Analysis of measured transverse beam echoes in RHIC

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Motivation

- Diffusion processes lead to emittance growth and so, to change of beam equlibrium

- Major contribution expected to be due to Intra Beam Scattering (IBS)

- In HESR beams with high currents, small momontum spread are desired
  $\longrightarrow$ strong sensitivity to diffusion, need to determine diffusion in HESR

- In RHIC, expexted diffusion times $\tau_{\mathrm{diff}} \sim$ hours
  $\longrightarrow$ direct measurement of emittance growth difficult

- Other possibility: measurement of transverse beam echoes provides possibility to measure diffusion within comparable short times $\sim 1000\ T_0$

- Additionally, echoes can be used to measure numerical diffusion[1]

[1] A. Al-Khateeb, O. Boine-Frankenheim, R. Hasse, and I. Hofmann, PRST AB 6, 014205 (2003)

Generation of transverse beam echoes

Need of:

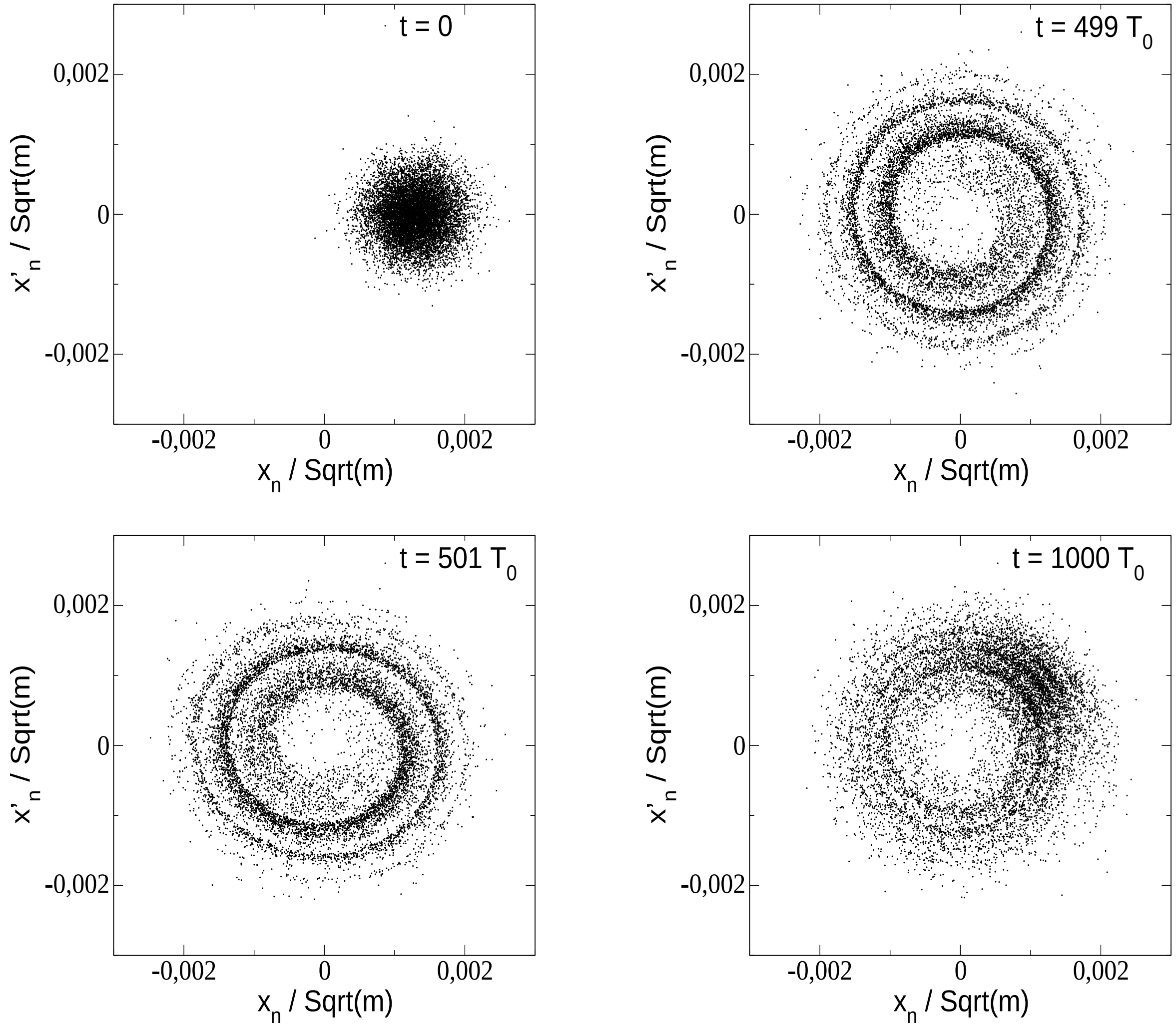
- **Focusing lattice**
- **Initial one turn dipole kick leading to displacement and dipole betatron oscillation of whole beam with the initial amplitude $\langle x \rangle_0$**
- **Nonlinear element, e.g. octupole, that leads to decoherence and decrease of amplitude of the dipole signal**
- **one turn quadrupole kick after certain time τ**

$$\Downarrow$$

After certain time coherent dipole oscillation ("beam echo") will briefly appear with an amplitude maximum at $t = 2\tau$

Generation of transverse beam echoes

Horizontal phase space diagram depending on time, $\tau = 500\ T_0$



Normalized phase space variables:

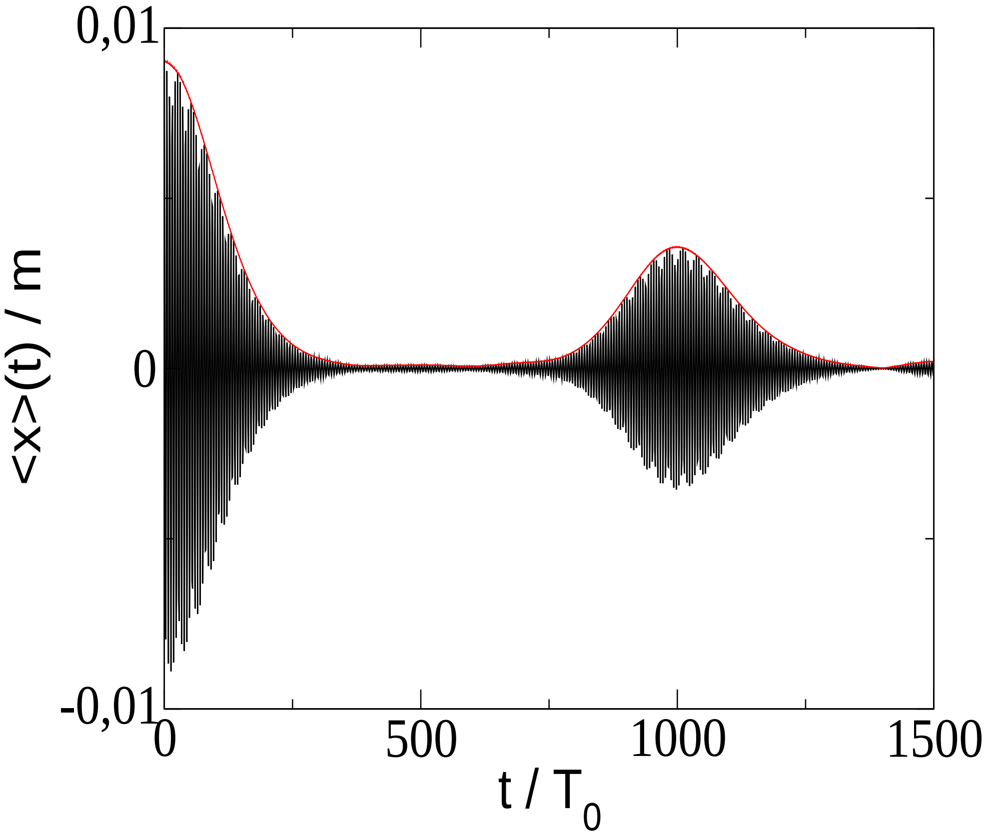
$$x_n = \frac{x}{\sqrt{\beta}}$$

$$x_n' = \frac{\beta x' + \alpha x}{\sqrt{\beta}}$$

Generation of transverse beam echoes

Amplitude of transversal oscillation $\langle x\rangle(t)$, conditions from O. Brüning et al.[1]:

$\tau = 500\ T_0$, maximum echo amplitude at $2\tau = 1000\ T_0$



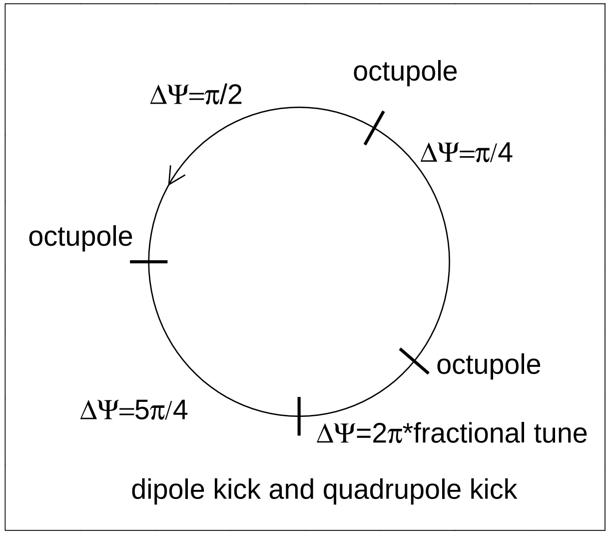
Diffusion leads to decrease of maximum echo amplitude.

[1]O. Brüning, W. Fischer, B. Parker, "On the possibility of Transverse Echoes in RHIC", C-A/AP/4 October 1999

Tracking method

- Configuration from O. Brüning et al.
- Rotational matrix for linear focusing parts with correct phase advances
- Usage of normalized coordinates

$$x_n = \frac{x}{\sqrt{\beta}} \text{ and } x_n^{'} = \sqrt{\beta}x' + \frac{\alpha}{\sqrt{\beta}}x$$



- Kicks for one turn dipole, one turn quadrupole, and octupoles, and a random kick for diffusion with Gauss distribution

$$\begin{pmatrix} x_{n,i+1} \\ x^{'}_{n,i+1} \end{pmatrix} = \begin{pmatrix} \cos\Delta\Psi & \sin\Delta\Psi \\ -\sin\Delta\Psi & \cos\Delta\Psi \end{pmatrix} \begin{pmatrix} x_{n,i} + \Delta x_n(D_0) \\ x^{'}_{n,i} + \Delta x^{'}_n(x_n, y_n, D_0) \end{pmatrix}$$

Diffusion coefficient D_0

- Used in a simple approximation

$$D_0 = \frac{\varepsilon_{\mathrm{rms}}}{\tau_{\mathrm{diff}}}$$

- $\varepsilon_{\mathrm{rms}}, \tau_{\mathrm{diff}}$ initial values being constant in space and time:

  $\longrightarrow$ strong simplification

  - τ_{diff} calculated from initial rms emittance of a Gaussian beam
  - but rms emittance is function of time
  - in addition, D_0 is intrinsically a function of emittances of single particles

Analytic formula for echo amplitude

Relative echo amplitude[1]**:**

$$A_{\text{echo,rel}} := \frac{\langle x \rangle_{\text{echo}}}{\langle x \rangle_0} = \frac{q}{\alpha^3}\frac{\tau}{\tau_{\text{dec}}}$$

with:

$$\alpha = \frac{2}{3}\frac{\tau}{\tau_{\text{diff}}}\left(\frac{\tau}{\tau_{\text{dec}}}\right)^2 + 1$$

τ – quadrupole kick time and τ_{diff} – diffusion time

$\tau_{\text{dec}} \approx \dfrac{T_0}{4\pi\mu}$ – and decoherence time[2] with

μ – detuning due to nonlinearity for $x = \sigma_{\text{rms}}$

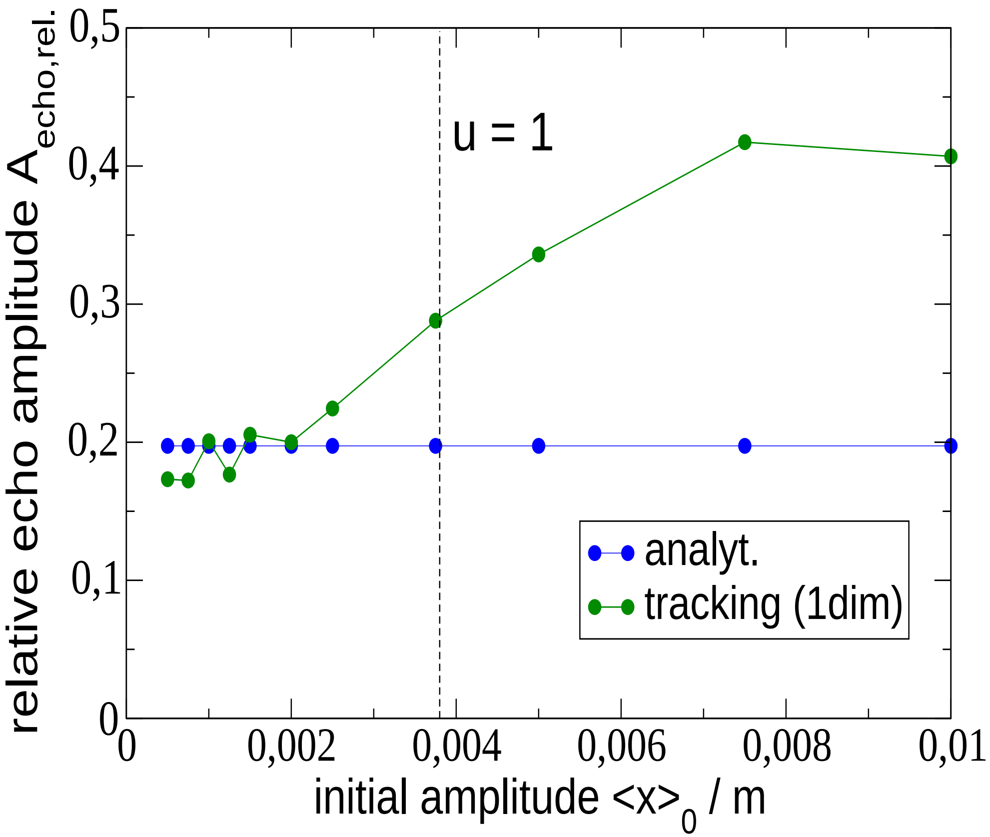
$q = \dfrac{\beta_x}{f}$ – normalized quadrupole strength

[1] A. W. Chao, Chapter 2 "Echo effect" in Lecturenotes: http://www.slac.stanford.edu/~achao/lecturenotes.html

[2] G. Stupakov "Echo" in "Handbook of accelerator physics" ed. by A. W. Chao and M. Tigner, World Scientific (1999)

Echo amplitudes: tracking vs. analytic formula

Echo amplitudes without diffusion, detuning $\mu = 0.0014$



- **Analytic formula valid for**

$$u = \frac{\langle x \rangle_0^2}{2\beta\varepsilon_{\rm ms}} \ll 1$$

- $u = 1$ **for** $\langle x \rangle_0 = 3.8$ **mm**

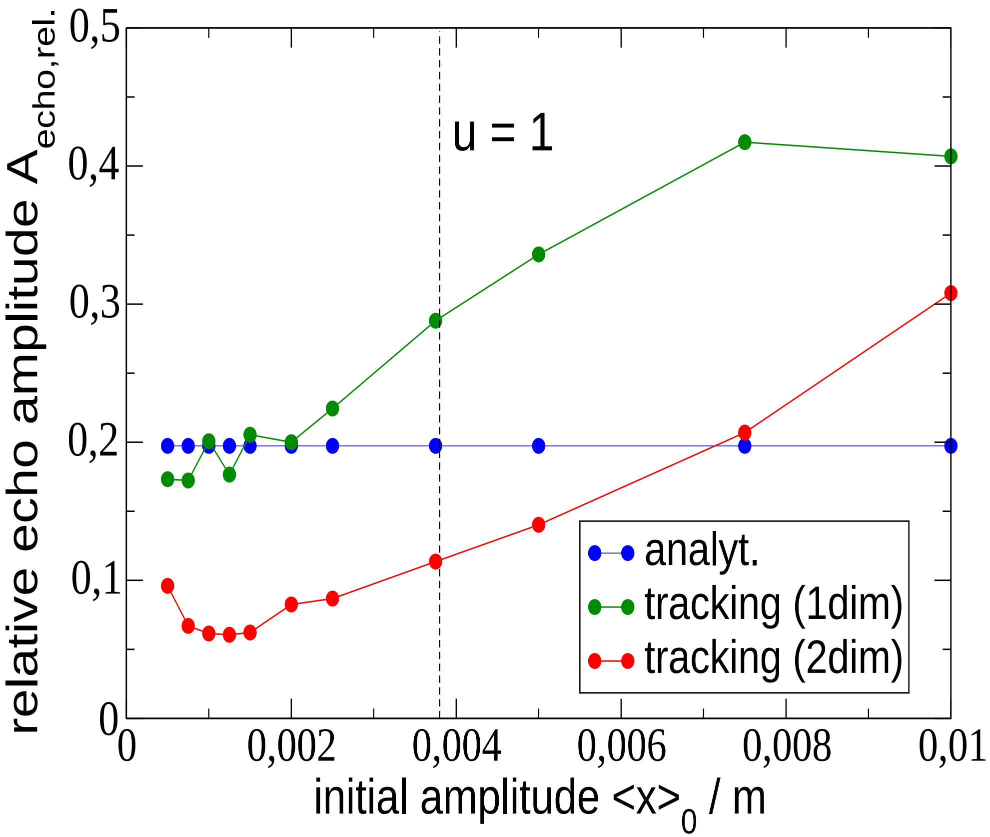
- **Particle number used in tracking:**

  $n_{\rm part} = 10000 \ldots 50000$

- **For smaller initial amplitudes** $\langle x \rangle_0$ **larger particle numbers necessary**

Echo amplitudes: tracking vs. analytic formula

Echo amplitudes without diffusion, detuning $\mu = 0.0014$



- "1 dimensional": no vertical extension $\longrightarrow$ unrealistic
- Coupling of directions leads to decrease of amplitudes in tracking calculations
- Analytic formula underestimates echo amplitudes for initial amplitudes used in experiments

RHIC experiments

- Application to existing experiments done in RHIC[1,2]:
  - Experiments with Au^{79+}, Cu^{29+}, and p
- This work:
  - Usage of IBS rates from realistic IBS models to describe experiments
  - Here, restriction to description of experiments with Au^{79+}

[1]W. Fischer, T. Satogata, R. Tomas, "Measurement of Transverse Echoes in RHIC", Proc. of PAC 2005

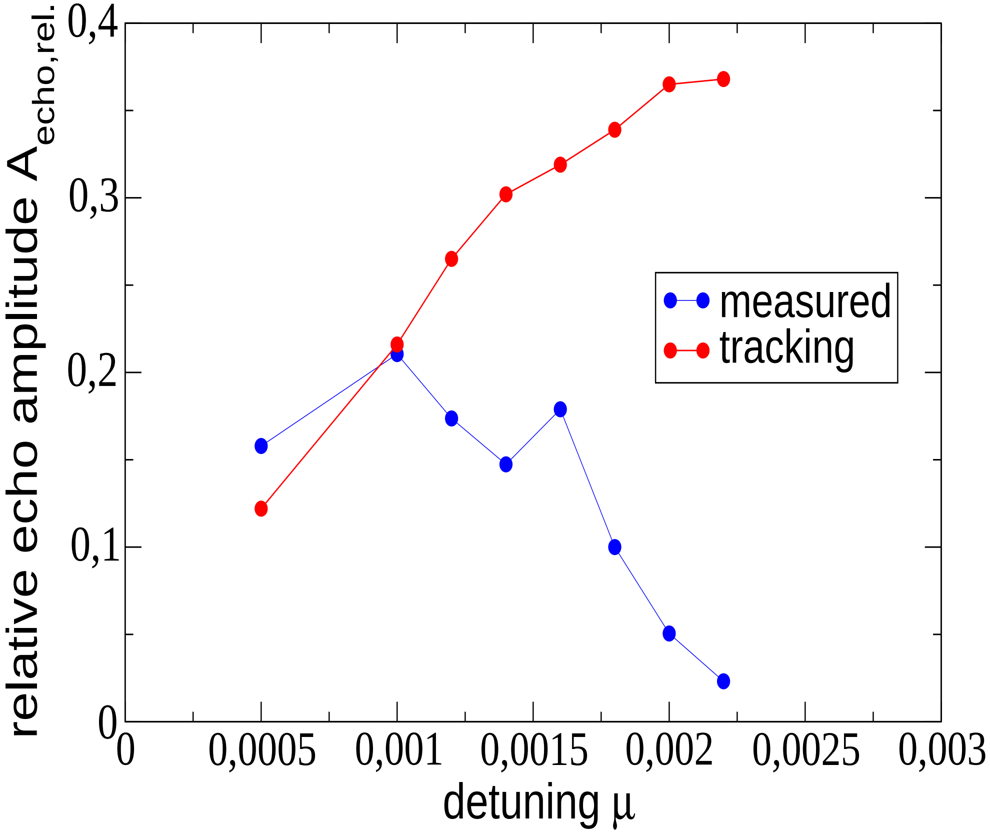
[2]W. Fischer, "Transverse echo measurements in RHIC", Proc. of COOL 2005, Galena, Illinois

Parameters for Au experiments

mass / charge number A/Z	179/79
relativistic γ	10.5
circumference/m	3834
revolution time $T_0/\mu\text{s}$	12.8
rms emittance, unnorm. $\varepsilon_{\text{rms}}/\text{mm mrad}$	0.16
rms beam size, σ/mm	2.5
initial dipole displacement $\langle x \rangle_0/\text{mm}$	10
detuning μ, for $x = \sigma$	0.0005 ... 0.0022, (0.0014)
normalized quadrupole strength q	0.025
quadrupole kick time τ/T_0	450
bunch intensity $N_b/10^9$	0.1 ... 1.0

1. Example: Rel. echo amplitude $A_{\mathrm{echo,rel}}$ vs. μ

Results from experiments and tracking for Au^{79+}

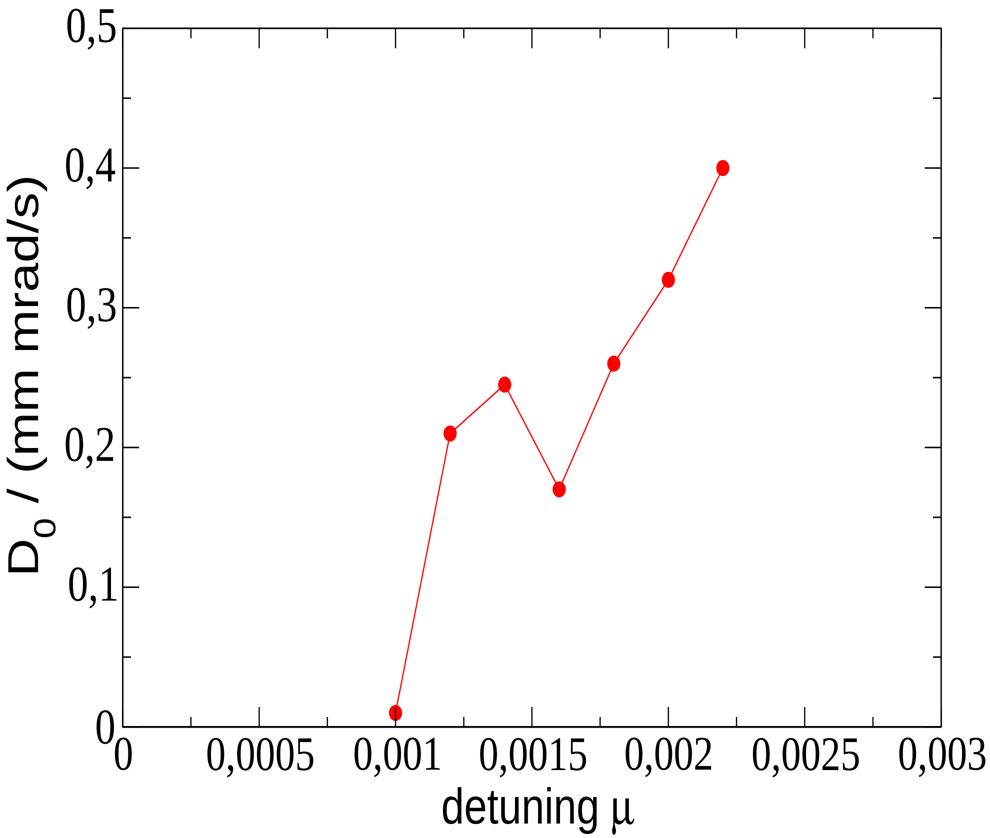


- **Calculated amplitudes yielded without diffusion**
- **For small detuning μ, calculated amplitudes smaller than measured ones**

  $\longrightarrow$ can not reproduce them
- **For large μ, further increase of echo amplitudes in contrast to measured ones**

Diffusion coefficients D_0 vs. μ for Au^{79+}

Calculated diffusion coefficients D_0 corresponding to measured rel. amplitudes

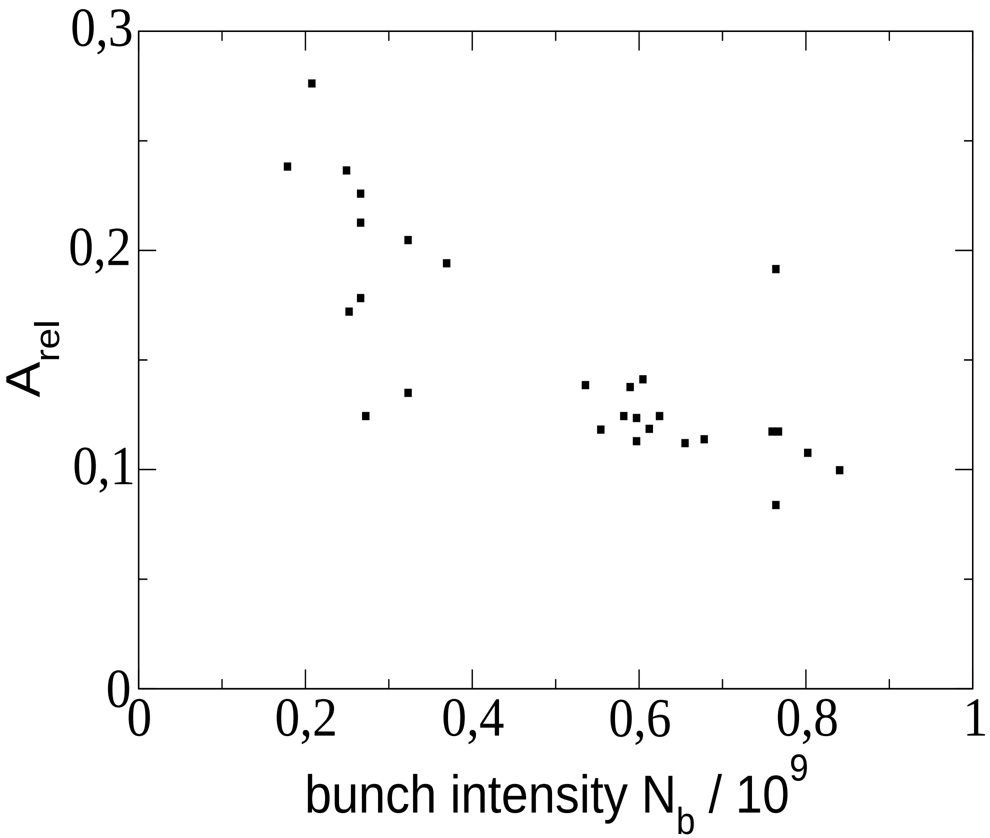


- Finite diffusion coefficients
- Adjusted to reproduce measured echo amplitudes
- For $\mu < 0.001$, no reasonable D_0 possible

$D_0 \sim (0.1 \ldots 1)$ mm mrad/s corresponds to $\tau_{\mathrm{diff}} \sim (1.6 \ldots 0.16)$ sec.

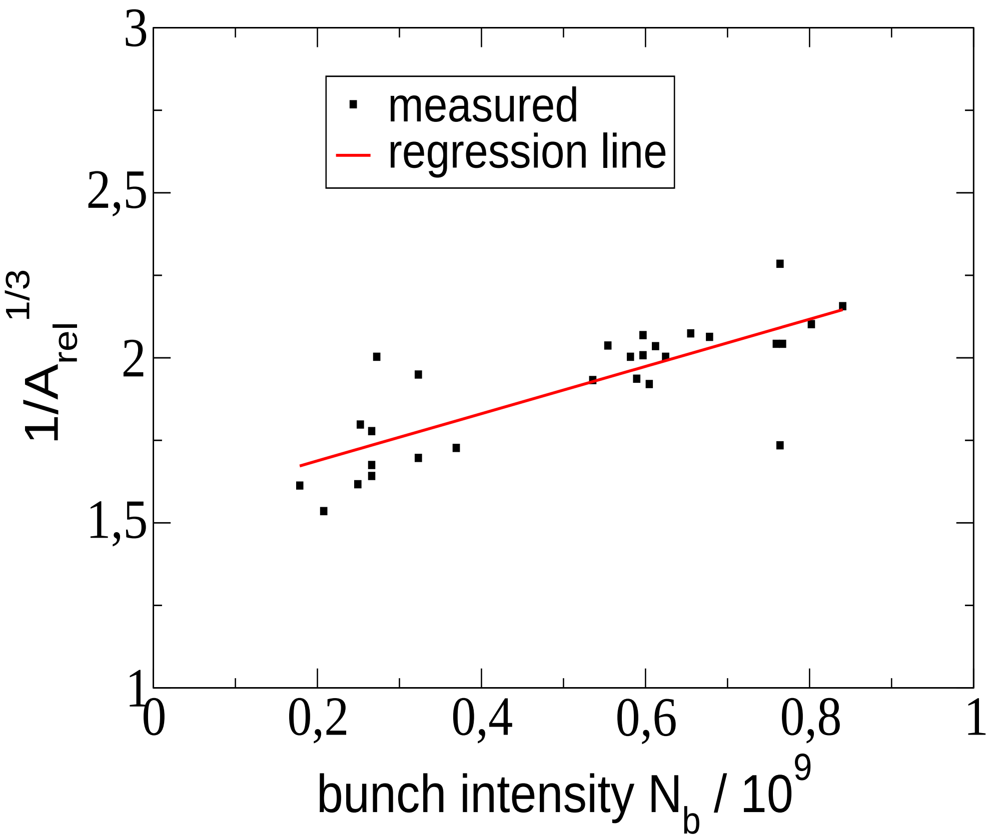
2. Example: $A_{\mathrm{echo,rel}}$ vs. bunch intensity N_b

Results from Au^{79+} experiment



- 28 data points instead of 7
  $\longrightarrow$ possibly better statistics
- As expected for IBS, echo amplitude decreases with bunch intensity

$A_{\mathrm{echo,rel}}^{-1/3}$ vs. N_b



- **From analytic formula follows**

$$A_{\mathrm{echo,rel}}^{-1/3} = \left(\frac{\tau_{\mathrm{dec}}}{q\tau}\right)^{1/3} \left(1 + \frac{2}{3}\frac{\tau^3}{\tau_{\mathrm{dec}}^2}\frac{1}{\tau_{\mathrm{ibs}}}\right)$$

- **IBS diffusion time** $\tau_{\mathrm{diff}} \propto N_b^{-1}$, **hence**

$$A_{\mathrm{echo,rel}}^{-1/3} = c_1 + c_2 N_b$$

- **Linear regression and inserting gives**

$$\tau_{\mathrm{dec}} = 41\, T_0 \text{ and } \frac{\tau_{\mathrm{ibs}}}{T_0} = 5.5 \cdot 10^{13}/N_b$$

For $N_b = 10^8$ we find $\tau_{\mathrm{ibs}} = 550000\, T_0 = 7$ s.

Remarks

Diffusion coefficients used are not realistic

- Diffusion coefficients used correspond to diffusion times

$$\tau_{\rm diff} \sim (10 \dots 0.1)\ \text{sec} \approx (800000 \dots 8000)\ T_0$$

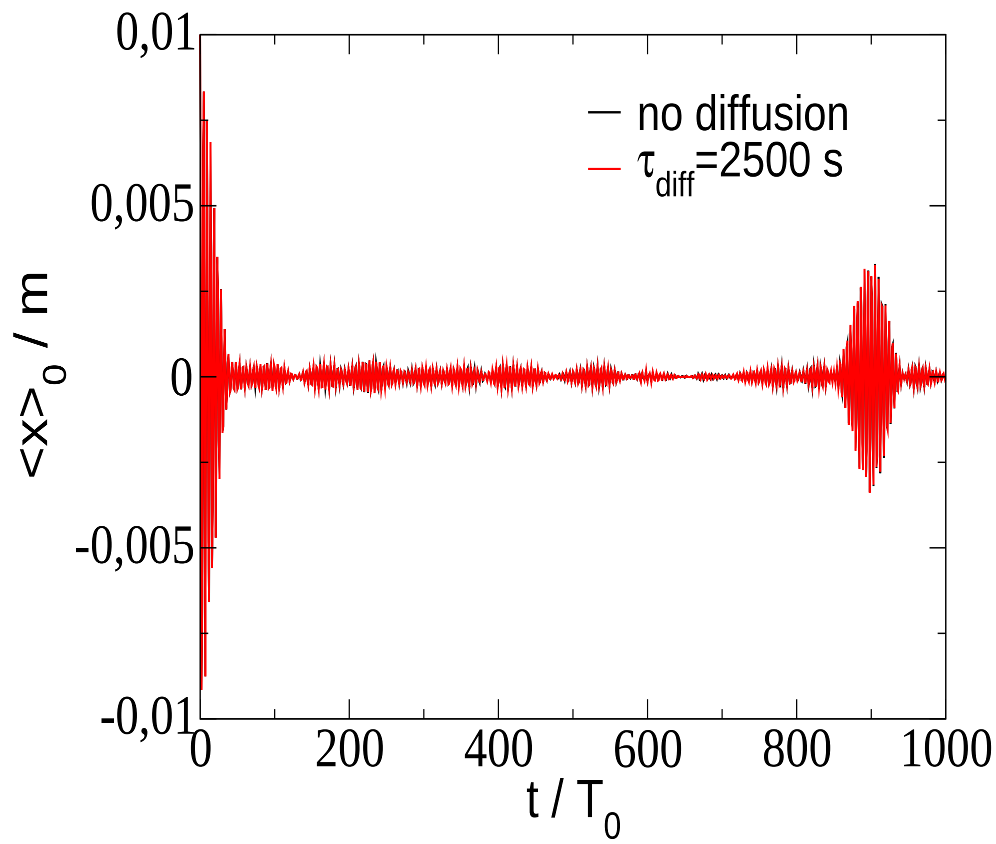
with $T_0 \approx 12.8\ \mu$s

- MAD-X using Bjorken-Mtingwa model applied to bunches with particle number $N_b = 10^8 \dots 10^9$, length $l_{b,\rm rms} \sim 1$ m, and energy uncertainty $\sigma_{\rm e} = 10^{-3}$ gives

$$\tau_{\rm diff} \sim (25000 \dots 2500)\ \text{sec} \approx (2 \cdot 10^9 \dots 2 \cdot 10^8)\ T_0$$

Tracking with realistic diffusion time

Tracking with $\tau = 450\ T_0,\ \tau_{\text{dec}} = 57\ T_0,\ \tau_{\text{diff}} = 2 \cdot 10^8\ T_0$:



No visible influence of diffusion

Discussion

Analytic formula

$$A_{\text{echo,rel}} = \frac{q}{\alpha^3}\frac{\tau}{\tau_{\text{dec}}} \qquad \text{with} \qquad \alpha = \frac{2}{3}\frac{\tau}{\tau_{\text{diff}}}\left(\frac{\tau}{\tau_{\text{dec}}}\right)^2 + 1$$

and same parameters as in tracking calculation

$$\alpha = 1.0000935 \quad \rightarrow \quad \frac{1}{\alpha^3} = 0.9997.$$

Example: For reduction of amplitude to $50\ \%$, it must be $\tau^3 \approx 0.4\tau_{\text{diff}}\tau_{\text{dec}}^2$

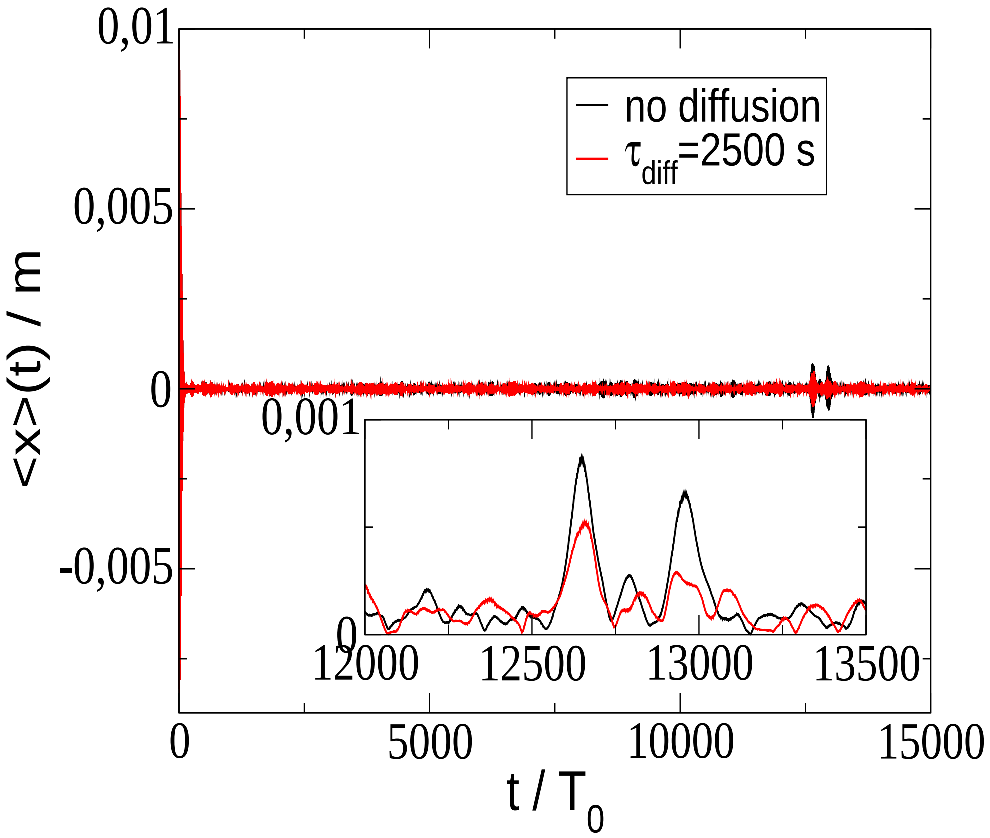
$$\tau_{\text{dec}} \approx 1\ T_0 \text{ with } \tau = 450\ T_0 = \text{const} \quad \text{unrealistic}$$

or

$$\tau \approx 6400\ T_0 \text{ with } \tau_{\text{dec}} = 57\ T_0 = \text{const}$$

Long term tracking with realistic diffusion time

$\tau = 6400\ T_0,\ \tau_{\rm dec} = 57\ T_0,$ and $\tau_{\rm diff} = 2500\ \text{s} = 2 \cdot 10^8\ T_0$



- **Echo appears at expected position**

  $t_{\rm echo} \approx 12800\ T_0$

- **Reduction of amplitude due to diffusion** $\sim 50\ \%$

- **In experiment, echoes only for**

  $400\ T_0 < \tau < 550\ T_0.$

- **It remains, that experimental data can be explained only with very large diffusion coefficients.**

Search for other mechanisms necessary, which decrease echo amplitudes.

Conclusion

- Usage of transverse beam echoes was presented as method to estimate diffusion processes in an ion beam. Application to experiments in RHIC.
- Calculations applying analytic formula of Chao and tracking method using rotation matrix.
- Calculations show, that diffusion coefficients according to realistic IBS times are much too small to describe experimental echo amplitudes
- To detect effects with realistic IBS times, echo measurements had to be extended to longer times
- Search for other mechanisms influencing amplitudes and
- Investigation of consequence of approximations used for the diffusion coefficients are necessary.