Simulation studies of coherent instability thresholds with space charge

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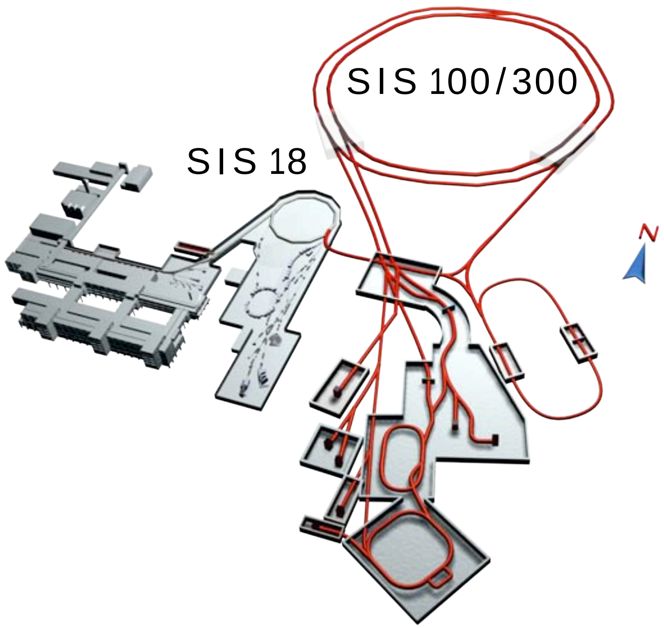
The FAIR Synchrotrons

Design beam parameters in SIS 18 and SIS 100



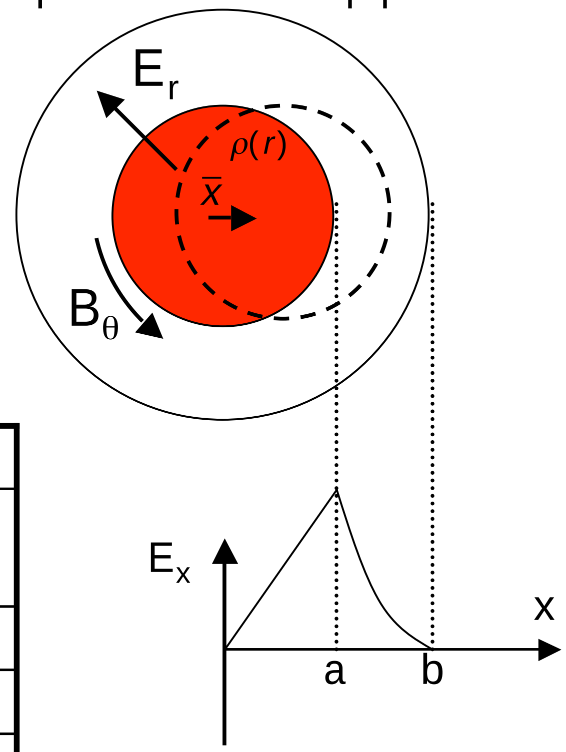
	SIS 18 today	SIS 18 in 2008	SIS 18 in 2012	SIS 100
Reference Ion	U^{73+}	U^{73+}	U^{28+}	U^{28+}
Maximum Energy	1 GeV/u	1 GeV/u	0.2 GeV/u	2.7 GeV/u
Maximum Intensity	$3x10^{9}$	$2x10^{10}$	$2x10^{11}$	$5x10^{11}$
Repetition Rate	0.3 Hz	1 Hz	2.7 Hz	1 Hz

Space charge and image fields

Transverse incoherent space charge tune shift:

$$\boxed{\Delta Q \propto -\frac{q^2 N R}{m B_f \beta_0^2 \gamma_0^3 \varepsilon}}$$

transverse beam profile inside a pipe



Transverse coherent tune shift ('space charge impedance')

$$\Delta\Omega_c \propto -i \frac{q^2 N}{m \gamma_0 Q_0} Z_\perp \qquad \boxed{Z_\perp^{sc} = -i \frac{Z_0 R}{\beta_0^2 \gamma_0^2 b^2}}$$

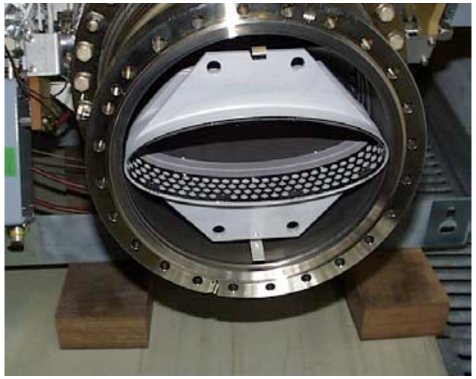
(Ω: frequency of coherent beam center oscillations)

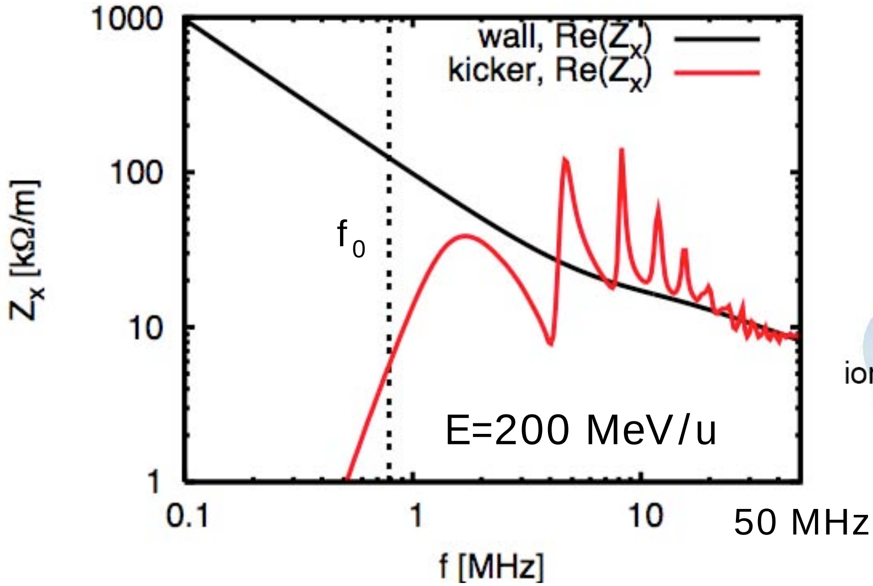
	E [GeV]	ΔQ_v	impedances	remarks
SNS	1	-0.15*	image, e-cloud, kicker, wall	1 ms accumulation uncontrolled loss < 10^{-4}
CERN PS	1.4	-0.25	kickers	
SPS	26	-0.07	e-cloud, kickers	
SIS 18	0.01	-0.5*	image, wall, kicker, rf	$\Delta p/p$ preservation
SIS 100	0.2	-0.3*	image, wall, kicker, rf	1 s accumulation, $\Delta p/p$ preservation uncontrolled loss < 10^{-2}

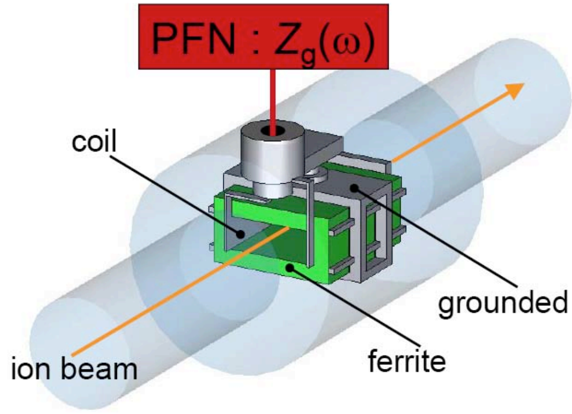
*performance goals !

Transverse impedance sources in SIS 18/100

SIS 18 beam pipe







SIS 18 kicker module

- ✓ (0.3 mm thin) resistive (stainless steel) beam pipe
  - o e.g. A. Al-khateeb, R. Hasse et al., under preparation
- ✓ Ferrite loaded injection/extraction kickers
  - o B. Doliwa, et al., this conference
- ✓ other sources: e.g. 'Electron-clouds'
  - o G. Rumolo, et al., this conference

coherent (dipole) frequency shift:

$$\Delta\Omega_c \propto -i\frac{q^2N}{m\gamma_0 Q_0} Z_\perp(\Omega)$$

growth rate

$$\tau_I = \left(\omega_0 \Im \Delta\Omega_c\right)^{-1}$$

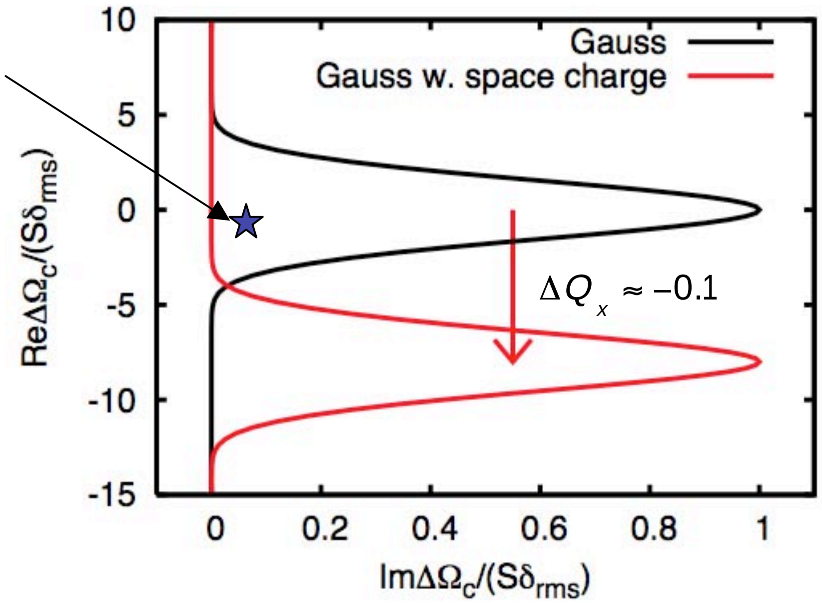
Impedance Budget Estimates (coasting beam)

Example: SIS 18 kicker impedances

Stability threshold (200 MeV/u) with $S = \omega_0(\xi - (n-Q)\eta_0)$

$$\Delta\Omega_c \propto -i \frac{q^2 N}{m\gamma_0 Q_0} Z_\perp^{kicker}(\Omega = 10\,\text{MHz})$$

$$\boxed{\tau_I = \left(\omega_0 \Im \Delta\Omega_c\right)^{-1} \approx 3\text{ ms}}$$



Potential cures for SIS 18/100:

- increase momentum spread by a factor 2 (not an option !)
- increase tune spread with octupoles, space charge : (self-consistent ?) simulations
- rf or barrier buckets: mode coupling, 3D simulation studies !
- active damping system (< 50 MHz, 1 ms reaction time)

Simulation codes for instability studies

Used presently at GSI

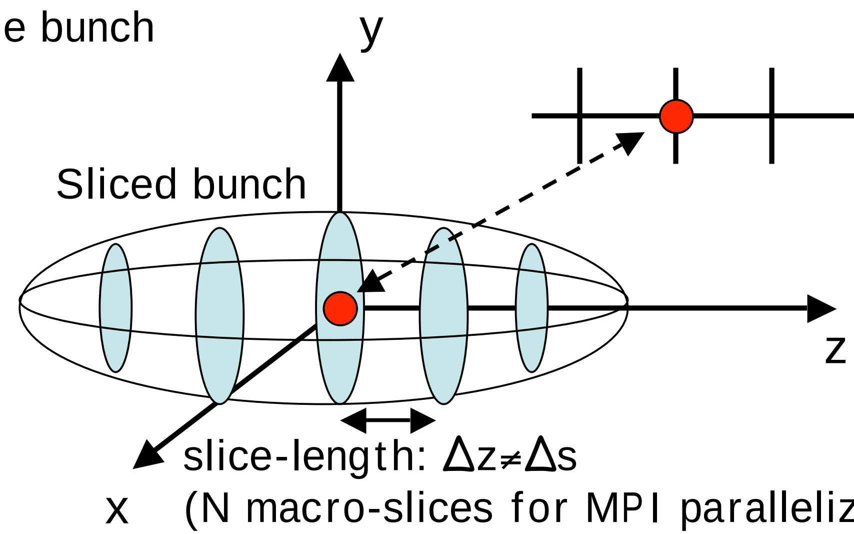
Code:	HEADTAIL	SIMBAD/UAL	PATRIC (JASCC ?)
Authors:	G. Rumolo/F. Zimmermann, CERN	N. D'Imperio/ A.Luccio/N. Malitsky, BNL	O. Boine-Frankenheim/V. Kornilov, GSI
Tracking:	One-turn map	e.g. TEAPOT	MAD-X sectormaps
Space charge and image fields:	Laslett tune shifts (non-self-consistent)	2/2.5/3D schemes	2D Poisson solvers, '2.5D slicing'.
Impedances:	Wake fields	Impedance kicks	Impedance kicks
Language	C	C++/MPI	C++/MPI/Python
Other options:	e.g. electron clouds	All UAL options, 'design code'	Electron cooling, intra-beam scattering

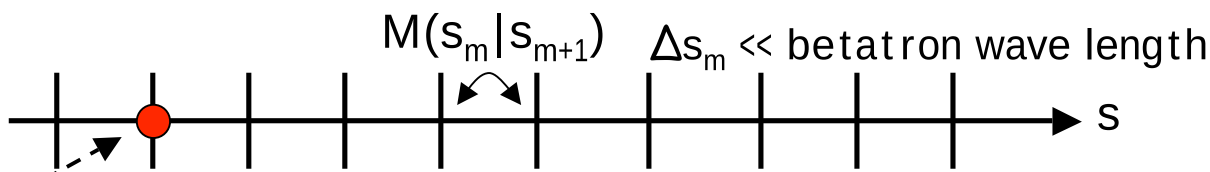
PATRIC

‘2.5 D’ sliced space charge kicks

z: longitudinal position in the bunch

s_m: position in the lattice





The transfer maps M are ‘sector maps’ taken from MADX.

2D space charge field for each slice:

$$\rho(x,y,z,s_m) = \sum_j Q_j S(\vec{x}-\vec{x}_j) \quad \text{(3D interpolation)}$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = \frac{\rho(x,y,\{z,s_m\})}{\varepsilon_0} \quad \text{(fast 2D Poisson solver)}$$

$$E_z \approx -\frac{g}{4\pi\varepsilon_0\gamma_0^2}\frac{\partial\rho_L(z,\{s_m\})}{\partial z} \quad (\rho_L \text{ line density})$$

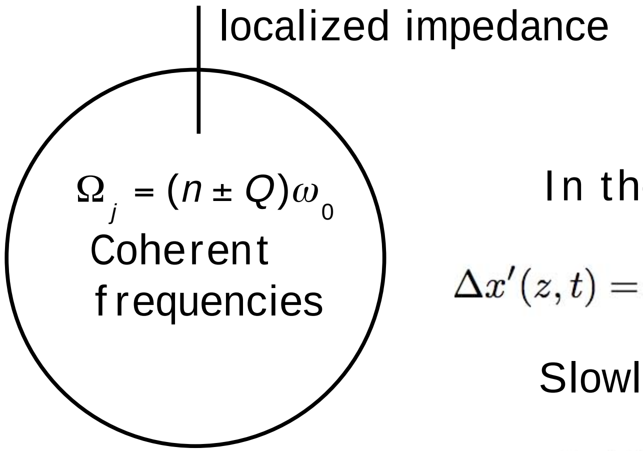
space charge kick:

$$\begin{pmatrix} x_j \\ x'_j \\ y_j \\ y'_j \\ z_j \\ \delta_j \end{pmatrix}_{s_{m+1}} = M(s_m|s_{m+1}) \begin{pmatrix} x_j \\ x'_j + \dfrac{qE_x(x_j,y_j,z_j,\{s_m\})}{mv_0^2\gamma^3}\Delta s \\ \vdots \end{pmatrix}_{s_m}$$

Transverse impedance kicks in PATRIC

see also V.Danilov, J. Holmes, PAC 2001

Dipole moment times current: $\psi(t)$



Impedance kick:

$$\Delta x' = \frac{\int F_\perp ds}{\beta_0^2 E_0} = \mathrm{Re}\left(\frac{iq}{\beta_0 E_0}\sum_j \psi_j Z_\perp(\omega_j)\exp(i\omega_j t)\right)$$

In the bunch frame (Δs=L for localized impedance):

$$\Delta x'(z,t) = \frac{\Delta s}{L}\frac{q}{\beta_0 E_0}\mathrm{Re}\left(i\exp(\pm iQ\omega_0 t)\sum_n \psi_n(t) Z_\perp[(n\pm Q)\omega_0]\exp(inz/R)\right)$$

Slowly varying dipole amplitude:

$$\psi_n(t) = \exp(\mp iQ\omega_0 t)\int_0^L \psi(z,t)\exp(-inz/R)dz$$

Numerical implementation:

$$\psi(z,t_m) = \beta_0 c \sum_j Q_j S(z - z_j) x_j$$

$$\psi_n(t_m) = \mathrm{FFT}[\psi(z_j,t_m)]$$

$$\Delta x'(z_j,t_m) = \frac{q}{\beta_0 E_0}\mathrm{Re}\left(\mathrm{FFT}^{-1}[i\psi_n(t_m)Z_\perp((n\pm Q)\omega_0)]\right)$$

Coasting beam (n=0):

$$\psi(z,t) = I_0 \bar{x}(z,t)$$

$$\Delta x'(t) = \frac{\Delta s}{L}\frac{qI_0}{\beta_0 E_0}\left(-\mathrm{Im}(Z_\perp)\bar{x}(t) + \mathrm{Re}(Z_\perp)\frac{\dot{\bar{x}}(t)}{Q\omega_0}\right)$$

Stability boundaries from PATRIC simulation scans

coasting beams with space charge

1D example: longitudinal instability and momentum spread

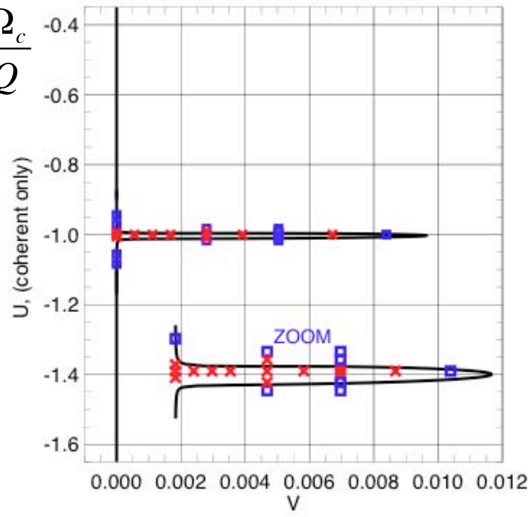
O. Boine-F., I. Hofmann, PRST-AB 2003

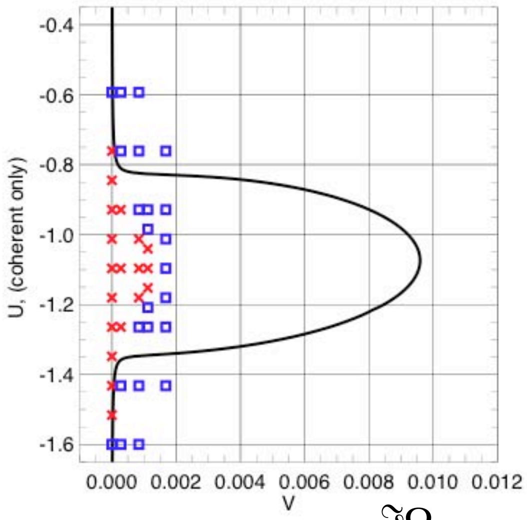
2D example: Transverse resistive wall instability

only octupole

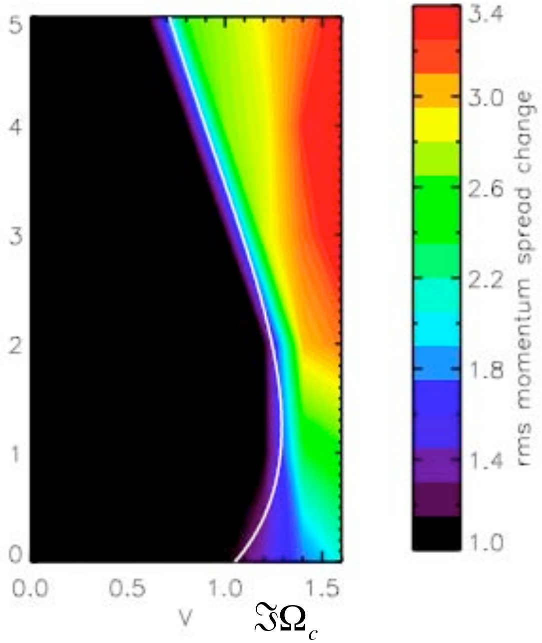
oct. + nonl. space charge

$\frac{\Re\Omega_c}{\Delta Q}$





$\frac{\Re\Omega_c}{S}$



V.Kornilov, 2006

$\frac{\Im\Omega_c}{\Delta Q}$

$\frac{\Im\Omega_c}{S}$

Issues:

- Computer time and resources consuming endeavor, only possible in 1D and 2D(?)
- Artificial noise can spoil the results: codes are not 'collisionless'
- How does the (2D) results change if we 'freeze' space charge.

Self-consistent vs. frozen space charge kicks
for coherent (dipole) instability thresholds

- In parameter scans required for stability boundary studies full 2.5D simulations with self-consistent space charge and sufficient macro-particle number and grid resolution cannot be performed over the time scales of interest (> ms) in SIS 18/100.

- A possible solution are frozen (nonlinear) space charge fields:

Parabolic beam profile and rigid 'offset' dipole oscillations: $\rho(x,y) \approx \frac{2I(z)}{\pi a^2 v_0}\left(1 - \frac{(x-\bar{x})^2 + y^2}{a^2}\right)$

Resulting nonlinear transverse space charge electric fields, moving with the beam center:

$$E_x(x,y) \approx \frac{Ix}{\pi\varepsilon_0 v_0 2a^2}\left(2 - \frac{1}{a^2}((x-\bar{x})^2 + y^2)\right) \qquad E_y(x,y) \approx \frac{Iy}{\pi\varepsilon_0 v_0 2a^2}\left(2 - \frac{1}{a^2}((x-\bar{x})^2 + y^2)\right)$$

Questions to be answered:

- ✓ Do we need self-consistent space charge in order to predict instability thresholds ?
- ✓ How to decide, what are appropriate tests ?

‘Computer Beam Transfer Function’ (CBTF)
2D coasting beam (n=0 mode)

- Determine and compare the properties of different codes/approximations.
- Obtain stability boundaries in one simulation run, instead of time consuming parameter scans

Perturbation: Dipole kicks with bandwidth limited noise:

$$\Delta\theta(t) = \Delta\theta_0 \sum_j \sin(\Omega_j t + \varphi_j)$$

$$\Omega_j = \Omega_c \pm \xi_j \Delta\Omega \qquad \varphi_j = \pi\chi_j$$

random numbers: ξ_j , χ_j

Resulting offset:

$$\bar{x}(t) = \frac{1}{N}\sum_{l=0}^{N} x_l(t)$$

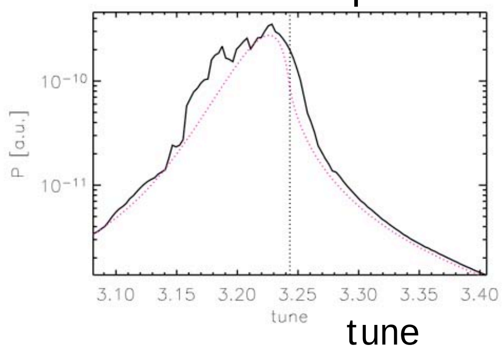
$$\bar{x}(\Omega) = \mathrm{FFT}[\bar{x}(t)]$$

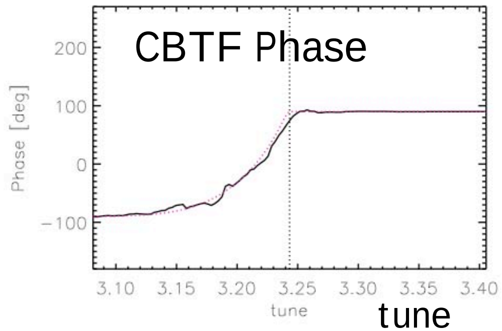
$$(CBTF)(\Omega) = \frac{\bar{x}(\Omega)}{\Delta\theta(\Omega)}$$

$(CBTF)^{-1}$ = Stability diagram

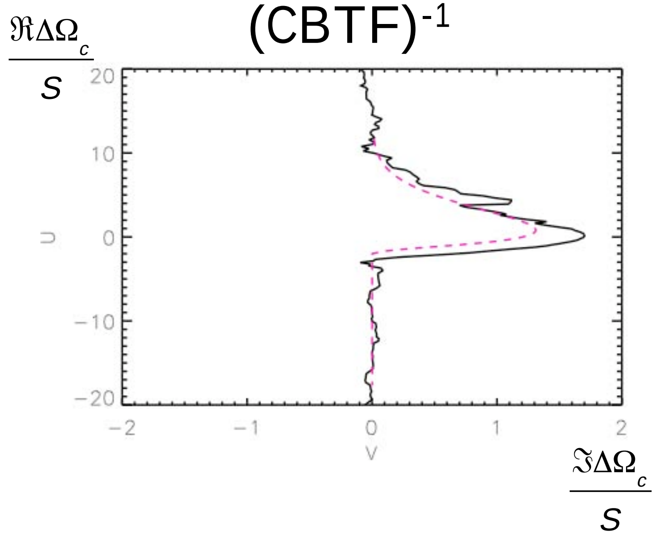
Example: BTF of a Gaussian betatron amplitude distribution + octupoles (2D, 50k particles).

CBTF Amplitude





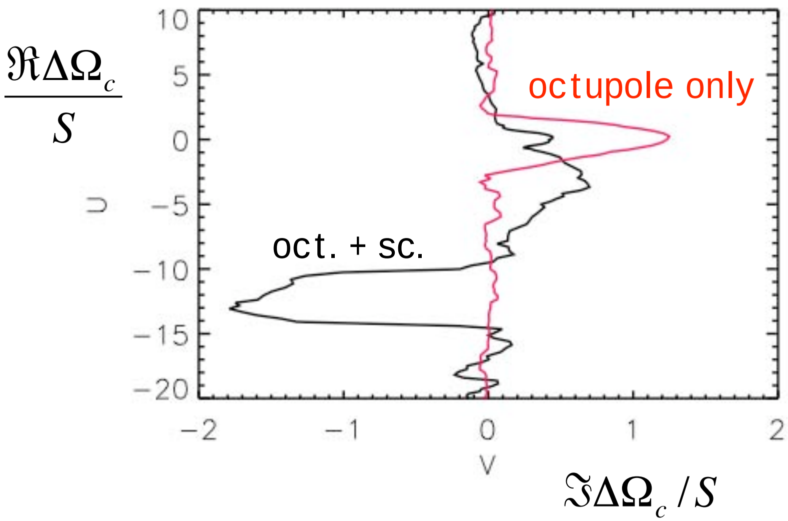
$(CBTF)^{-1}$

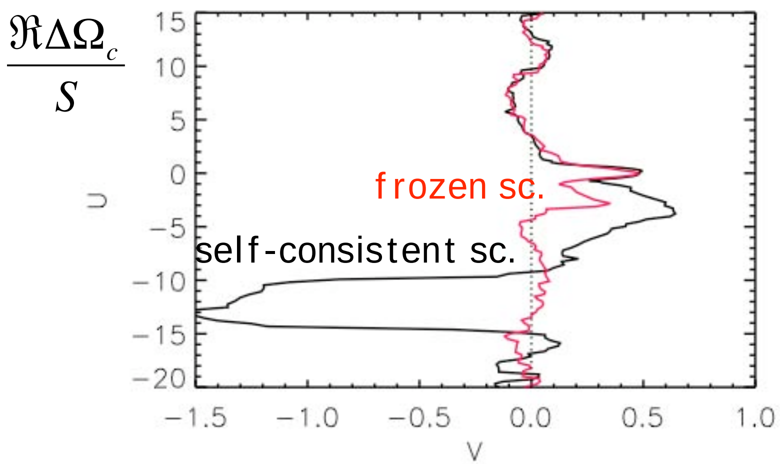


Stability boundary from CBTF with space charge ?

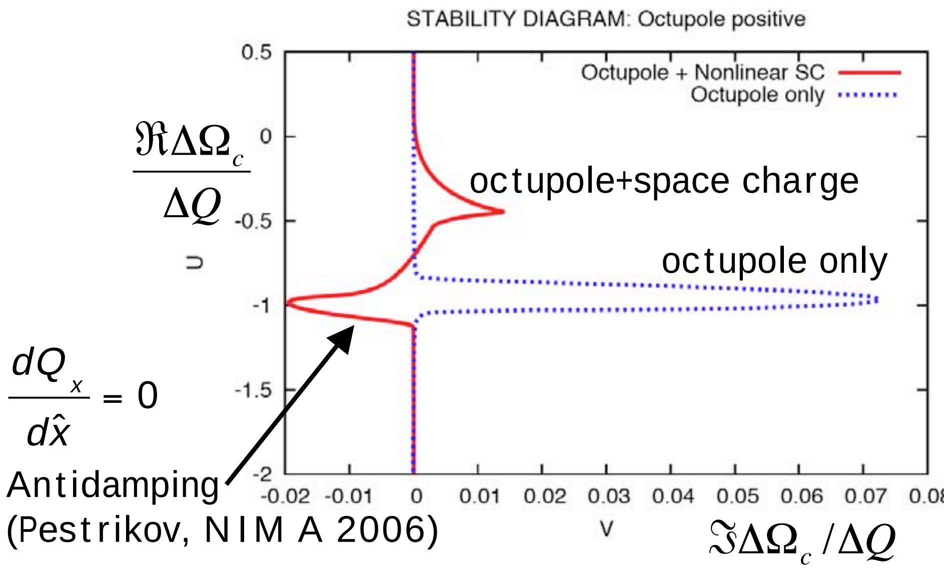
A ‘challenging’ example: octupole and (nonlinear) space charge

CBTF results (very preliminary !):





Stability boundary obtained from analytic (non-self-consistent) dispersion relation (Möhl, Schönauer, 1979)



These studies are being performed in parallel to parameter scans.

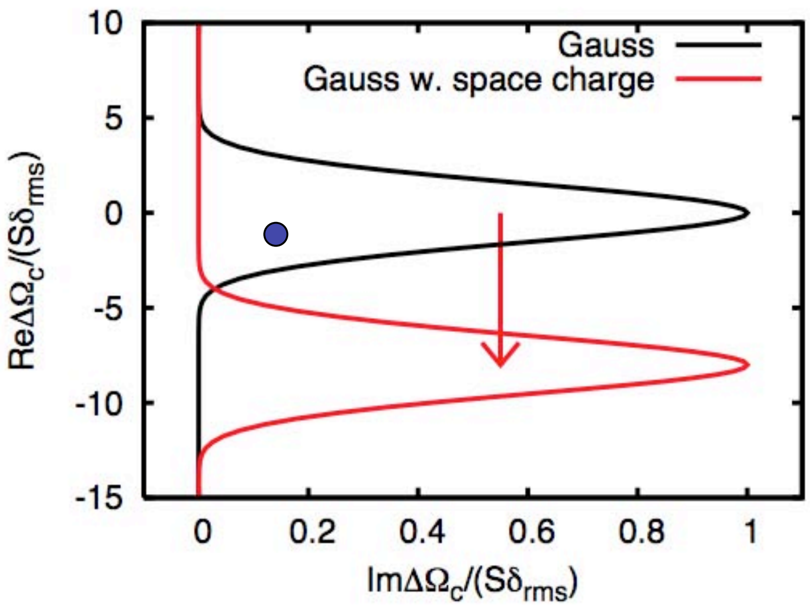
They seem to indicate that only with frozen space charge an instability develops without resistive impedance

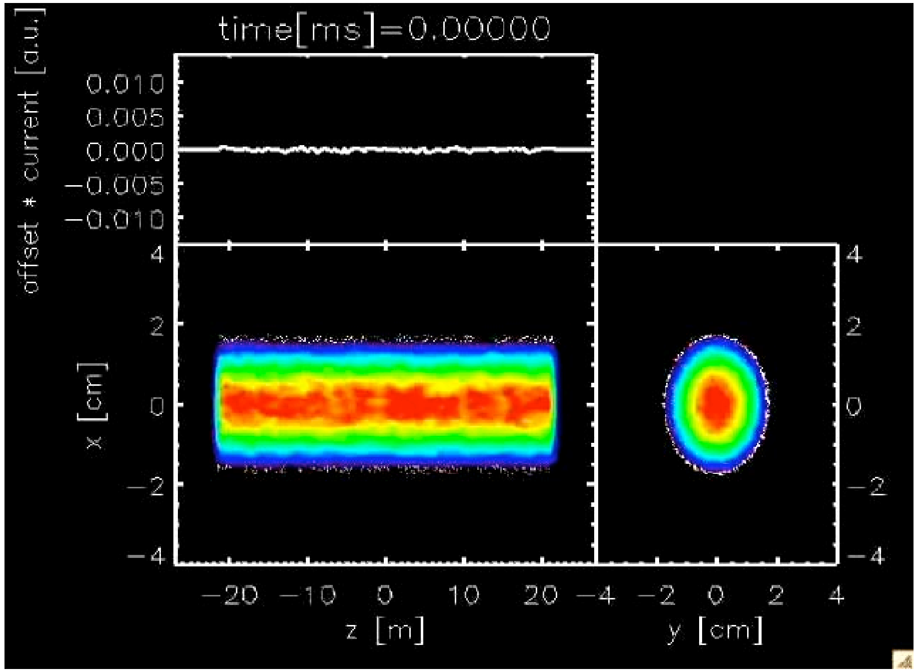
Stabilization mechanisms in long bunches
relevant in SIS 18/100

Bunching (here barrier bucket) can strongly increase the instability thresholds.
'Slow mode damping at the bunch end.'

Better understanding of the mechanisms with space charge is required:
2.5D simulation studies can help.

coasting beam stability diagram





'Preliminary' conclusions

The space charge and impedance conditions in SIS 18/100 are special and require detailed parameter studies:

- Long time scales.
- Long bunches in single rf or barrier buckets.
- Relatively large space charge tune spreads. 'Thick beams'.
- Broadband impedance spectrum: wall, kickers, collimators, e-clouds (?),..

✓ Potential cures: Octupoles, bunching, (reactive/resistive) broadband feedback systems

The 2.5D space charge/impedance module in PATRIC is presently being benchmarked against analytic examples ('Schottky' noise spectrum, CBTF, stability boundaries) and also against other codes (UAL/SIMBAD, HEADTAIL).

Besides the SIS 18/100 impedance budget, one important outcome of the studies will be e.g. the importance of self-consistent space charge for the stability boundaries.