt{\lambda}}\int_{-\infty}^{\infty} E(x) e^{-\frac{2\pi i}{\lambda}\theta x} dx$$

Fourier optics propagation through all optical elements. Drift, crystal, lens, etc.

Wigner function representation and propagation (1)

$$W(x,\theta) = \frac{1}{\lambda}\int_{-\infty}^{\infty} E^*(x - \frac{\phi}{2})E(x + \frac{\phi}{2})e^{\frac{-2\pi i}{\lambda}\phi\theta}d\phi$$

We normalize the wavefronts:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(x,\theta)dxd\theta = 1$$

$$\int_{-\infty}^{\infty} E^*(x)E(x)dx = 1$$

$$\int_{-\infty}^{\infty} E^*(\theta)E(\theta)d\theta = 1$$

This is analogous to quantum mechanics, but

$\hbar \longrightarrow \lambda\!\!\!^-$

$$E^*(x)E(0) = \frac{1}{\lambda}\int_{-\infty}^{\infty} W\left(\frac{x}{2},\theta\right)e^{\frac{2\pi i}{\lambda}x\theta}d\theta$$

Reconstruct E field in coherent case

$$\begin{aligned} < x^2 > &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 W(x,\theta)dxd\theta \\ < \theta^2 > &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \theta^2 W(x,\theta)dxd\theta \\ < x\theta > &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\theta W(x,\theta)dxd\theta \end{aligned}$$

$$\begin{aligned} < x^2 > &= \int_{-\infty}^{\infty} x^2 E^*(x)E(x)dx \\ < \theta^2 > &= \int_{-\infty}^{\infty} \theta^2 E^*(\theta)E(\theta)d\theta \end{aligned}$$

Requires calc of angular representation

Partial coherence included by a convolution with incoherent source

Wigner function representation and propagation (2)

$$H(x, \theta; z)$$

Evolution under a general Hamiltonian

$$\frac{\partial W(x,\theta)}{\partial z} = [W, H]_*$$

(Note that we ignore absorption in this formulation.)

Moyal Bracket

$$[f, g]_* = \frac{1}{i\lambda}(f * g - g * f)$$

$$* = e^{\frac{i\lambda}{2}\left(\overleftarrow{\partial}_x \overrightarrow{\partial}_\theta - \overleftarrow{\partial}_\theta \overrightarrow{\partial}_x\right)}$$

Moyal Star

To first order…

$$* = 1 + \frac{i\lambda}{2}\left(\overleftarrow{\partial}_x \overrightarrow{\partial}_\theta - \overleftarrow{\partial}_\theta \overrightarrow{\partial}_x\right) + O(\lambda^2)$$

Consider quadratic Hamiltonian $H = \frac{1}{2} S_{jk} Z_j Z_k$

Then, Moyal bracket reduces to Poisson bracket and evolution is that of classical Mechanics!

Wigner function representation and propagation (3)

Thus, evolution under a (z-dependent) quadratic Hamiltonian is that of classical mechanics.

Linear map

$$\vec{z}_f = M\vec{z}_i, \qquad \vec{z} = \begin{pmatrix} x \\ \theta \end{pmatrix}$$

$$\boxed{W_f(\vec{z}) = W_i(M\vec{z})}$$

Classical evolution!
-linear transport

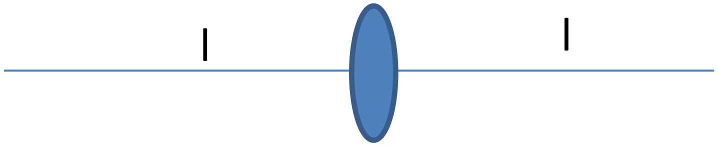
Gaussian
Wigner function

$$\boxed{\Sigma_f = M\Sigma_i M^T}$$

Only need moments!

Transfer Matrix Calculation

Compute transfer matrix for
4 crystal beamline



$$M_d = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

Ideal lens
Focal length f

$$M_f = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

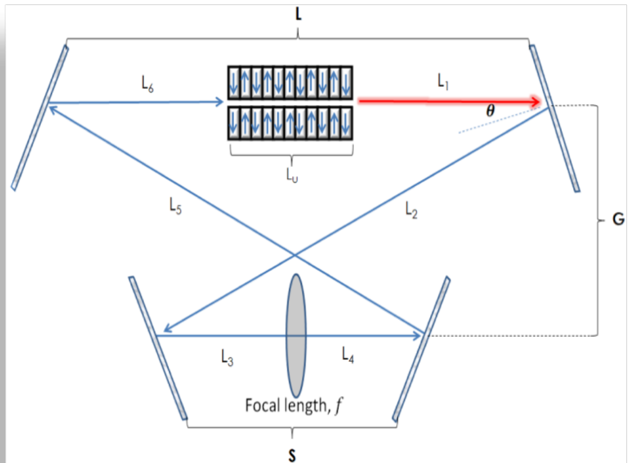
ignore crystals!
--treat absorption independently

$$\begin{aligned} M_T &= M_d M_f M_d \\ &= \begin{pmatrix} 1-\frac{l}{f} & 2l-\frac{l^2}{f} \\ -\frac{1}{f} & 1-\frac{l}{f} \end{pmatrix} \end{aligned}$$

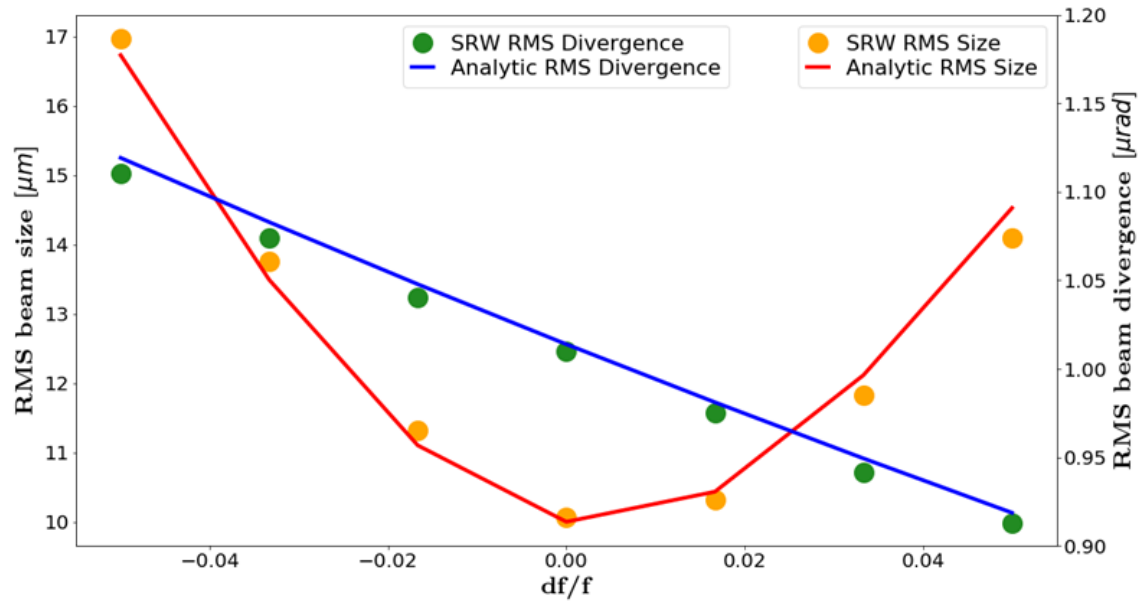
In case where $f = {}^{l}/_{2}$ $M_T = \begin{pmatrix} -1 & 0 \\ -\frac{2}{l} & -1 \end{pmatrix}$

Can also compute in Ray tracing code like SHADOW!

Gaussian results in SRW vs. analytical moment calculation



$$f = \frac{l}{2}\left(1 + \frac{df}{f}\right)$$



Crystals result in 96% transmission

$$R = \frac{\Phi_f}{\Phi_i} = \frac{\iint I_f(x,y)dxdy}{\iint I_i(x,y)dxdy}$$

Speed increase = 608

Non-Gaussian Wigner Function propagation

Hermite-Gauss modes: $$E_m(x) = E_{0,m} H_m\left(\frac{\sqrt{2}x}{w(z)}\right) e^{-i\frac{kx^2}{2q(z)}}$$

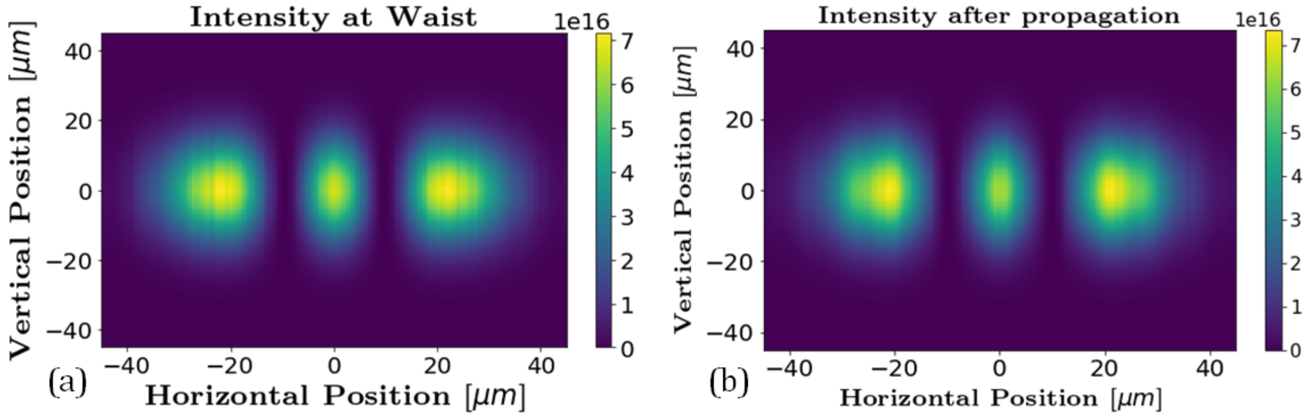
Complex beam parameter

$H_0(x) = 1$ — Gaussian

$H_1(x) = x$

$H_2(x) = 4x^2$-1

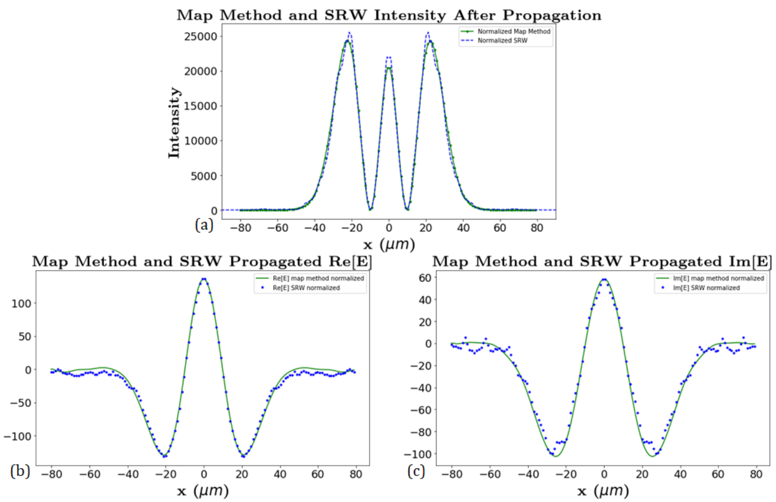
Consider mx=2, my=0



SRW propagation through 1:1 recirculation FELO beamline

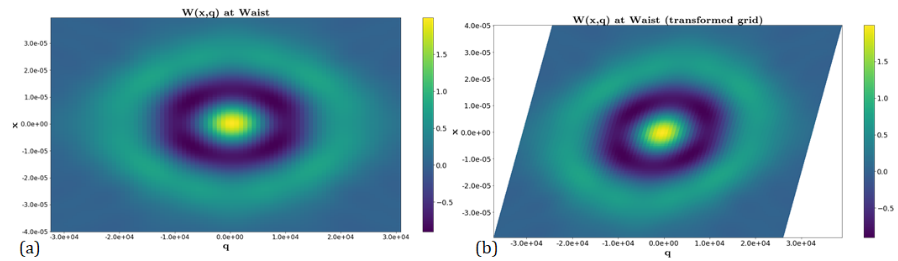
Non-Gaussian Wigner Function propagation

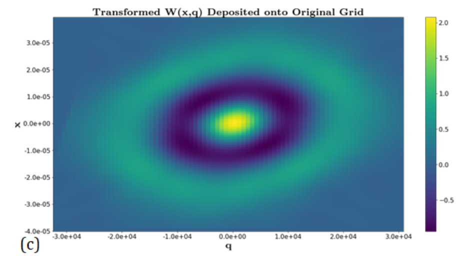
SRW propagation



43 seconds for SRW

Map method





Note negativity of WF!

38 seconds for map method

Good agreement between SRW and Map propagated electric fields

Conclusions and future work

We have demonstrated the use of transfer matrices for radiation Propagation in an FEL Oscillator beamline and cross checked against SRW simulations.

For Gaussian wavefronts, only moments are necessary. Moment Transport faster than SRW Gaussian propagation by ~ 600 !!

For non-Gaussian wavefronts, we computed the numerical Wigner function and propagated it with the transfer matrix. We found agreement with SRW result. As implemented, the two methods have comparable speed. Map method could be sped up and is expected to be faster for longer beamlines.

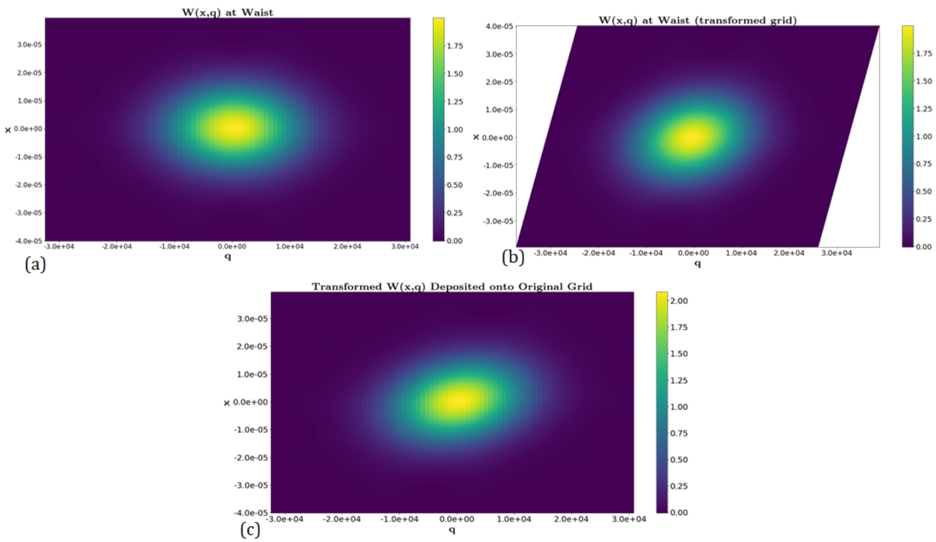
In progress/next steps:
Fast FEL calculation
4-D Wigner function
Compute matrix from general beamline with ray tracing code
- polarization
  - partial coherence (no change to map method)
  - treatment of apertures and other nonlinear elements

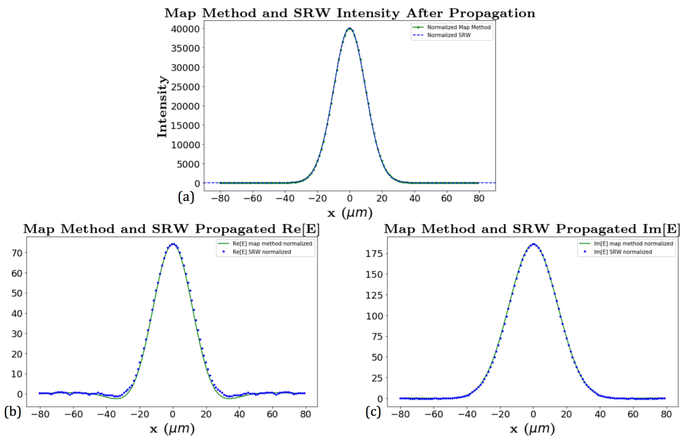
Apply this to synchrotron light source beamlines

Thanks for your attention!!

Extra Slides

Gaussian Wigner function evolution





Parameters

We have set up the four crystal beamline as described in reference [4]. Note however that we've used even simpler optics, with just a single ideal lens, located at the midpoint of the beamline. See Fig. 1 for the schematic. The parameters used for this example calculation are as follows. The undulator length, L_u is 10 meters. The total length, L, is 100 meters. The crystal diffraction angle, θ, is $\pi/8$. The length of the lower leg of the beamline, S, is 3 meters. The other parameters are then determined by geometric relations and are as follows. L_1 and L_6 are 45.0 meters. L_2 and L_5 are 72.8 meters. L_3 and L_4 are 1.5 meters. The distance between the two legs of the beamline, G, is 51.5 meters.

The reflecting crystals were chosen to be diamond with a d-spacing of 0.892 Å. The crystal thickness was 10 millimeters. The real and imaginary parts of the 0-th Fourier component of crystal polarizability were -0.217×10^{-4} and 0.280×10^{-7} respectively. The real and imaginary parts of the next Fourier component of crystal polarizability was -0.544×10^{-5} and 0.259×10^{-7} respectively.