Computation of Eigenmodes in Long and Complex Accelerating Structures by means of Concatenation Strategies

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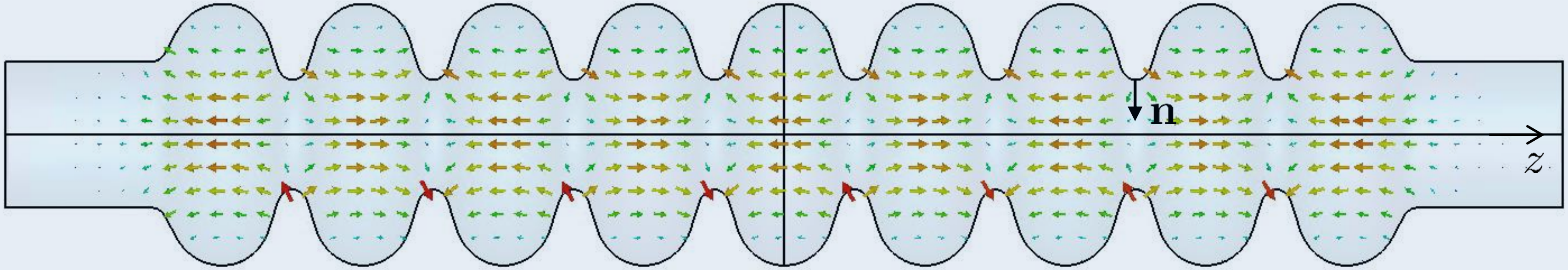
13th International Computational Accelerator Physics Conference

Key West, Florida, USA, 20th – 24th of October 2018

INTRODUCTION AND MOTIVATION



$$\Delta \mathbf{E}_i(\mathbf{r}) + \left(\frac{\omega_i}{c_0}\right)^2 \mathbf{E}_i(\mathbf{r}) = \mathbf{0} \quad \begin{cases} \mathbf{n} \cdot \mathbf{E}_i(\mathbf{r}) = 0 \text{ on } \partial\boldsymbol{\Omega}_{\text{PMC}} \\ \mathbf{n} \times \mathbf{E}_i(\mathbf{r}) = \mathbf{0} \text{ on } \partial\boldsymbol{\Omega}_{\text{PEC}} \end{cases}$$

Interaction between modes and charged particles quantified by the coupling impedance:

$$\frac{R_i}{Q_i} = \frac{1}{\omega_i W_i} \left| \int E_{z,i}(0,0,z) \exp(j\omega_i z/c_0)\, \mathrm{d}z \right|^2$$

$\mathbf{E}_i(\mathrm{r})$: electric field strength

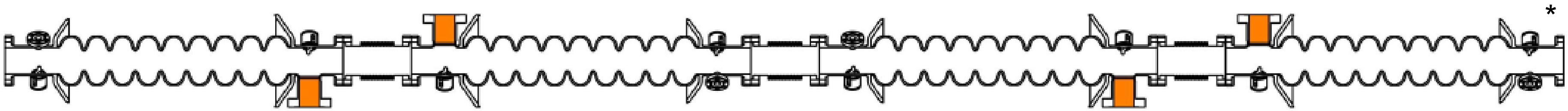
ω_i : angular frequency

c_0 : speed of light

W_i : energy stored

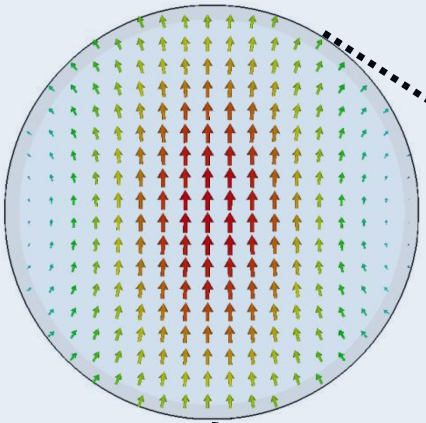
$\partial\boldsymbol{\Omega}_{\text{PEC}}$: perfect electric conducting boundary

$\partial\boldsymbol{\Omega}_{\text{PMC}}$: perfect magnetic conducting boundary



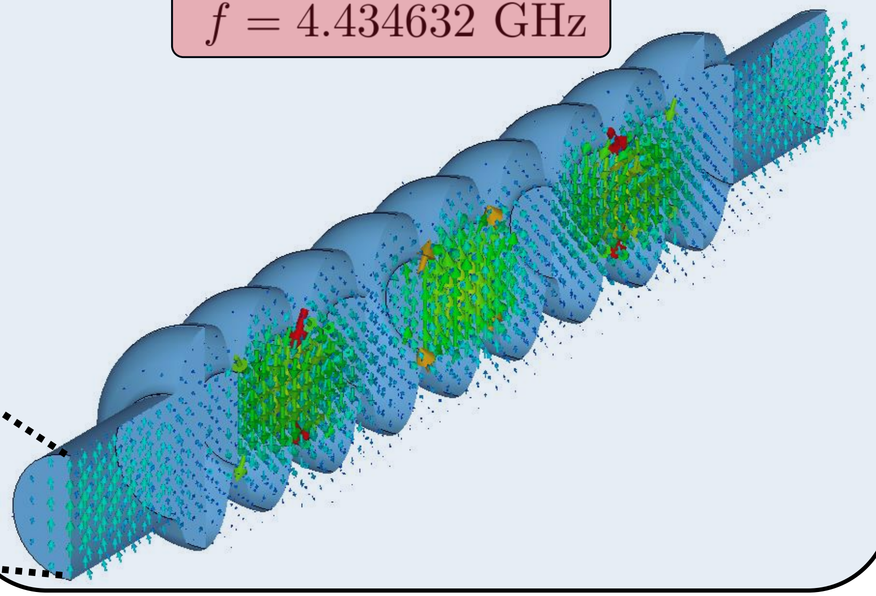
fundamental waveguide mode

$$f_{\mathrm{co,TE}_{11}} = 4.39246\ \mathrm{GHz}$$



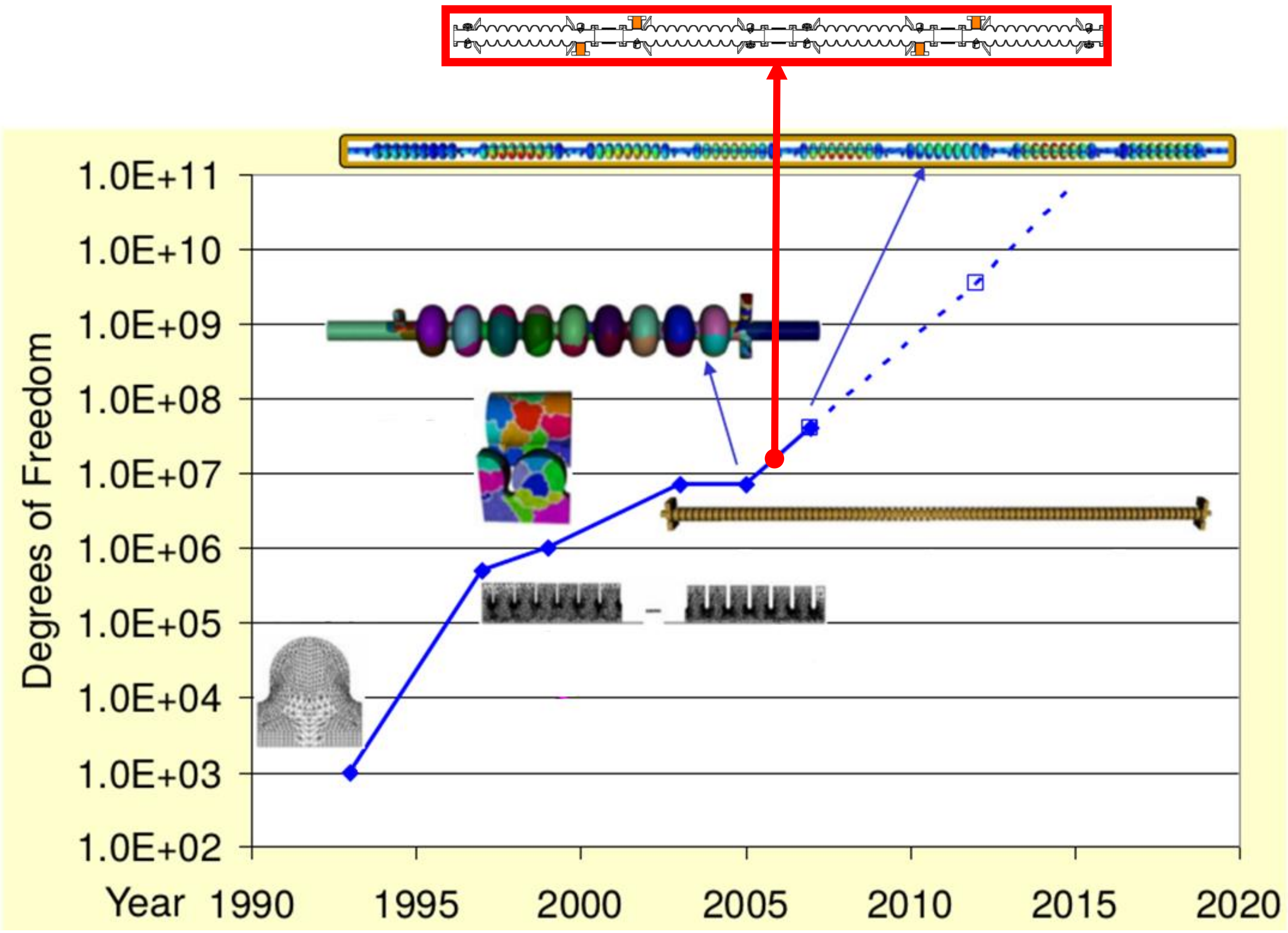
electric field of third dipole mode in resonator

$$f = 4.434632\ \mathrm{GHz}$$

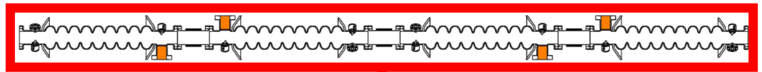


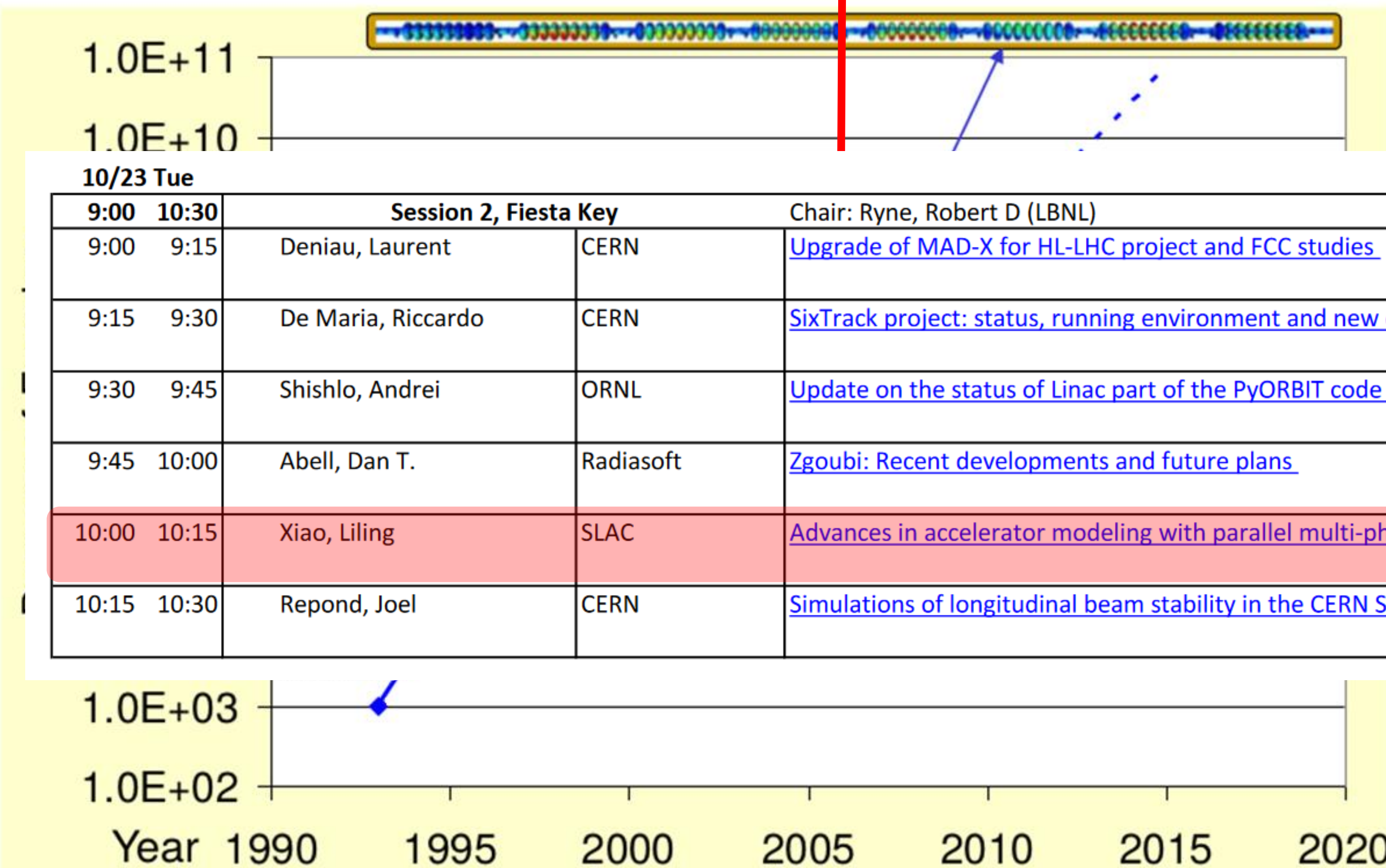
- higher order modes are not required to be confined in the individual resonators
- consideration of entire cavity chain for a reasonable RF analysis is needed

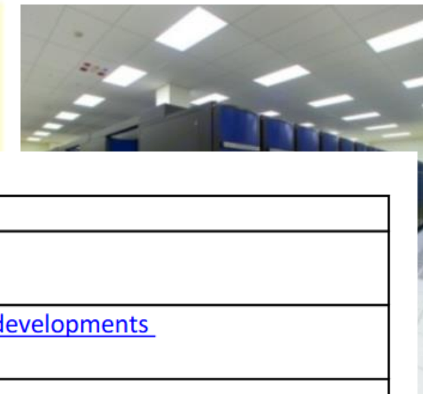
*E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008

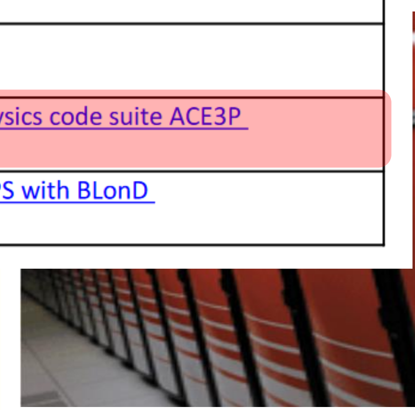


Adapted from Liling Xiao, Lixin Ge, Kwok Ko, Kihwan Lee, Zenghai Li, Cho-Kuen Ng: “Superconducting Cavity Imperfection Study for Project X Linac Using ACE3P”, ComPASS All-Hands Meeting LBNL, Sept. 27 -28, 2012 und Kwok Ko et. al: “Advances in Parallel Electromagnetic Code for Accelerator Science and Development”, Proceedings of the Linear Accelerator Conference 2010, pp. 1028 - 1032, Tsukuba, Japan 2010









10/23 Tue

9:00	**10:30**	**Session 2, Fiesta Key**		Chair: Ryne, Robert D (LBNL)
9:00	9:15	Deniau, Laurent	CERN	Upgrade of MAD-X for HL-LHC project and FCC studies
9:15	9:30	De Maria, Riccardo	CERN	SixTrack project: status, running environment and new developments
9:30	9:45	Shishlo, Andrei	ORNL	Update on the status of Linac part of the PyORBIT code
9:45	10:00	Abell, Dan T.	Radiasoft	Zgoubi: Recent developments and future plans
10:00	10:15	Xiao, Liling	SLAC	Advances in accelerator modeling with parallel multi-physics code suite ACE3P
10:15	10:30	Repond, Joel	CERN	Simulations of longitudinal beam stability in the CERN SPS with BLonD

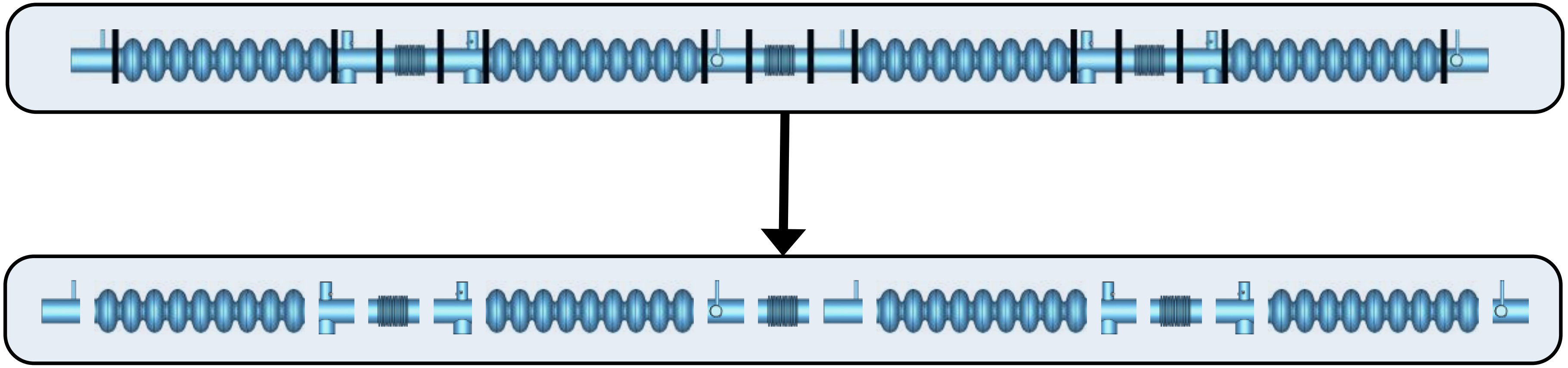
Adapted from Liling Xiao, Lixin Ge, Kwok Ko, Kihwan Lee, Zenghai Li, Cho-Kuen Ng: “Superconducting Cavity Imperfection Study for Project X Linac Using ACE3P”, ComPASS All-Hands Meeting LBNL, Sept. 27 -28, 2012 und Kwok Ko et. al: “Advances in Parallel Electromagnetic Code for Accelerator Science and Development”, Proceedings of the Linear Accelerator Conference 2010, pp. 1028 - 1032, Tsukuba, Japan 2010

ARE WE ABLE TO AVOID THE NEED OF SUPERCOMPUTERS?

COMPLEMENTARY APPROACH: STATE-SPACE COMPUTATIONS*

*T. Flisgen, “Compact State-Space Models for Complex Superconducting Radio-Frequency Structures Based on Model Order Reduction and Concatenation Methods”, PhD thesis, University of Rostock, 2015

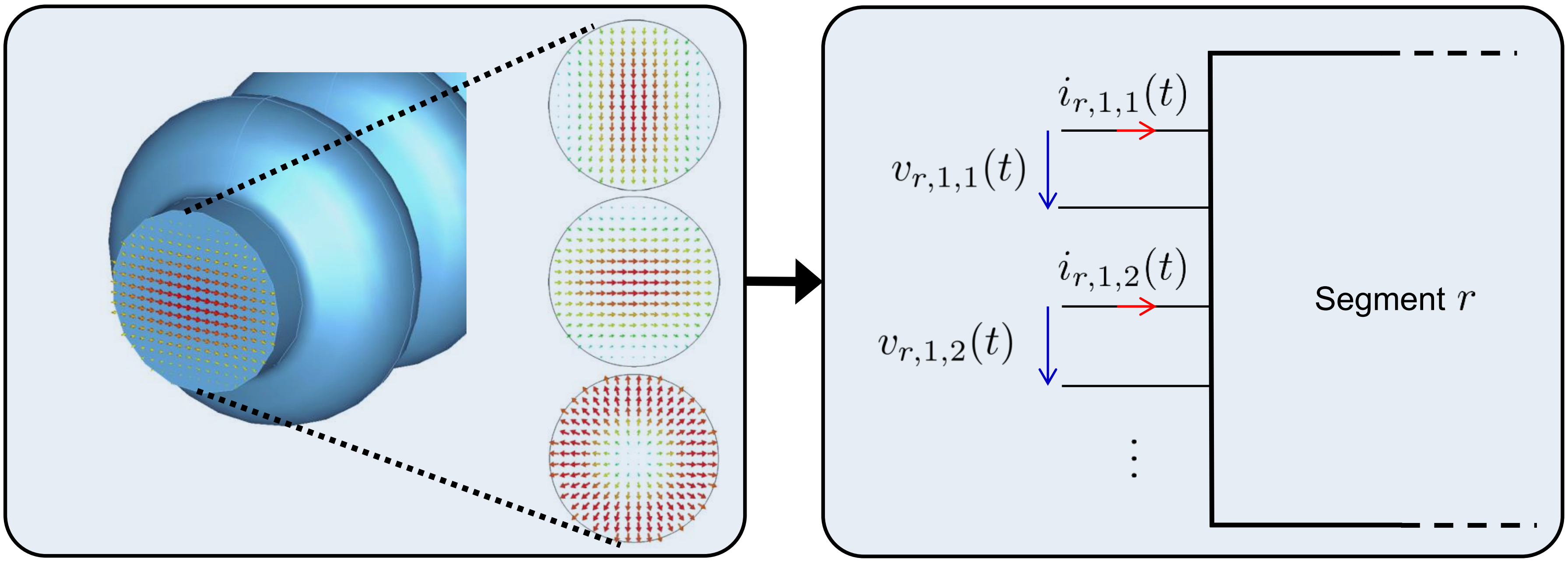
1. Decomposition of the Structure at Regions of Constant Cross Section



Important properties:

- (numerical) treatment of segments is computationally less demanding
- single treatment of identical segments
- segments with simple geometry can be treated semi-analytically, which is very fast
- employment of symmetry of segments is feasible

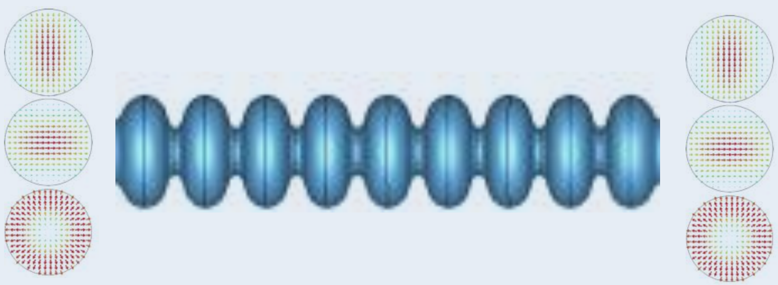
2. Consideration of Segments as Blocks with Terminals



E-field of first three 2D port modes

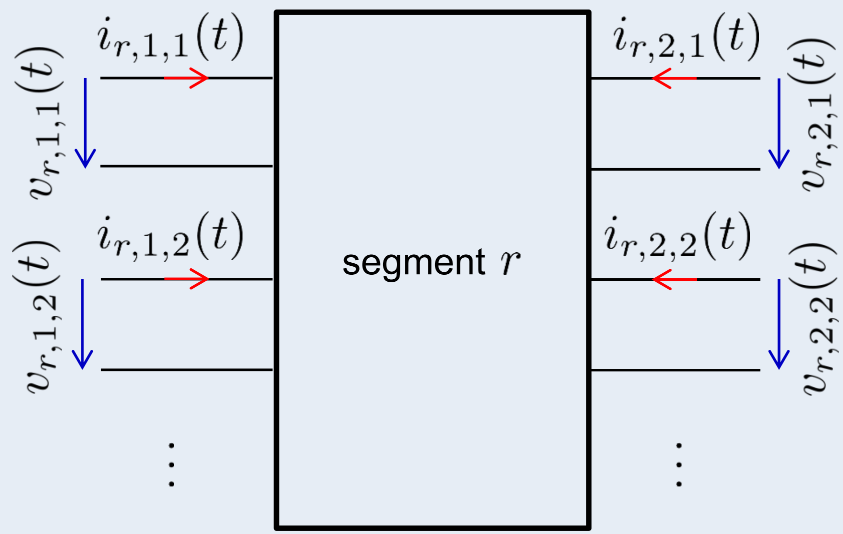
- Modal voltages $v_{r,p,m}(t)$ correspond to tangential electric fields of 2D port modes
- Modal currents $i_{r,p,m}(t)$ correspond to tangential magnetic fields of 2D port modes

3. Generation of Second-Order State-Space Equations for Segments



wave equation (PDE):

$$\Delta \mathbf{E}(\mathbf{r}, t) = \varepsilon\mu \frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r}, t) + \mu \frac{\partial}{\partial t}\mathbf{J}(\mathbf{r}, t)$$



state-space system (ODE):

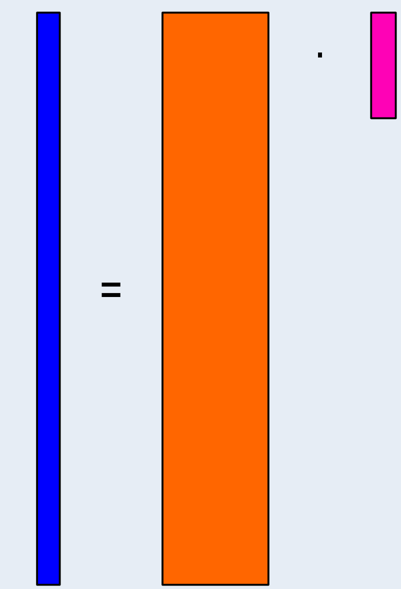
$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_r(t) = \mathbf{A}_r \mathbf{x}_r(t) + \mathbf{B}_r \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_r(t)$$

$$\mathbf{v}_r(t) = \mathbf{B}_r^{\mathrm{T}} \mathbf{x}_r(t)$$

4. Model-Order Reduction for State-Space Systems

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_r(t) = \mathbf{A}_r\mathbf{x}_r(t) + \mathbf{B}_r\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_r(t)$$

$$\mathbf{v}_r(t) = \mathbf{B}_r^{\mathrm{T}}\mathbf{x}_r(t)$$



$$\mathbf{x}_r(t) = \mathbf{W}_r \cdot \mathbf{x}_{\mathrm{rd},r}(t)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_{\mathrm{rd},r}(t) = \underbrace{\mathbf{W}_r^{\mathrm{T}}\mathbf{A}_r\mathbf{W}_r}_{\mathbf{A}_{\mathrm{rd},r}}\mathbf{x}_{\mathrm{rd},r}(t) + \underbrace{\mathbf{W}_r^{\mathrm{T}}\mathbf{B}_r}_{\mathbf{B}_{\mathrm{rd},r}}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_r(t)$$

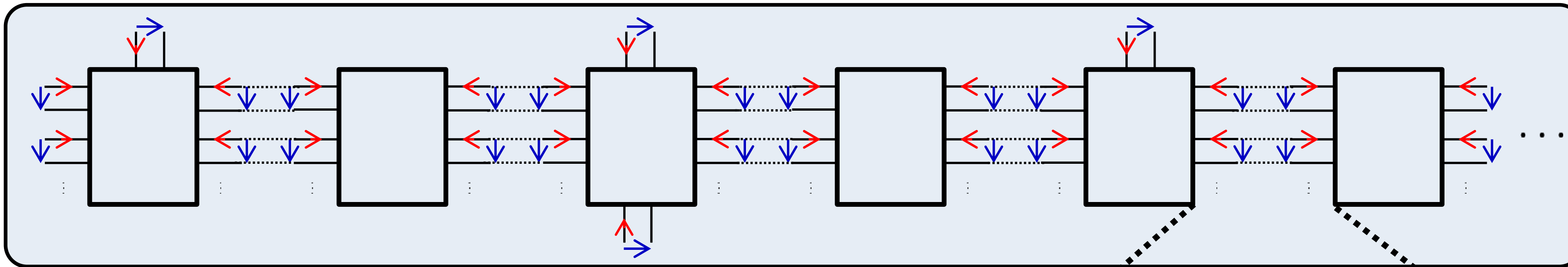
$$\mathbf{v}_r(t) = \underbrace{\mathbf{B}_r^{\mathrm{T}}\mathbf{W}_r}_{\mathbf{B}_{\mathrm{rd},r}^{\mathrm{T}}}\mathbf{x}_{\mathrm{rd},r}(t)$$

state vector: $\mathbf{x}_r(t) \in \mathbb{R}^{N_{\mathrm{s}}}$, $N_{\mathrm{s}} \approx 10^5$

orthogonal projection matrix: $\mathbf{W}_r \in \mathbb{R}^{N_{\mathrm{s}} \times N_{\mathrm{srd}}}$, $\mathbf{W}_r^{\mathrm{T}}\mathbf{W}_r = \mathbf{I}$

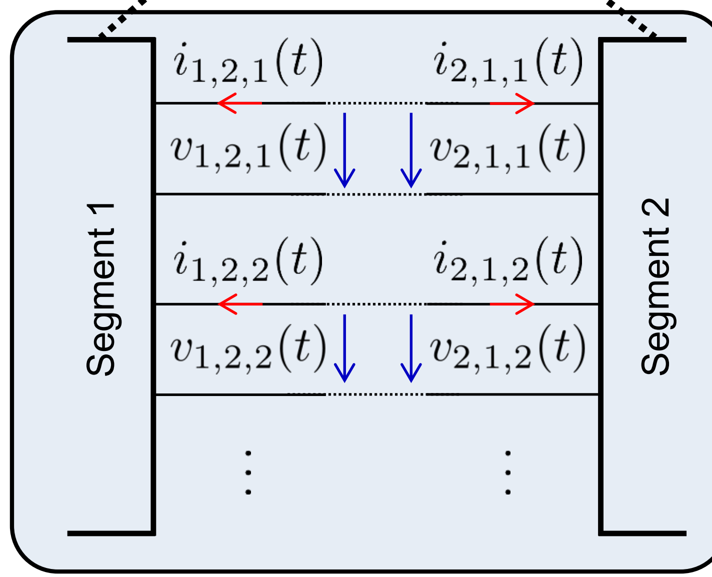
reduced state vector: $\mathbf{x}_{\mathrm{rd},r}(t) \in \mathbb{R}^{N_{\mathrm{srd}}}$, $N_{\mathrm{srd}} < 10^2$

5. Concatenation of Reduced-Order State-Space System (SSC)



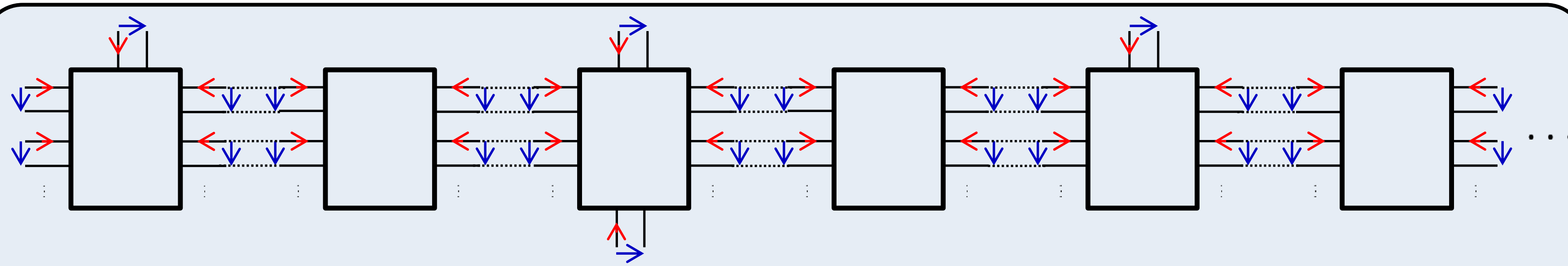
$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_{\mathrm{rdc}}(t) = \mathbf{A}_{\mathrm{rdc}}\mathbf{x}_{\mathrm{rdc}}(t) + \mathbf{B}_{\mathrm{rdc}}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}(t)$$

$$\mathbf{v}(t) = \mathbf{B}_{\mathrm{rdc}}^{\mathrm{T}}\mathbf{x}_{\mathrm{rdc}}(t)$$



- Coupling constraints according to Kirchhoff's laws
- Arbitrary topologies and number of 2D port modes supported

6. Computation of RF Properties by Means of the Reduced-Order Model

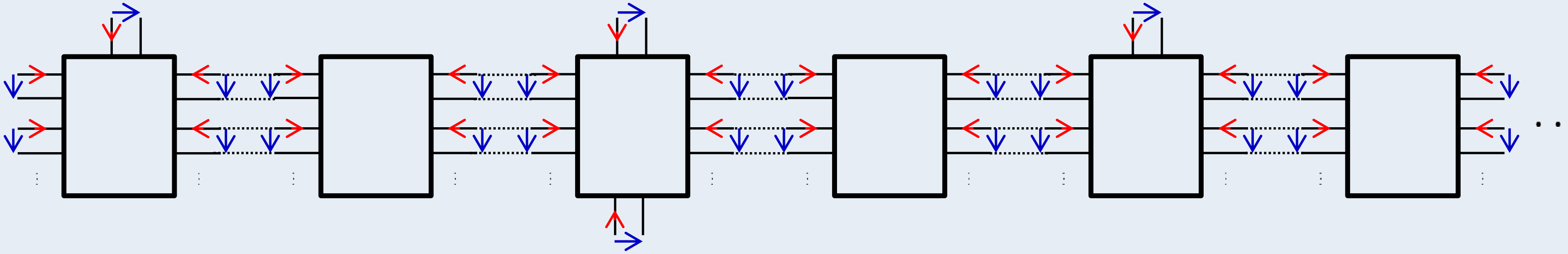


$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_{\mathrm{rdc}}(t) = \mathbf{A}_{\mathrm{rdc}}\mathbf{x}_{\mathrm{rdc}}(t) + \mathbf{B}_{\mathrm{rdc}}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}(t)$$

$$\mathbf{v}(t) = \mathbf{B}_{\mathrm{rdc}}^{\mathrm{T}}\mathbf{x}_{\mathrm{rdc}}(t)$$

- impedance- and scattering parameters
- 3D field distribution due to port excitations
- frequencies and field distributions of eigenmodes
- external quality factors

6. Computation of RF Properties by Means of the Reduced-Order Model



$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_{\mathrm{rdc}}(t) = \mathbf{A}_{\mathrm{rdc}}\mathbf{x}_{\mathrm{rdc}}(t) + \mathbf{B}_{\mathrm{rdc}}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}(t)$$

$$\mathbf{v}(t) = \mathbf{B}_{\mathrm{rdc}}^{\mathrm{T}}\mathbf{x}_{\mathrm{rdc}}(t)$$

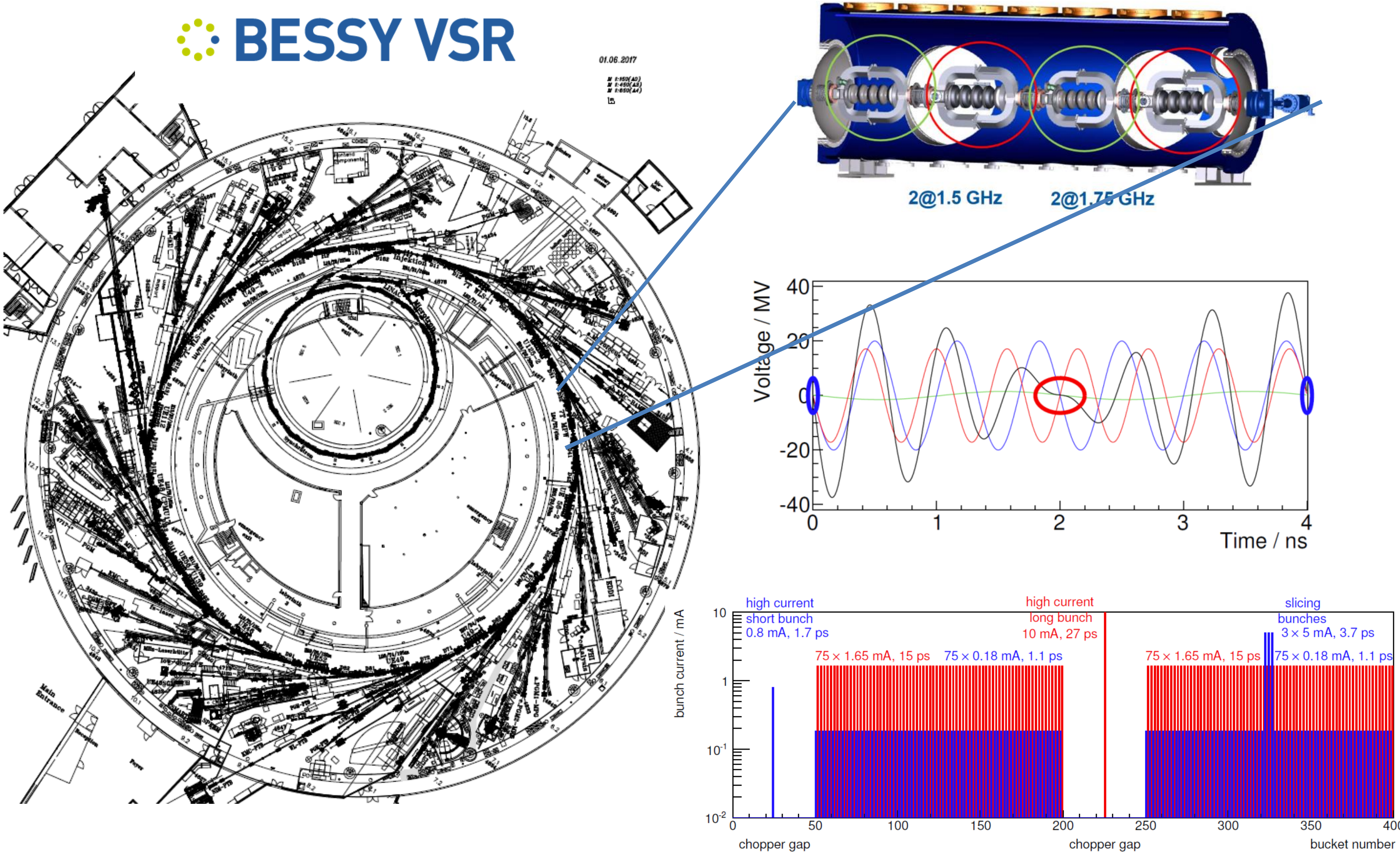
10/23 Tue

11:00	**12:30**	**Session 1, Grand Ballroom**			
11:00	11:30	(I)	Van Beeumen, Roel	LBNL	Parallel algorithms for solving nonlinear eigenvalue problems in accelerator cavity simulations
11:30	12:00	(I)	Pommerenke, Hermann	U Rostock	Efficient computation of lossy higher order modes in complex SRF cavities using reduced order models and nonlinear eigenvalue problem algorithms
12:00	12:15		De Gersem, Herbert	TU Darmstadt	Uncertainty quantification for the fundamental mode spectrum of the European XFEL Cavities
12:15	12:30		Fedurin, Mikhail	BNL	Electron beam longitudinal phase space restoration from the image after beam pass deflector cavity and spectrometer arm

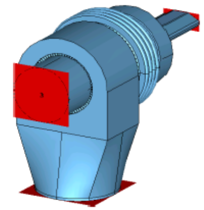
external quality factors

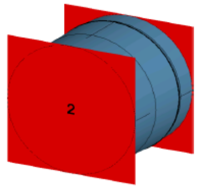
APPLICATION EXAMPLE:
EIGENMODE COMPENDIUM OF THE BESSY VSR COLD CHAIN

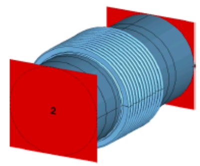
Work has been conducted by Johann Heller!

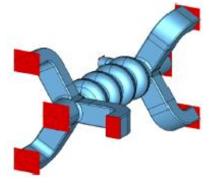


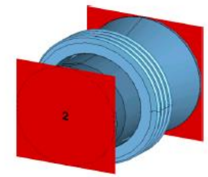
A. Jankowiak et al., “Technical Design Study BESSY VSR”, Helmholtz-Zentrum Berlin, Berlin 2015

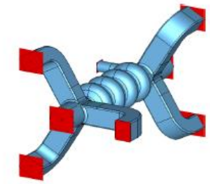

(1) Warm Taper Up

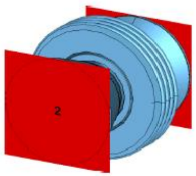

(2) Valve Up

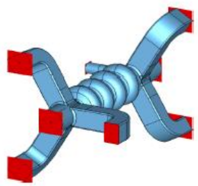

(3) Bellow 1

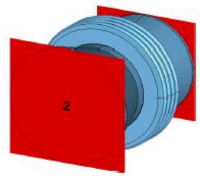

(4) Cavity 1 with f_π =1.5 GHz

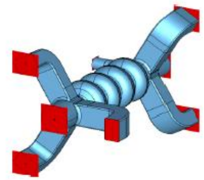

(5) Bellow 2

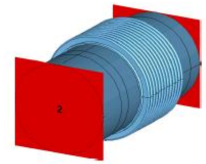

(6) Cavity 2 with f_π =1.75 GHz

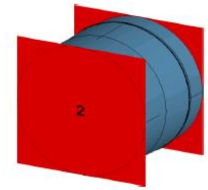

(7) Bellow 3


(8) Cavity 3 with f_π =1.75 GHz

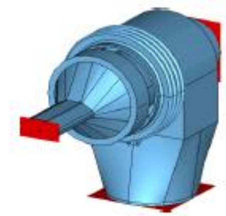

(9) Bellow 4


(10) Cavity 4 with f_π =1.5 GHz


(11) Bellow 5

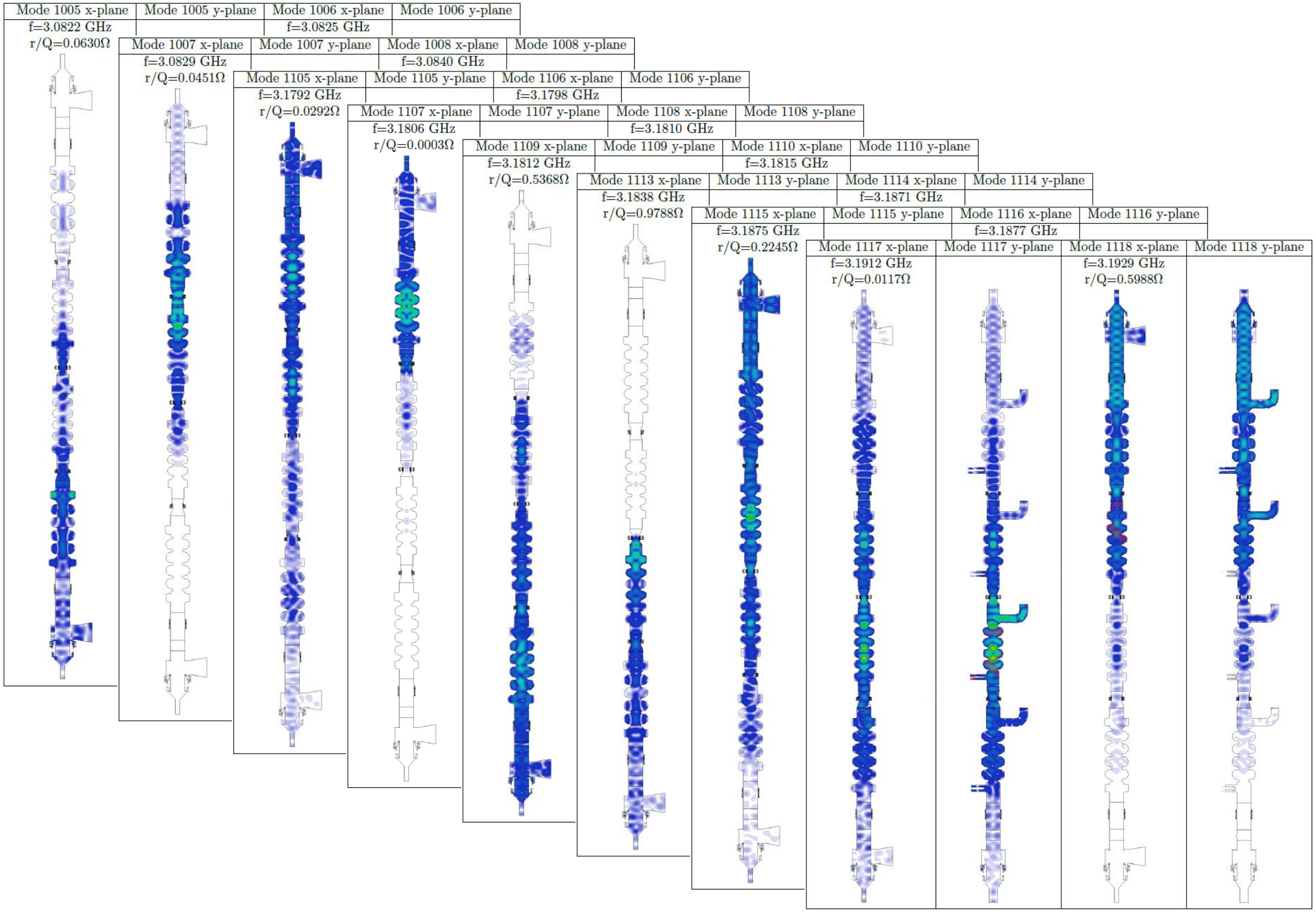

(12) Valve Down

Structure name	Mesh cells	DOFs	2D Port modes	Time System Creation	Time MOR	$\sum$ Time
Warm Taper Up	964.210	2,982.996	[8,25,16]	6 h 15 min	1 d 4 h 16 min	1 d 10 h 31 min
Valve Up	16.200	54.777	[16,16]	1 min	5 min	6 min
Bellow 1	89.216	287.595	[16,20]	4 min	52 min	56 min
Cavity 1	1,656.360	5,096.520	[20,7,7,1,7,7,7,20]	8 h 47 min	14 h 15 min	23 h 2 min
Bellow 2	350.208	1,096.095	[20,20]	21 min	4 h 15 min	4 h 36 min
Cavity 2	1.440.750	4,438.476	[20,7,7,1,7,7,7,20]	7 h 4 min	10 h 16 min	17 h 20 min
Bellow 3	322.560	1,010.685	[20,20]	25 min	4 h 21 min	4 h 41 min
Cavity 3	1.440.750	4,438.476	[20,7,7,1,7,7,7,20]	7 h 4 min	10 h 16 min	17 h 20 min
Bellow 4	350.208	1,096.095	[20,20]	21 min	4 h 15 min	4 h 36 min
Cavity 4	1.700.380	5,230.764	[20,7,7,1,7,7,7,20]	9 h 14 min	14 h 56 min	1 d 10 min
Bellow 5	89.216	287.595	[20,20]	4 min	52 min	56 min
Valve Down	16.200	54.777	[20,16]	1 min	5 min	6 min
Warm Taper Down	964.410	2,982.996	[16,25,8]	6 h 15 min	1 d 4 h 16 min	1 d 10 h 31 min
$\sum$	9.506.084	30.697.769				6 d 1 h 51 min


(13) Warm Taper Down

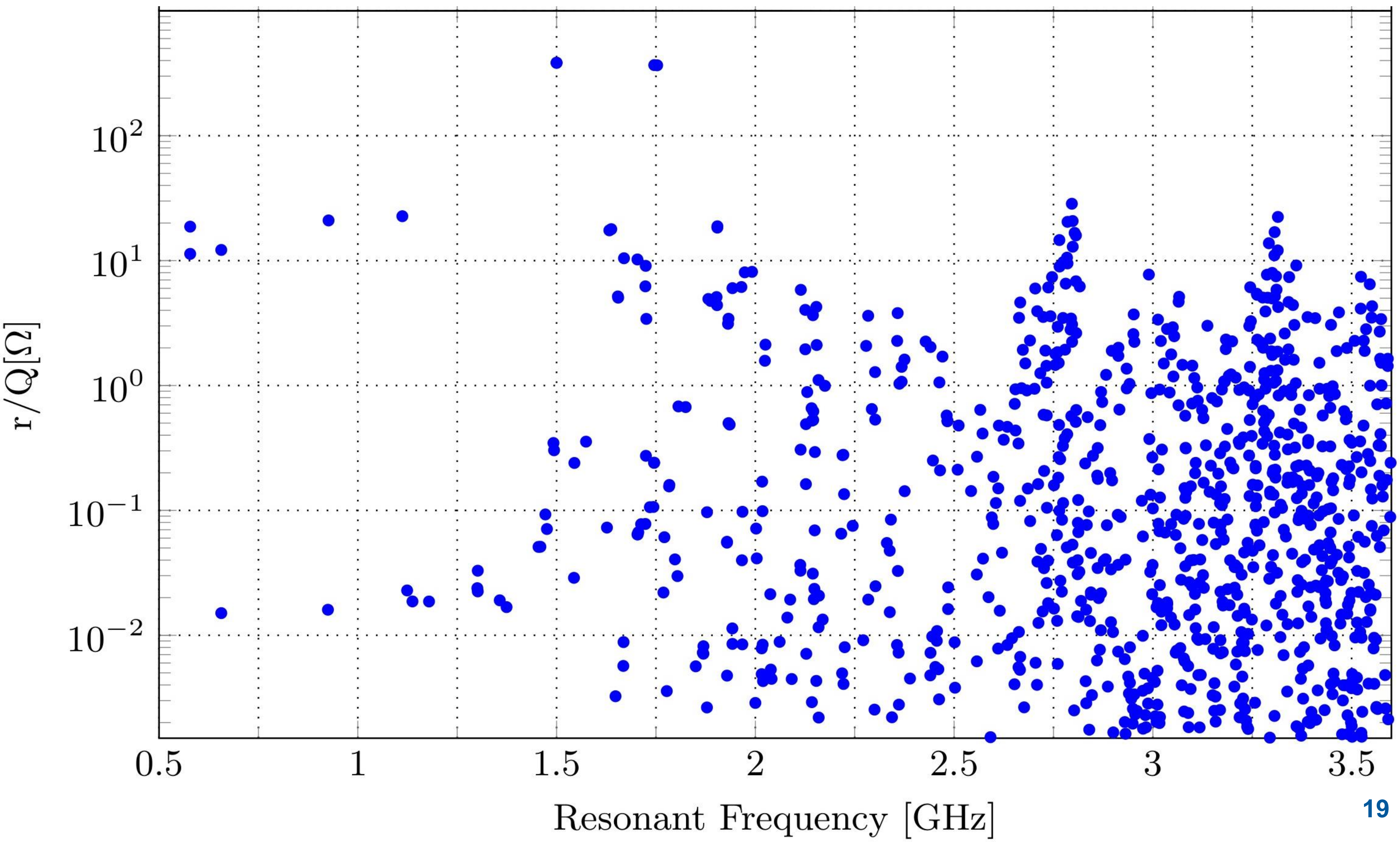
Computations performed on an Intel(R) Xeon(R) CPU E5-2687W @ 3.4 GHz with 256 GB of RAM with Windows Server 2012

MODAL ATLAS FOR THE COLD STRING GENERATED WITH SSC

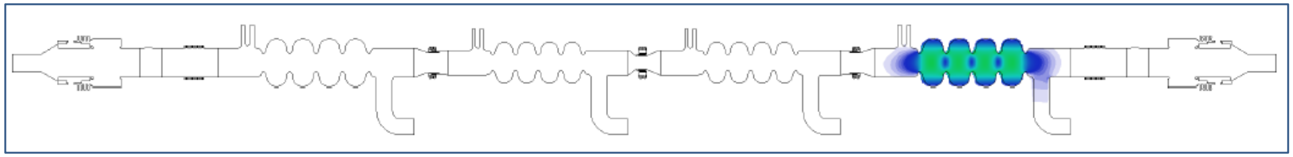


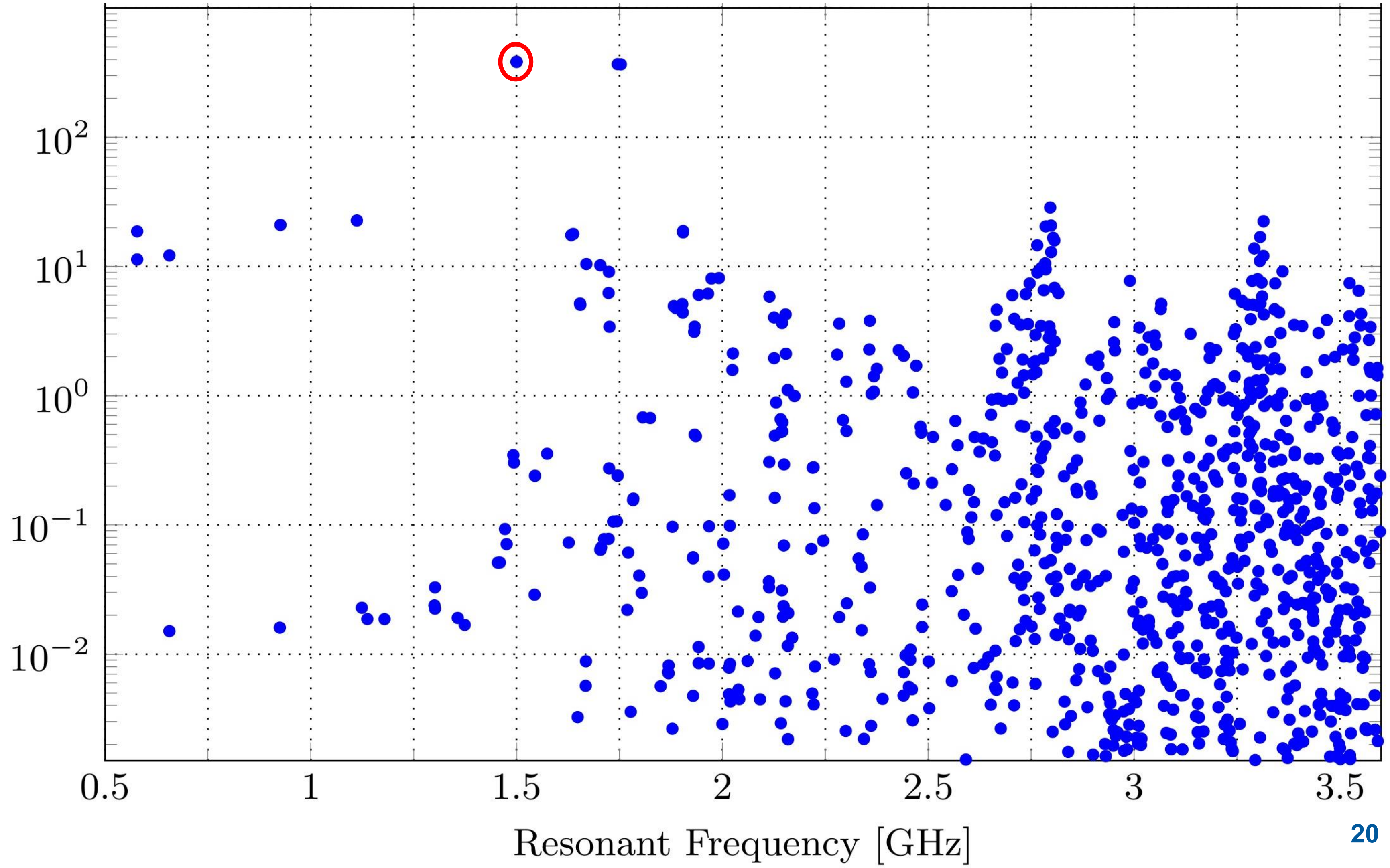
- computed 1,564 eigenmodes in the interval 0.5 GHz to 3.6 GHz
- assembled a modal compendium for the chain with resonant frequency, geometric impedance, and electric field distribution for each mode

COUPLING IMPEDANCES OF MODES IN THE CHAIN

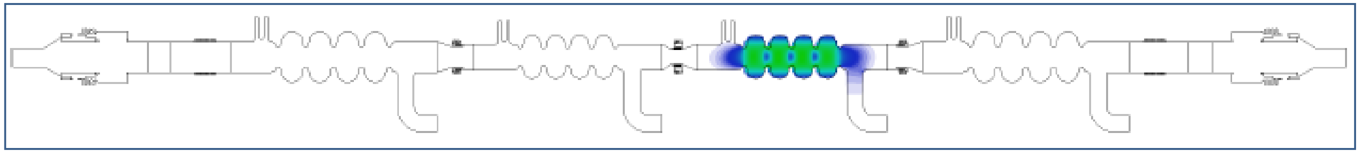


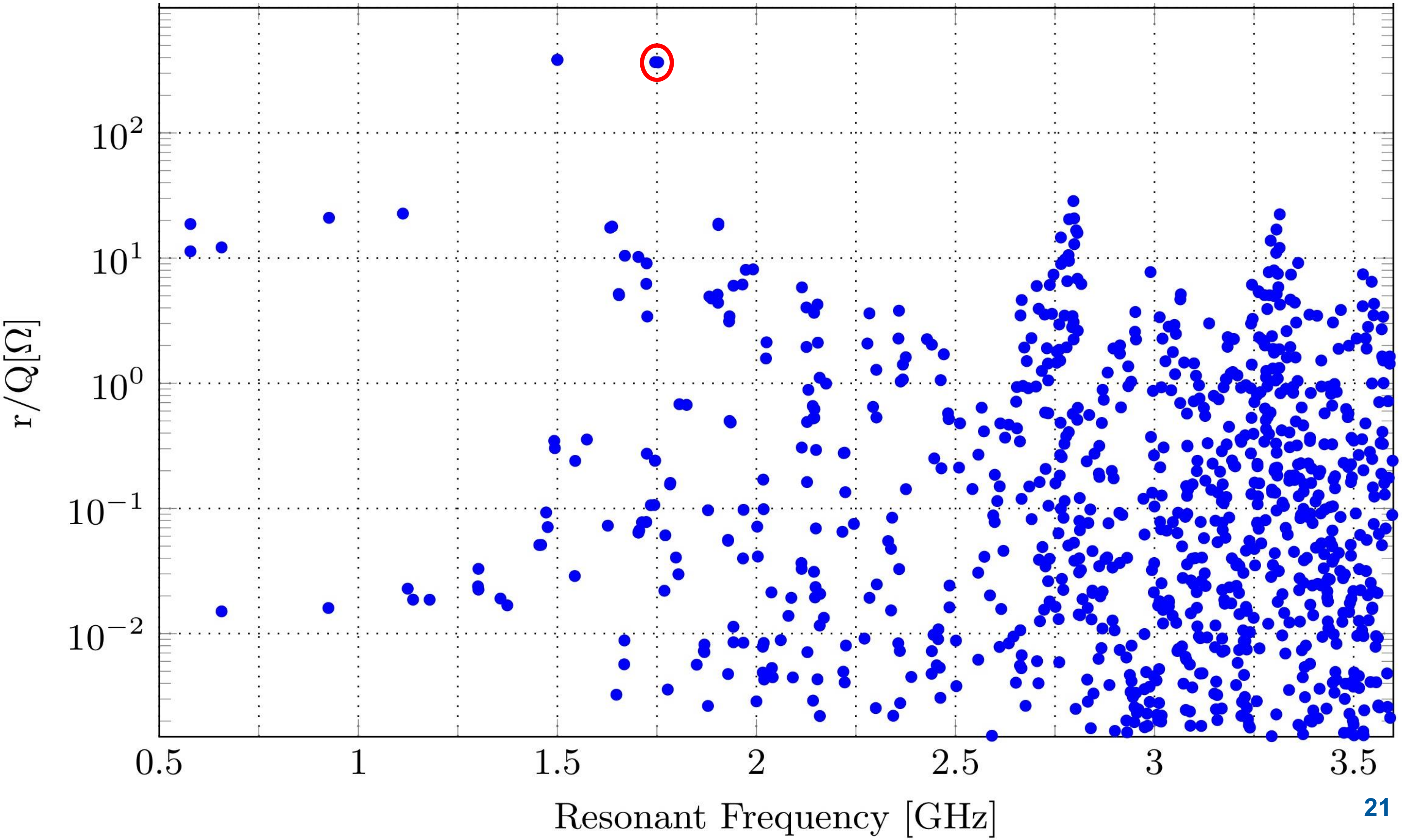
COUPLING IMPEDANCES OF MODES IN THE CHAIN

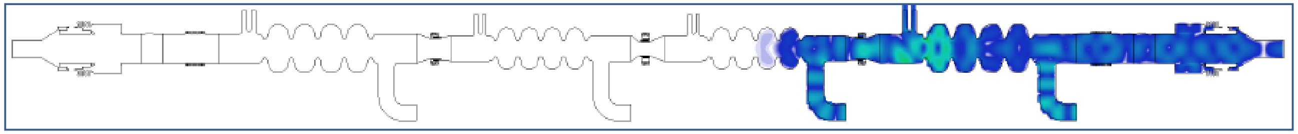


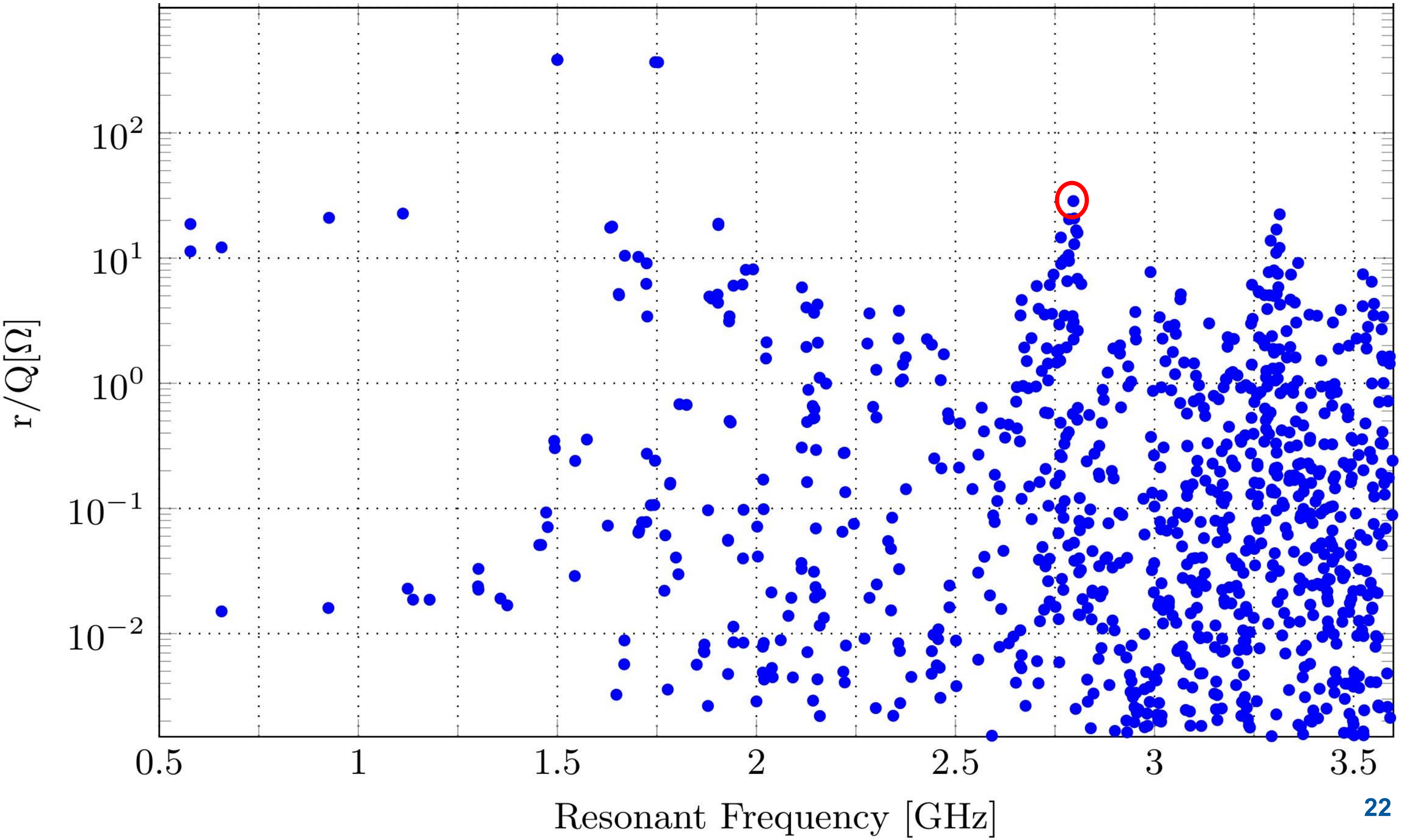


COUPLING IMPEDANCES OF MODES IN THE CHAIN

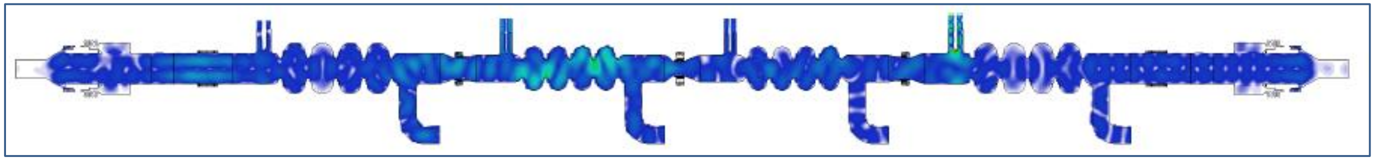


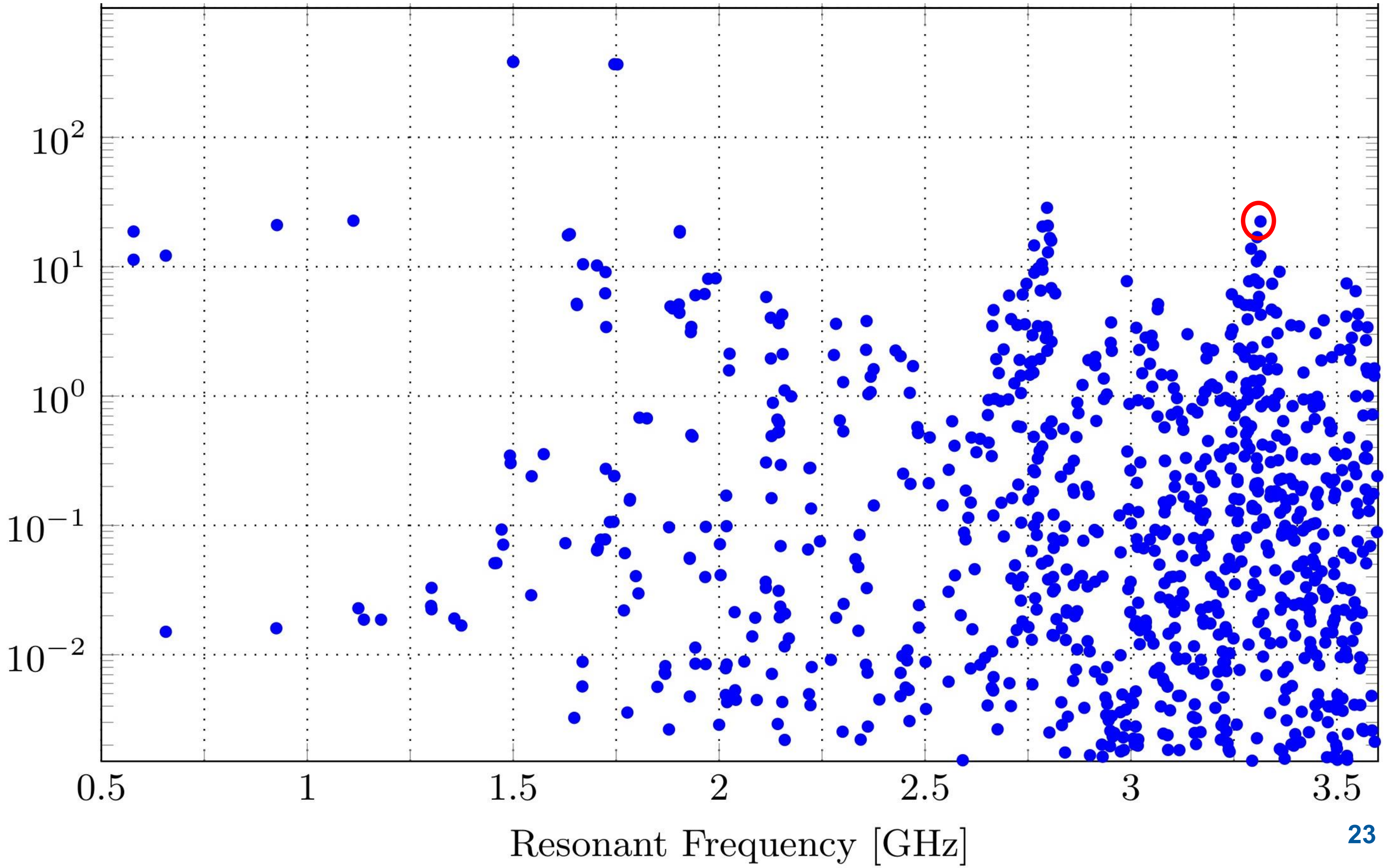


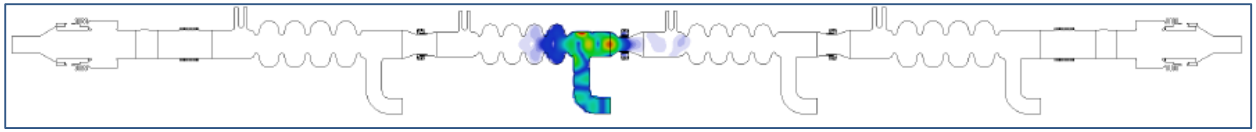


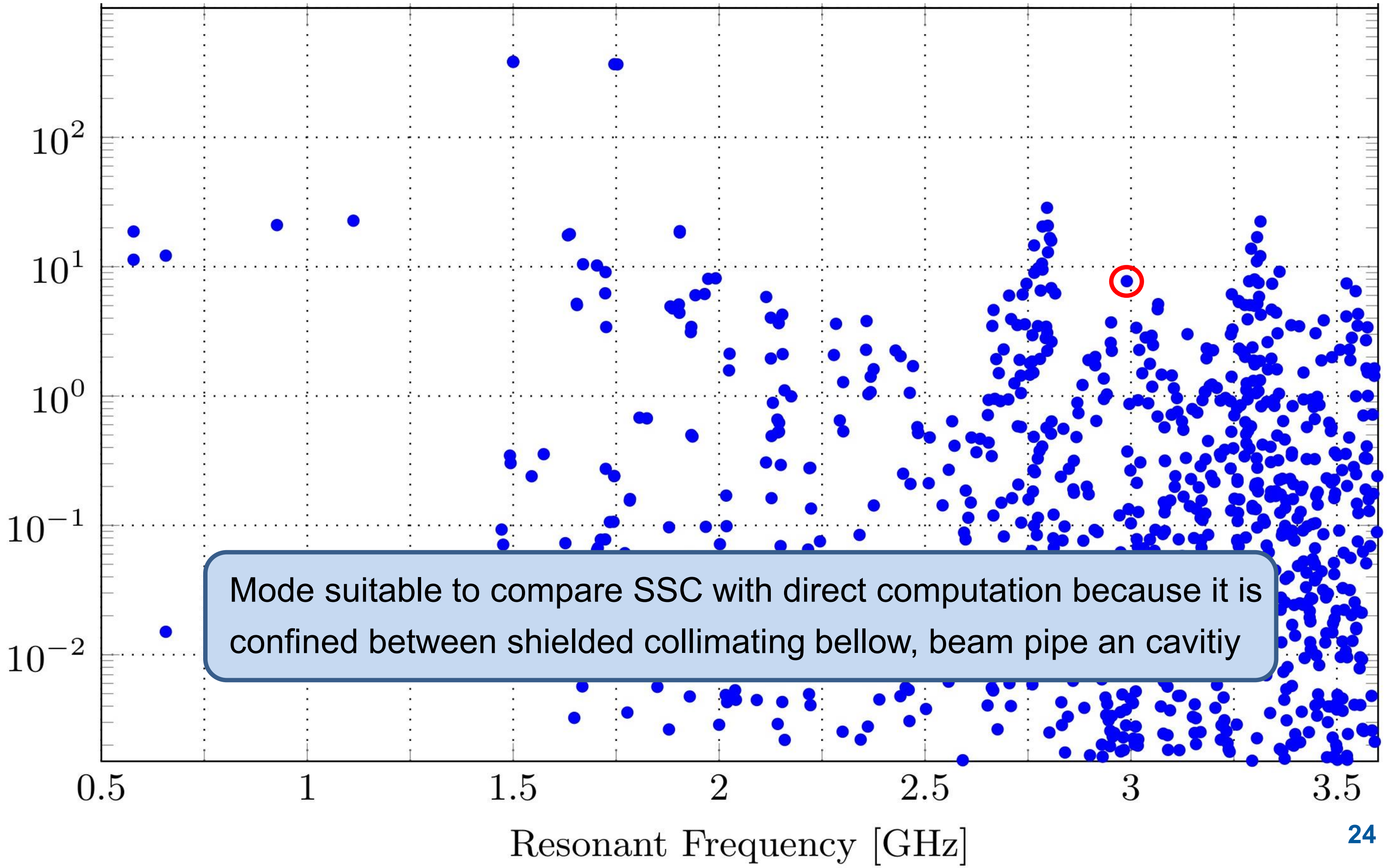

r/Q[Ω]
10^2
10^1
10^0
10^{-1}
10^{-2}
0.5
1
1.5
2
2.5
3
3.5
Resonant Frequency [GHz]

COUPLING IMPEDANCES OF MODES IN THE CHAIN

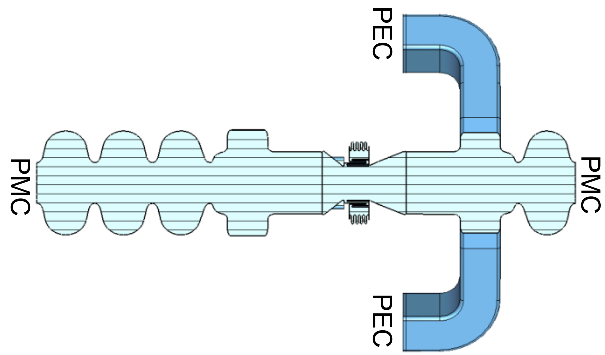




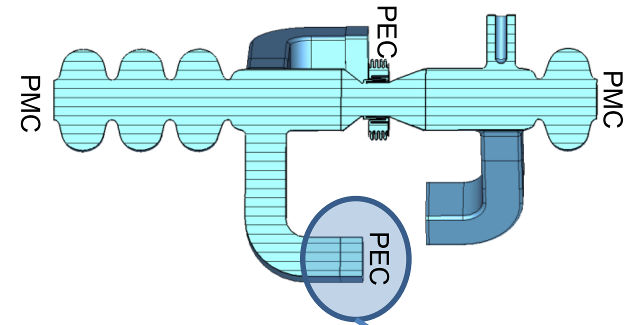


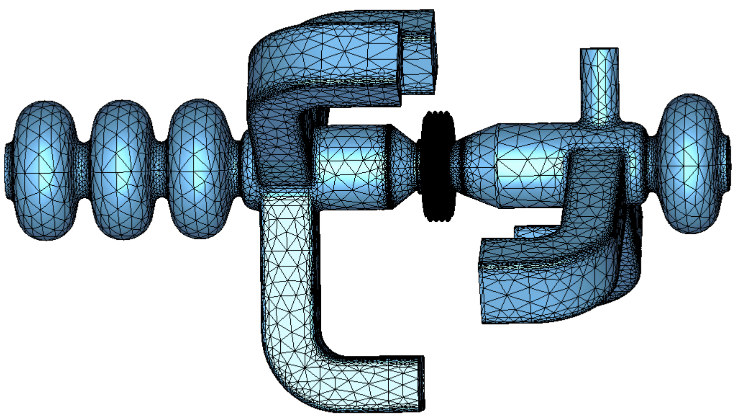


Mode suitable to compare SSC with direct computation because it is confined between shielded collimating bellow, beam pipe an cavitiy x-cutplane



y-cutplane





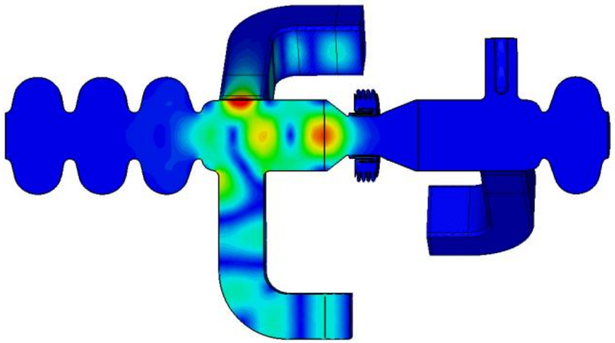
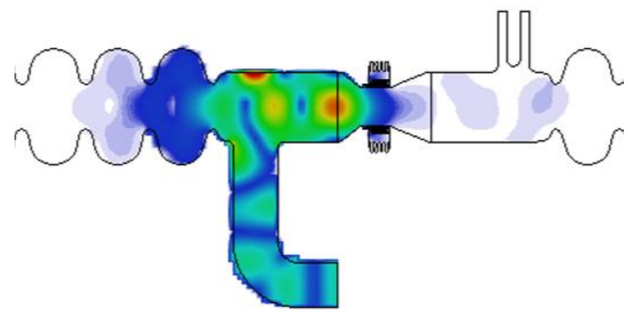
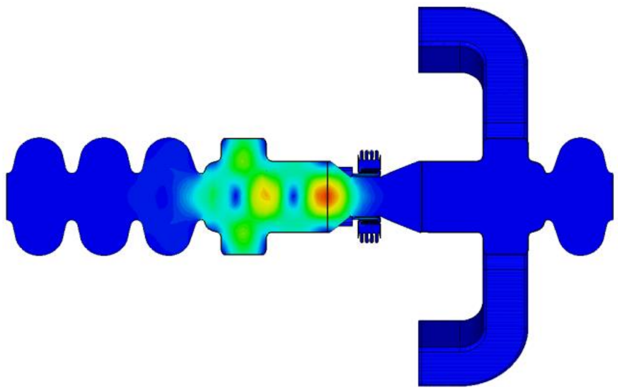
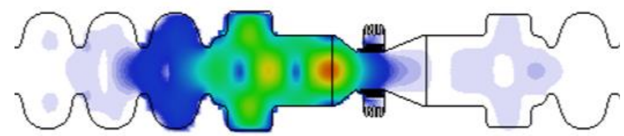
Artificially enhanced waveguides by

$$\Delta z = \frac{1}{4}\frac{c_0}{\sqrt{f_{888}^2 - f_{co}^2}}$$

to model a PMC condition at black line

- 218,444 tetrahedrons
- searched for 10 modes with frequencies larger than 2.98 GHz
- T_{comp} = 13 min

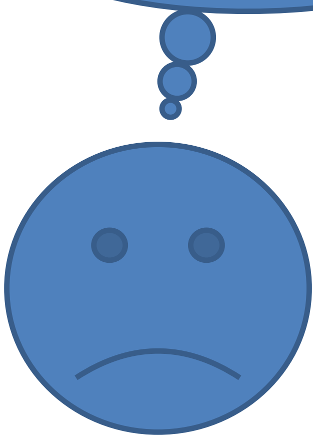
FIELD DISTRIBUTIONS OF MODE 888 – DIRECT ON TET VS. SSC ON HEX (CONT.)

	CST on TET	SSC on HEX
y-cutplane		
x-cutplane		

- results agree reasonably well
- differences are attributed to the different meshes (TET vs. HEX) and the different approaches (direct vs. SSC)

EXTERNAL QUALITY FACTORS BASED ON REFLECTIONS AT THE FPC

How do reflections of the dielectric windows of the fundamental power couplers influence the quality factors of resonant modes in the BESSY VSR chain?



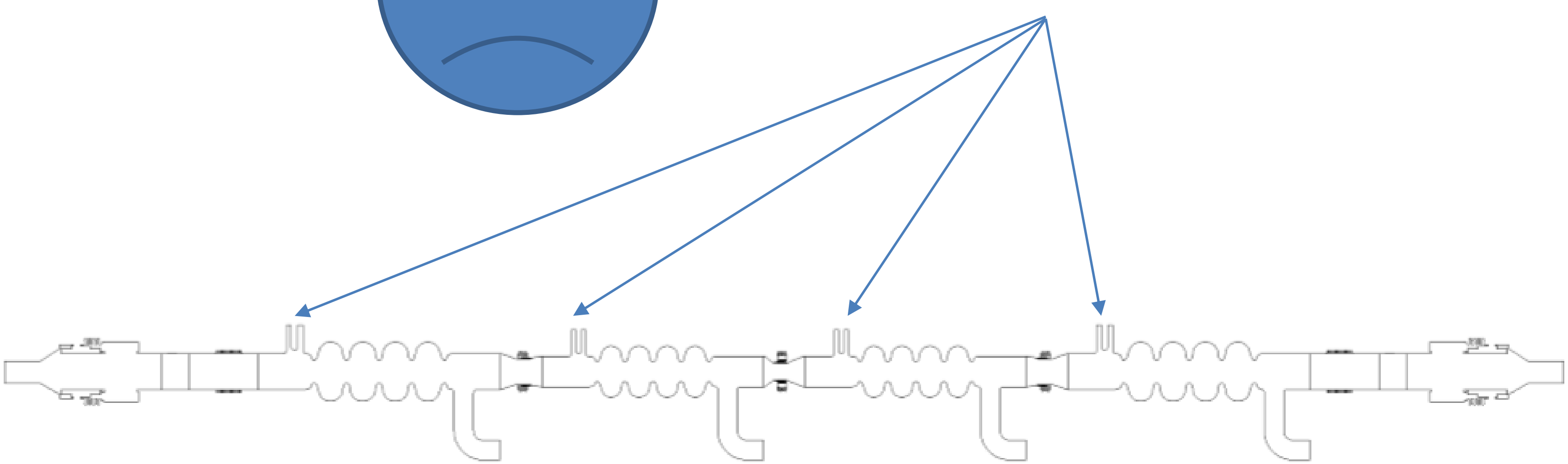


Figure of the BESSY VSR string courtesy of Johann Heller

Change of matching impedances at the fundamental input couplers while terminations at all other terminals remain untouched (=matched).

$$Z_{\mathrm{fpc}} = (1-p)\cdot Z_{\mathrm{TEM}}$$

$$0 \leq p \leq 1$$

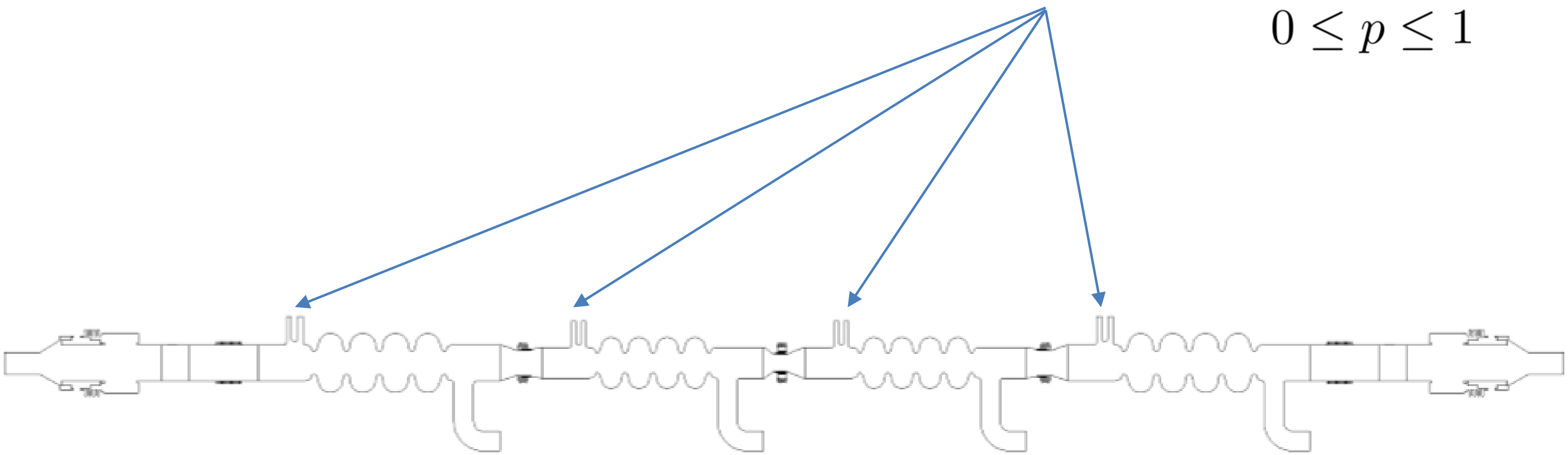
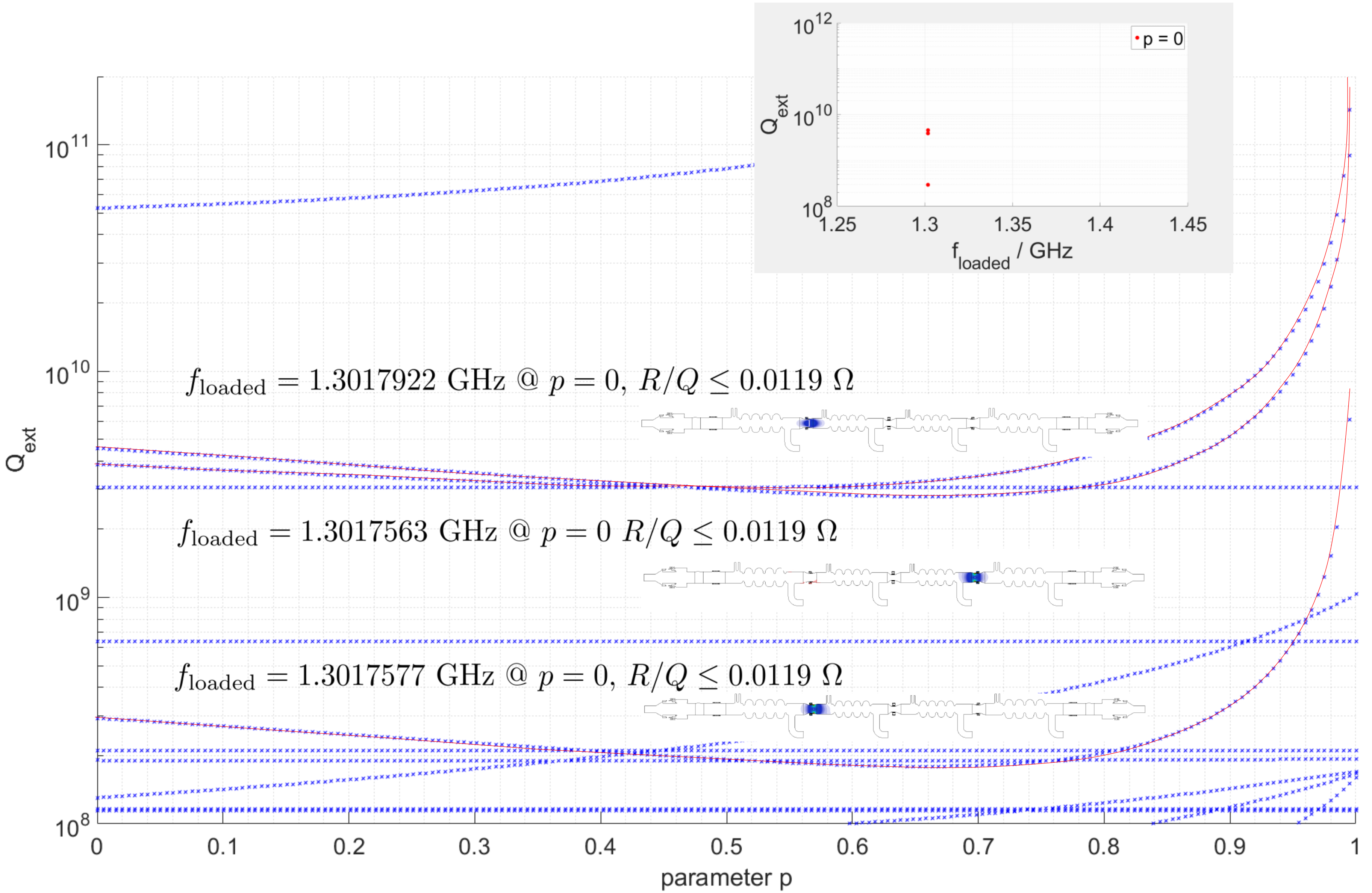


Figure of the BESSY VSR string courtesy of Johann Heller

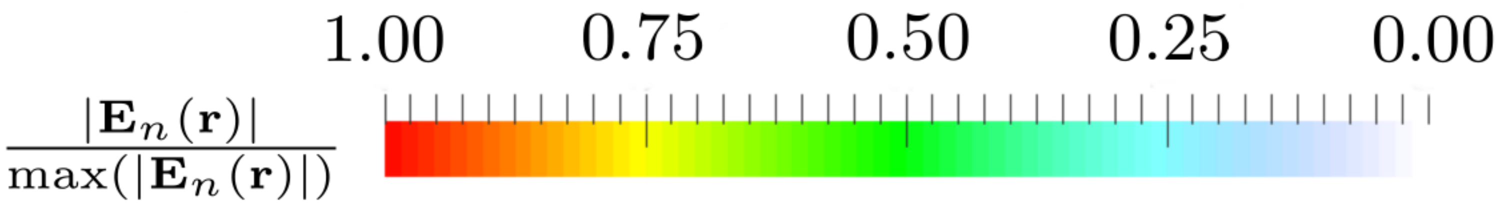
SENSITIVE MODES WITH COMPARABLY LARGE EXTERNAL Q



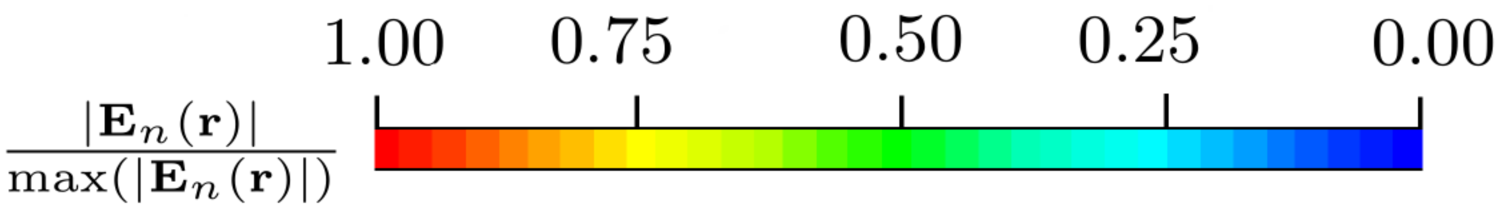
Figures of the modes courtesy of Johann Heller

SUMMARY

- SSC allows for solving large scale problems without using high performance computers
- The complexity of the field problem is reduced by decomposition and model order reduction
- It is not sufficient to exclusively focus on the cavitiy as relevant modes be localized along the chain or along parts of it or between cavites
- Comparison with a direct computation shows very good agreement in the resonant frequency and reasonable agreement in the R/Q
- Further comparisons of SSC with direct computations are available in the literature
- SSC is very flexible and e.g. allows for perturbation approaches


1.00
0.75
0.50
0.25
0.00
$\frac{|\mathbf{E}_n(\mathbf{r})|}{\max(|\mathbf{E}_n(\mathbf{r})|)}$

(a)


1.00
0.75
0.50
0.25
0.00
$\frac{|\mathbf{E}_n(\mathbf{r})|}{\max(|\mathbf{E}_n(\mathbf{r})|)}$

(b)