Simulation of the FAIR Synchrotron Magnets

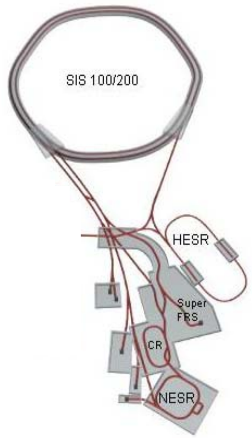
H. De Gersem[1], S. Koch[1] and T. Weiland[1]
E. Fischer[2] and G. Moritz[2]

ICAP 2006, Chamonix Mont-Blanc, 3. October 2006

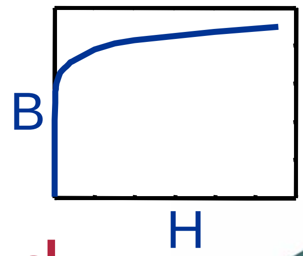
E. Fischer, R. Kurnyshov, G. Moritz, P. Shcherbakov, 3-D transient process calculations for fast cycling superferric accelerator magnets, IEEE Trans. Applied Superconductivity, Vol. 16, No. 2, June 2006, 407-410.

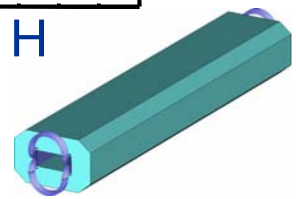
[1]Technische Universität Darmstadt
[2]Gesellschaft für Schwerionenforschung (GSI)

Outline

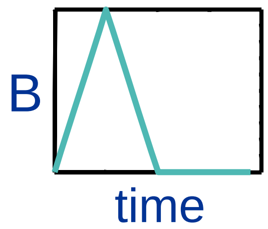


1. Motivation

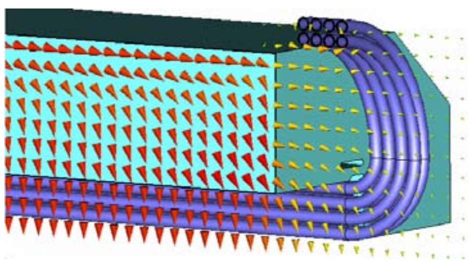


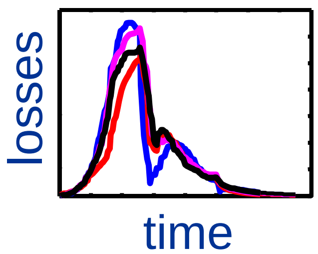


2. Modelling and Simulation



3. Results





4. Summary

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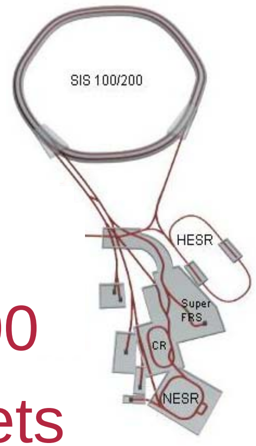
Outline



1. Motivation

- FAIR / SIS100
- dipole magnets
- AC losses

2. Modelling and Simulation

3. Results

4. Summary

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Heavy-Ion Accelerator

Gesellschaft für Schwerionenforschung

Pictures: GSI

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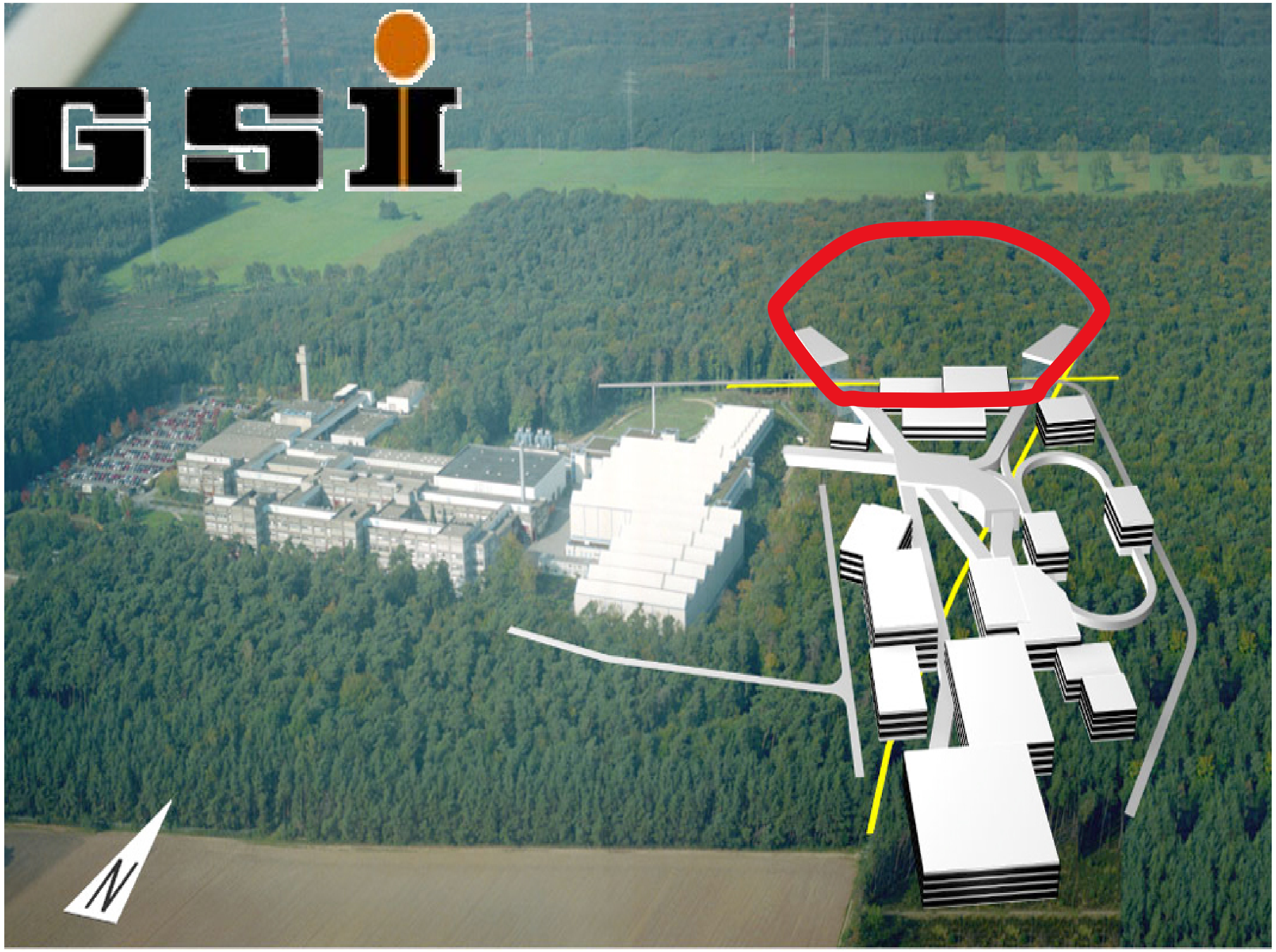
FAIR Project

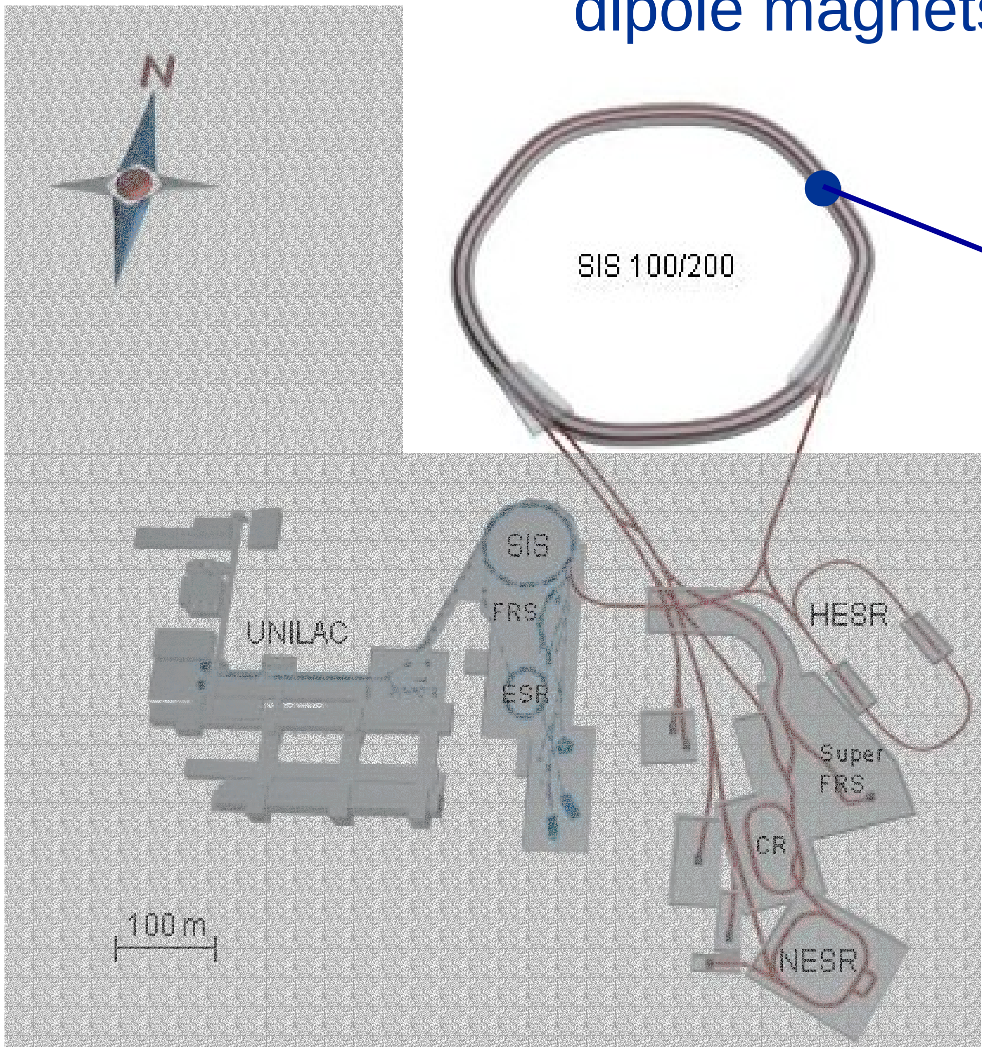
Facility for Antiproton and Ion Research

Gesellschaft für Schwerionenforschung

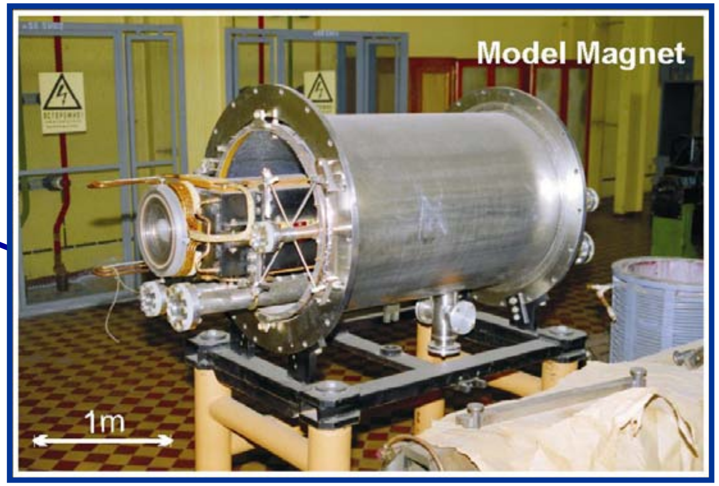


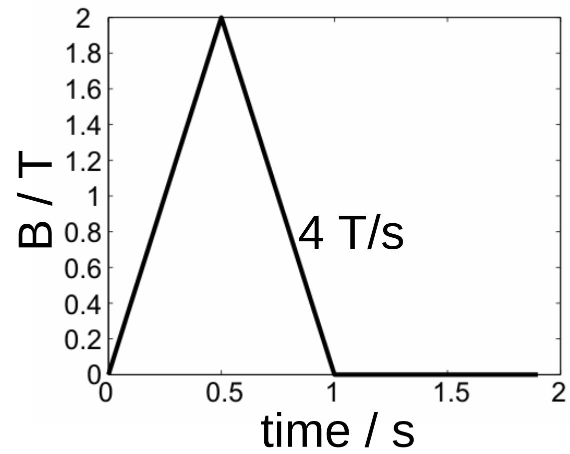
Pictures: GSI

SIS100 Synchrotron



dipole magnets





ramped excitation

Pictures: GSI

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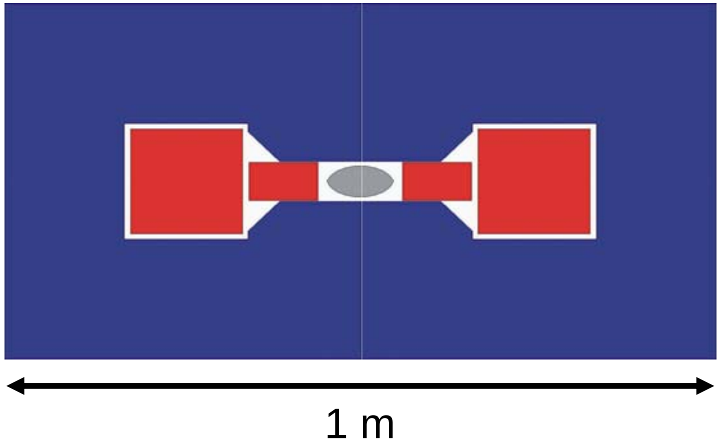
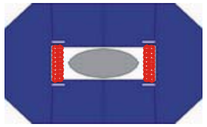
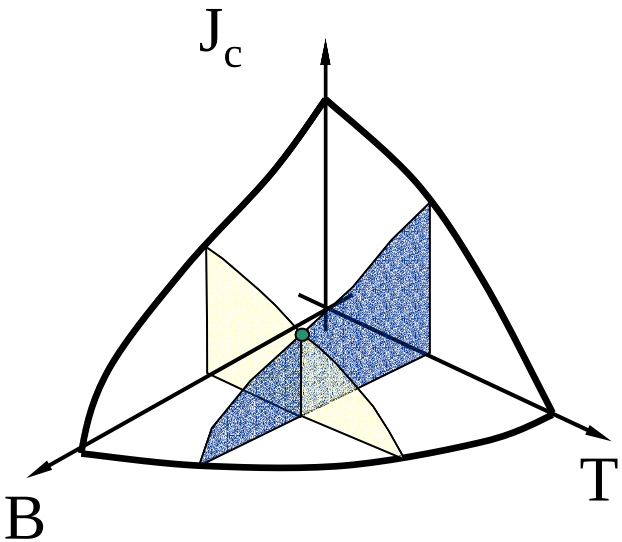
Superconductivity

conventional magnet



superconductive magnet





- + higher current density
- + smaller and lighter
- + no DC power loss
- + smaller operation cost
- \- AC power losses
- \- cooling (liquid He)
- \- complex design
- \- higher investment cost

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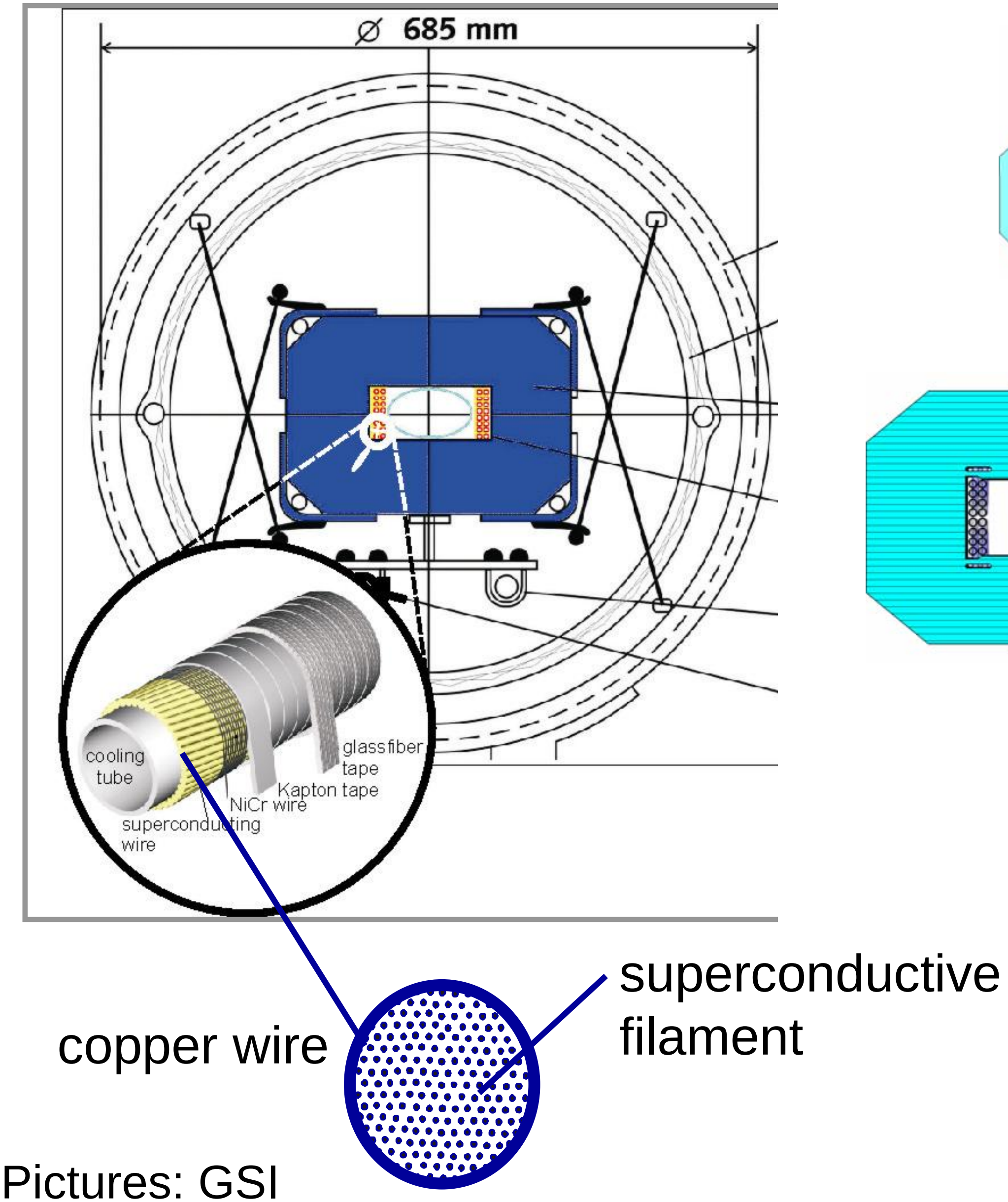
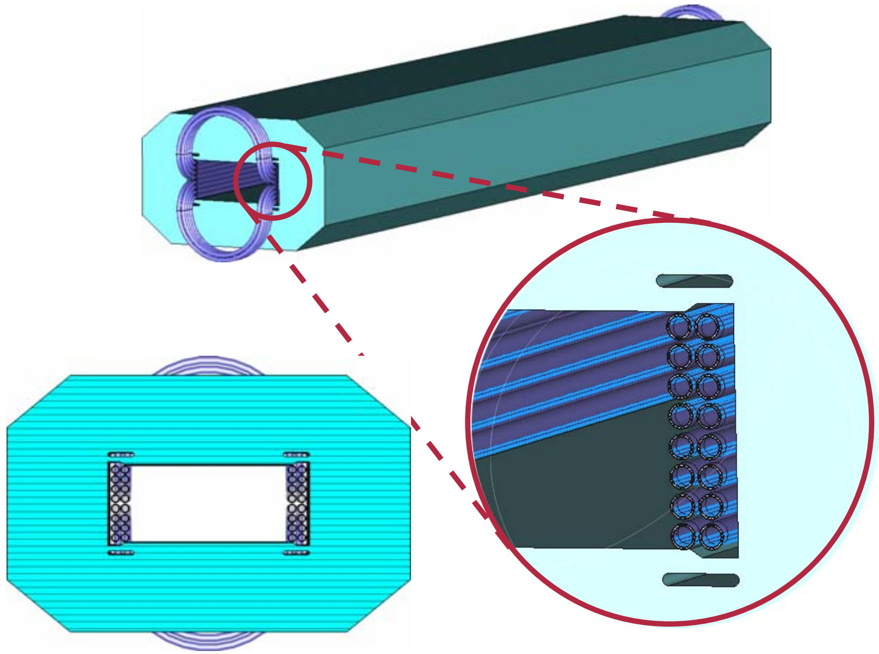
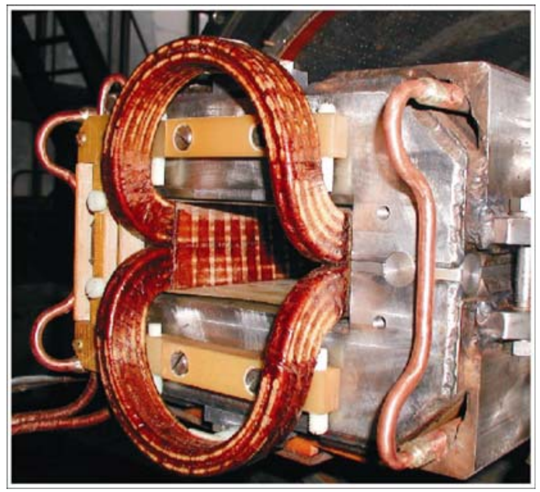
Nuclotron Magnet







Pictures: GSI

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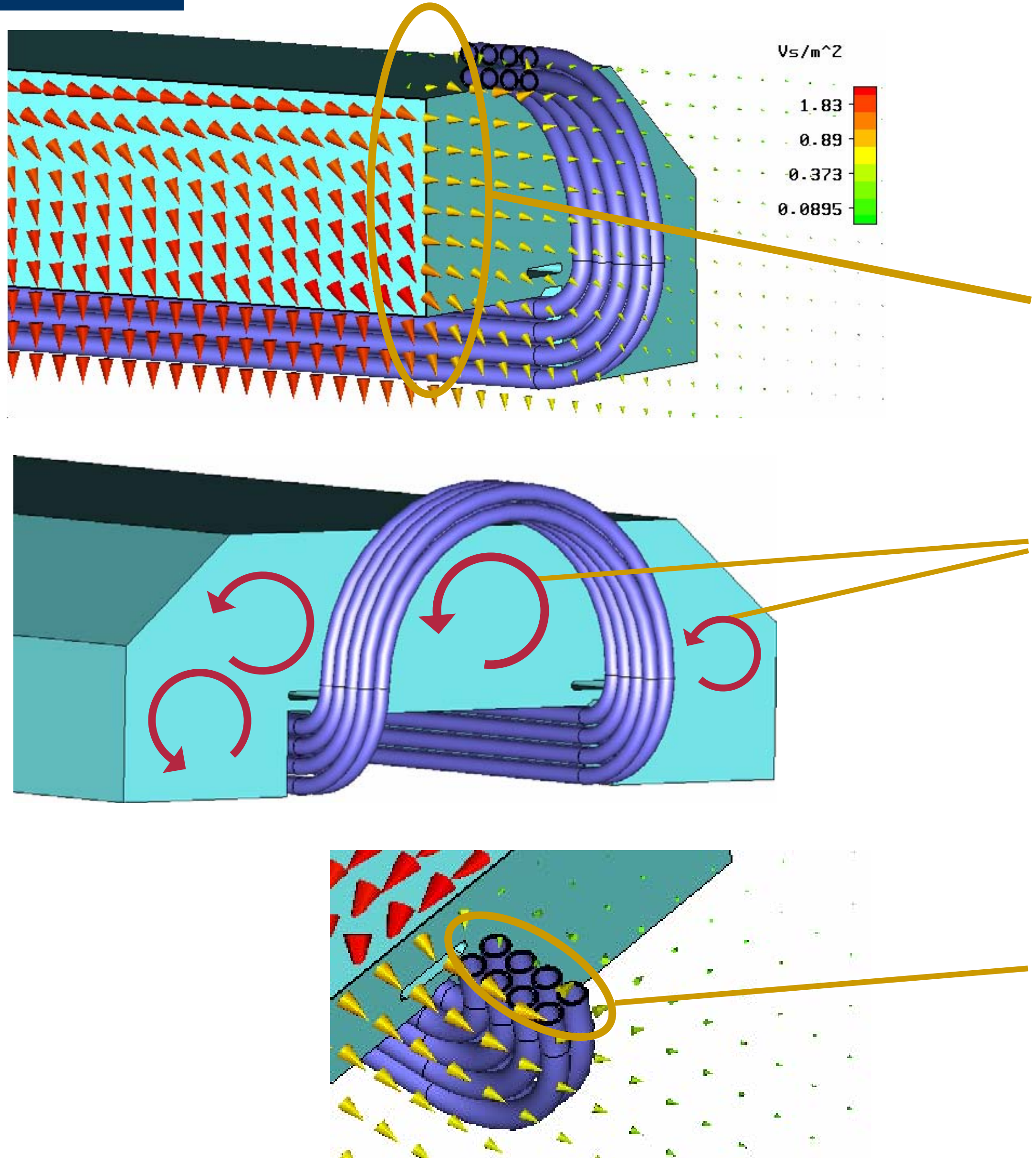
Time-Varying Magnetic Fields



flux perpendicular to the lamination

eddy currents
+ power losses

flux through super-conductive cable

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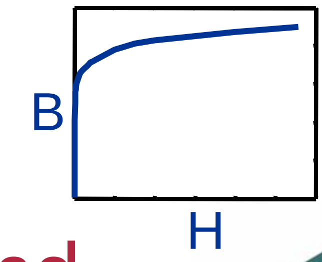
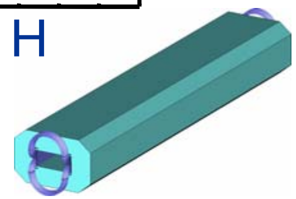
Outline

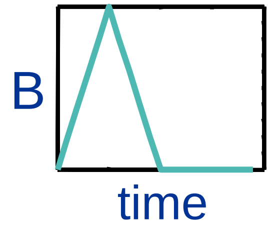
1. Motivation





2. Modelling and Simulation

- homogenisation
- nonlinearity
- cable magnetisation



3. Results

4. Summary

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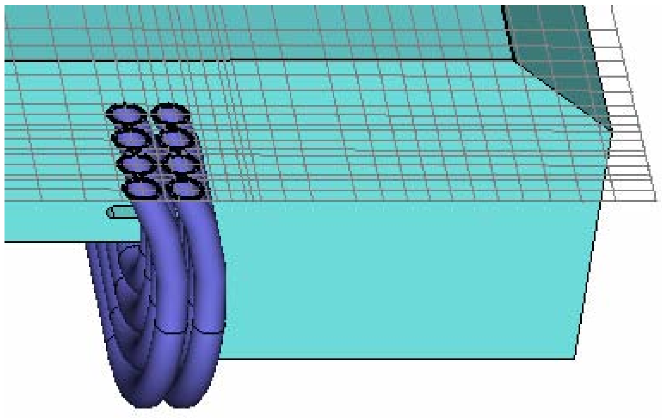
Transient FE/FIT Solver

$$\nabla\times(\overleftrightarrow{\nu}\nabla\times\mathbf{A})+\overleftrightarrow{\sigma}\frac{\partial\mathbf{A}}{\partial t}=\mathbf{J}_s$$

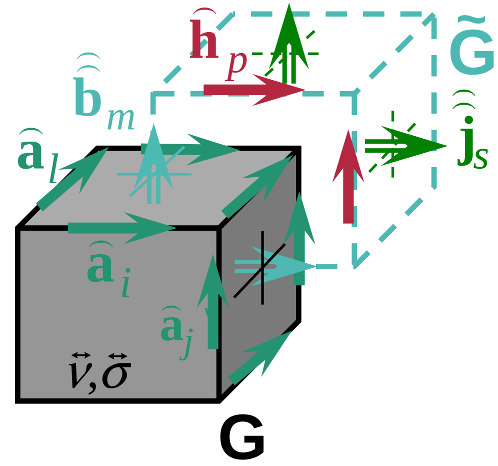


Finite Integration Technique (FIT)

$$\widehat{\widehat{\mathbf{b}}}=\mathbf{C}\widehat{\mathbf{a}}$$

$$\widehat{\mathbf{h}}=\mathbf{M}_{\overleftrightarrow{\nu}}\widehat{\widehat{\mathbf{b}}}$$

$$\widehat{\widehat{\mathbf{j}}}=\tilde{\mathbf{C}}\widehat{\mathbf{h}}$$



$$\tilde{\mathbf{C}}\mathbf{M}_{\overleftrightarrow{\nu}}\mathbf{C}\widehat{\mathbf{a}}+\mathbf{M}_{\overleftrightarrow{\sigma}}\frac{d\widehat{\mathbf{a}}}{dt}=\widehat{\widehat{\mathbf{j}}}_S$$

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Transient FE/FIT Solver

Transient FE/FIT solver

- time integrator: Singly Diagonally Implicit Runge-Kutta 3(2)
  + adaptive time step selection
- non-linear solver: Newton
- system solver: curl-curl algebraic multigrid (ML package)

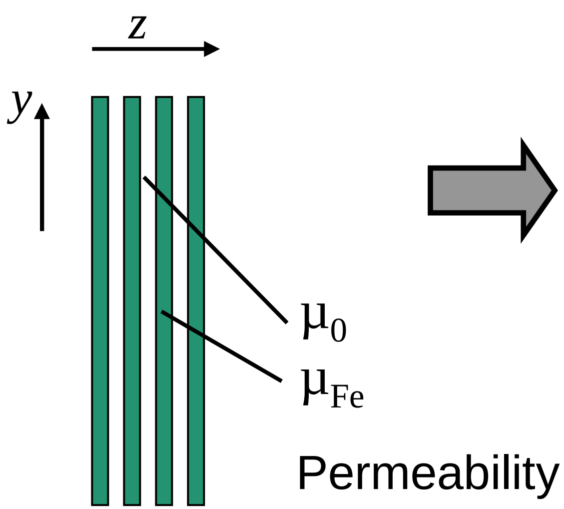
Typical model

- construction/visualisation by commercial tools (CST EMStudio, Matlab)
- 1 million degrees of freedom
- 100-1000 time steps
- one night of computation time

Tim Boonen (KU Leuven): help with the ML-package

Influence of the Lamination

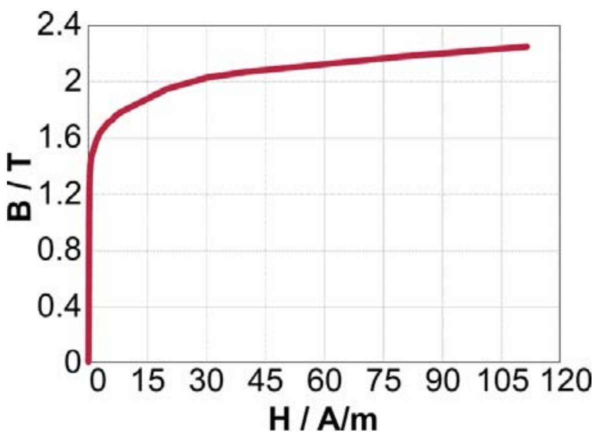
1. Lamination

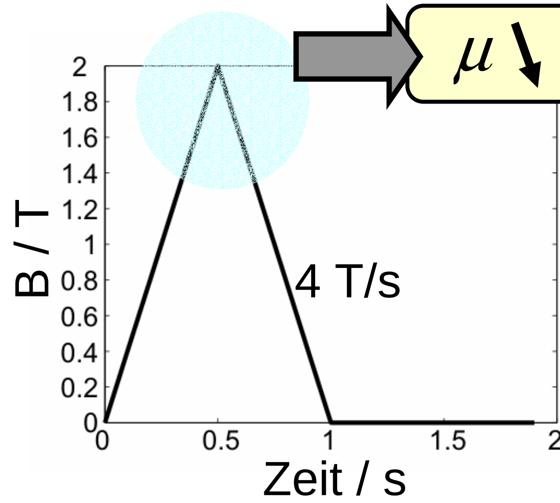


$\mu_{xy} \searrow$ $\mu_z \searrow$

$$\delta = \sqrt{\frac{2}{\omega \mu_{xy} \sigma_{xy}}} \nearrow$$

2. Ferromagnetic saturation





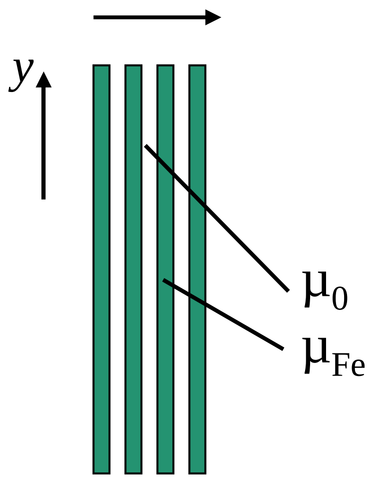
$\mu \searrow$ $\delta \nearrow$

larger skin depth

Homogenisation (1)

$$\nabla \times \left(\overleftrightarrow{\nu} \nabla \times \mathbf{A} \right) + \overleftrightarrow{\sigma} \frac{\partial \mathbf{A}}{\partial t} = \mathbf{J}_s$$

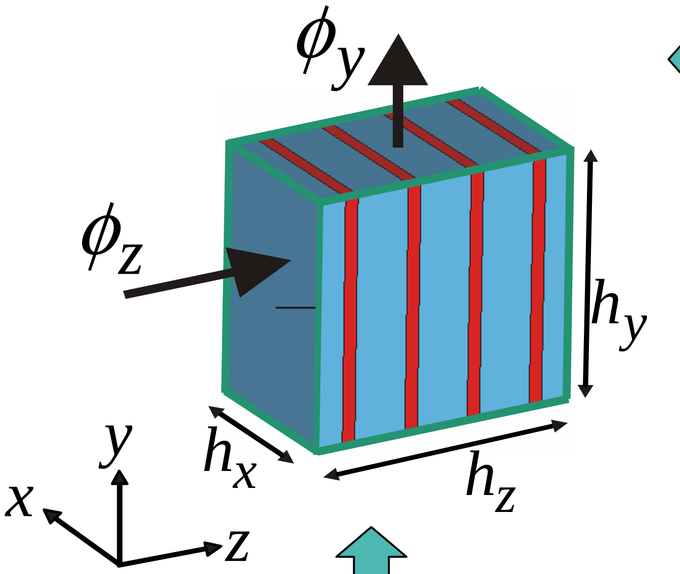
$$\overleftrightarrow{\sigma} = \begin{bmatrix} \sigma & & \\ & \sigma & \\ & & 0 \end{bmatrix}$$

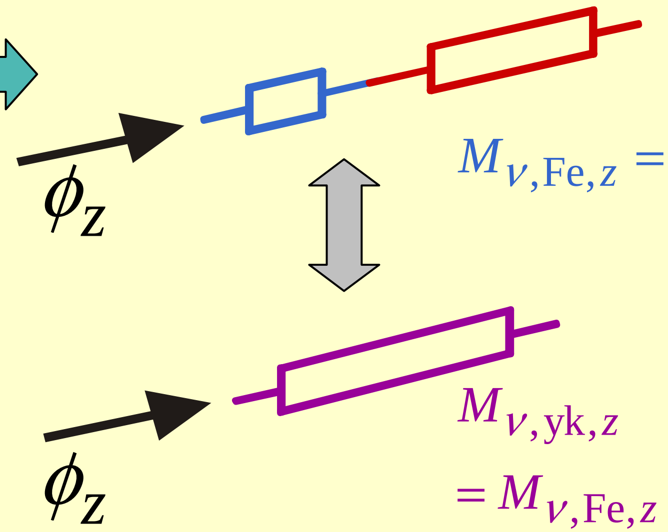


$$\overleftrightarrow{\nu} = \begin{bmatrix} \nu_{xy} & & \\ & \nu_{xy} & \\ & & \nu_z \end{bmatrix}$$

Homogenisation (2)

$$\nabla\times(\overleftrightarrow{\nu}\nabla\times\mathbf{A})=\mathbf{J}_s$$





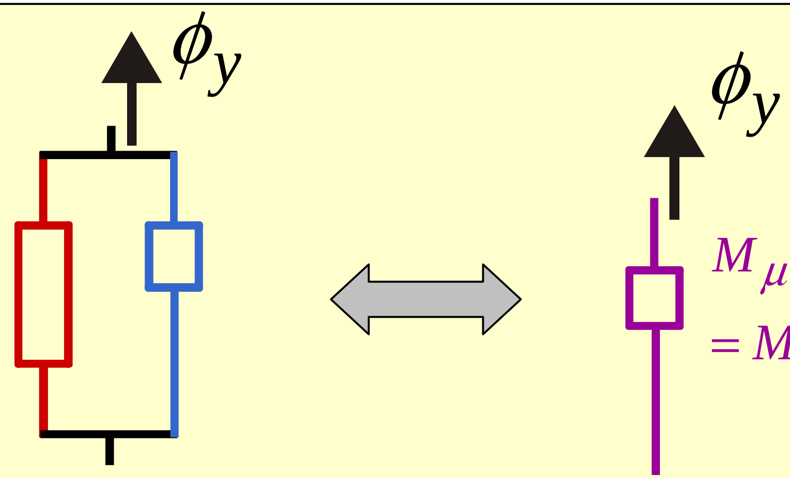
$$M_{\nu,\mathrm{ct},z}=\frac{\nu_{\mathrm{ct}}(1-\gamma_{\mathrm{st}})h_z}{h_x h_y}$$

$$M_{\nu,\mathrm{Fe},z}=\frac{\nu_{\mathrm{Fe}}\gamma_{\mathrm{st}}h_z}{h_x h_y}$$

$$M_{\nu,\mathrm{yk},z}=M_{\nu,\mathrm{Fe},z}+M_{\nu,\mathrm{ct},z}$$

$$M_{\mu,\mathrm{Fe},y}=\frac{\mu_{\mathrm{Fe}}\gamma_{\mathrm{st}}h_x h_y}{h_z}$$

$$M_{\mu,\mathrm{ct},y}=\frac{\mu_{\mathrm{ct}}(1-\gamma_{\mathrm{st}})h_x h_y}{h_z}$$



$$M_{\mu,\mathrm{yk},y}=M_{\mu,\mathrm{Fe},y}+M_{\mu,\mathrm{ct},y}$$

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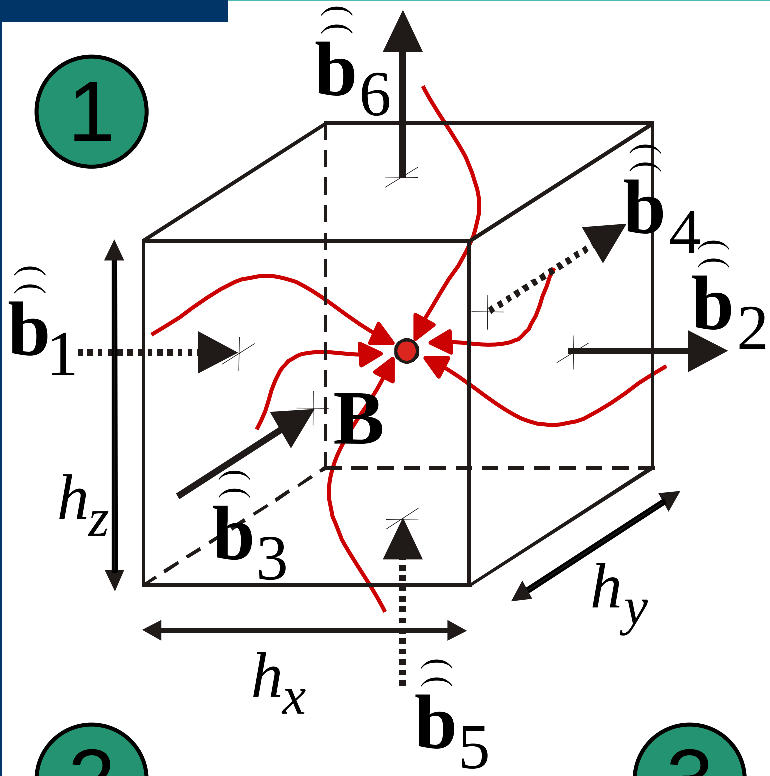
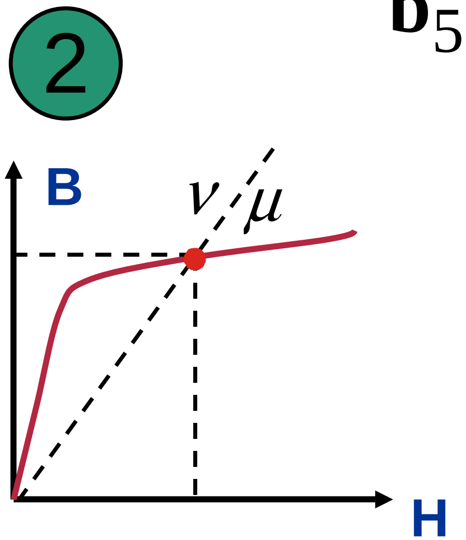
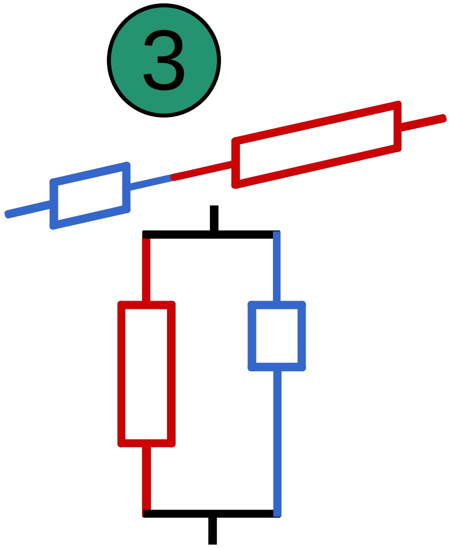
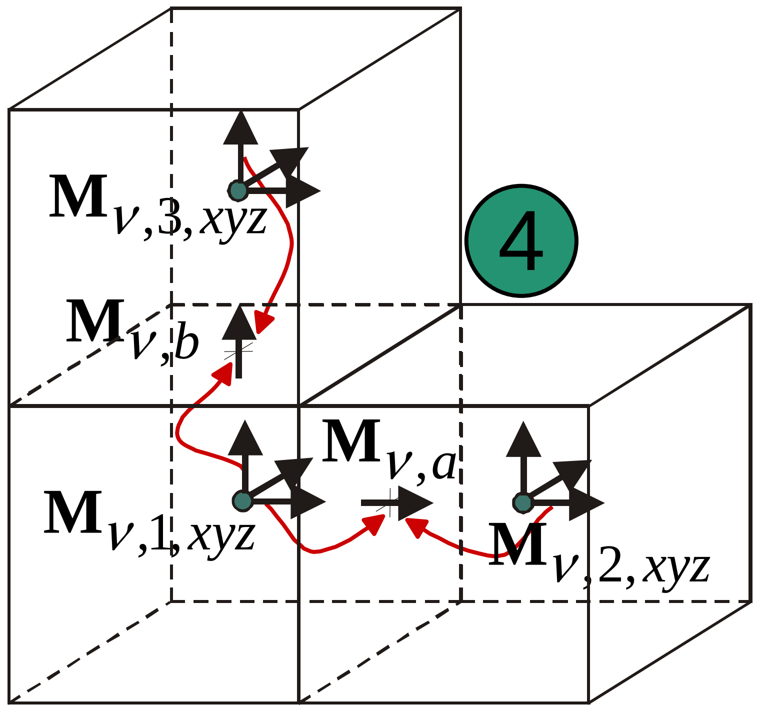
Reluctance Update

1. average **B** on primary grid cells
2. evaluate B-H characteristic
3. apply series/parallel connection
4. average $\mathbf{M}_{\nu,a}$ at the primary facets (= dual edges)









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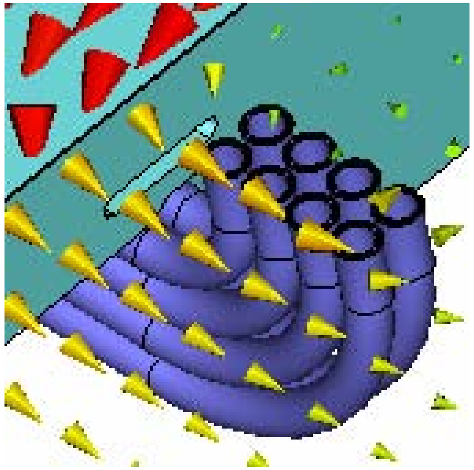
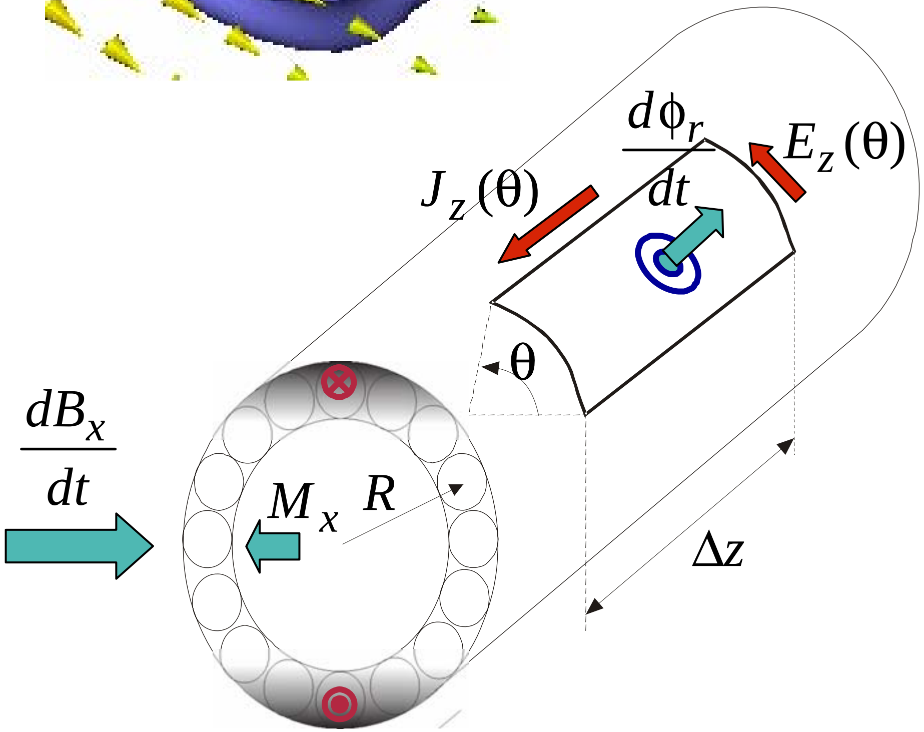
Cable Magnetisation (1)





time-varying magnetic field $\frac{dB_x}{dt}$

↓

induced electric field $E_z(\theta)$

↓

induced current density $J_z(\theta)$

↓

induced magnetisation M_x

cable magnetisation

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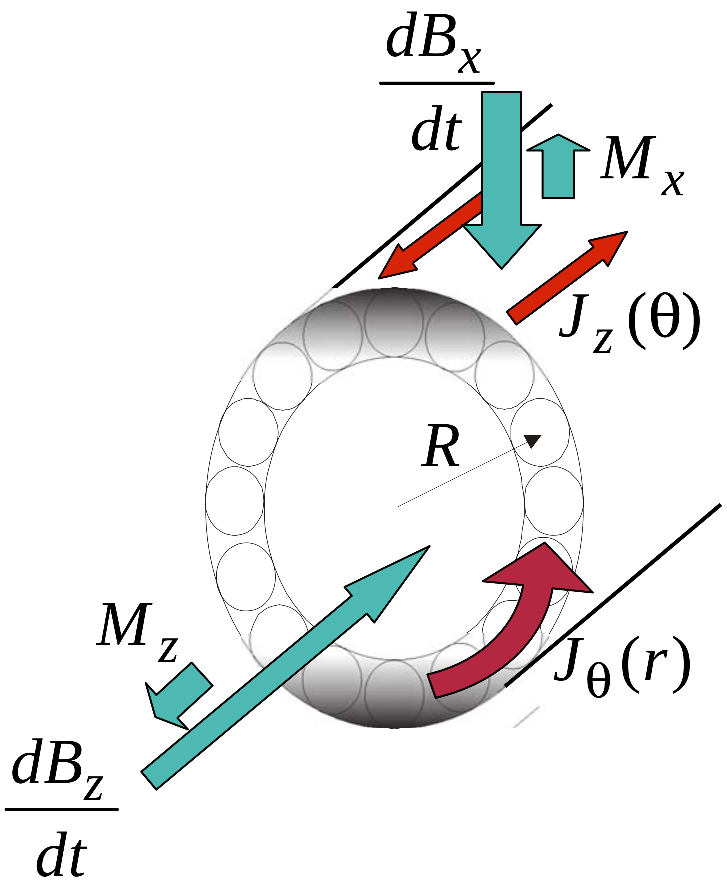
Cable Magnetisation (2)



$$\begin{cases} M_x = -\tau_{\mathrm{cb},xy} \dfrac{dB_x}{dt} \\ M_y = -\tau_{\mathrm{cb},xy} \dfrac{dB_y}{dt} \\ M_z = -\tau_{\mathrm{cb},z} \dfrac{dB_z}{dt} \end{cases}$$

$$\vec{\vec{\tau}}_{\mathrm{cb}} = \begin{bmatrix} \tau_{\mathrm{cb},xy} & & \\ & \tau_{\mathrm{cb},xy} & \\ & & \tau_{\mathrm{cb},z} \end{bmatrix}$$

magnetising current :

$$\mathbf{J}_{\mathrm{m}} = -\nabla \times \left(\nu_0 \vec{\vec{\tau}}_{\mathrm{cb}} \nabla \times \frac{\partial \mathbf{A}}{\partial t} \right)$$

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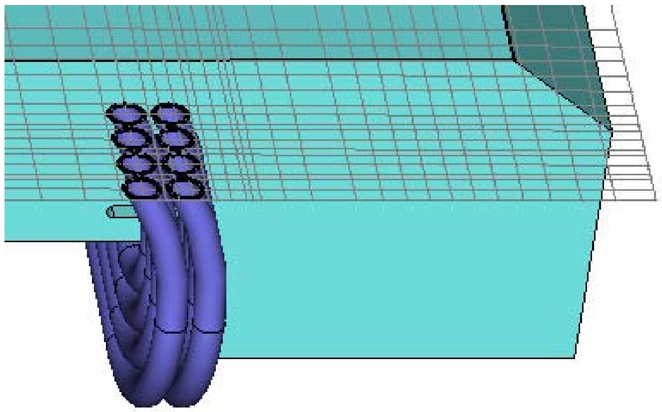
Transient FE/FIT Solver

$$\nabla \times (\vec{\vec{\nu}} \nabla \times \mathbf{A}) + \vec{\vec{\sigma}} \frac{\partial \mathbf{A}}{\partial t} + \nabla \times \left(\nu_0 \vec{\vec{\tau}}_{\mathrm{cb}} \nabla \times \frac{\partial \mathbf{A}}{\partial t} \right) = \mathbf{J}_s$$

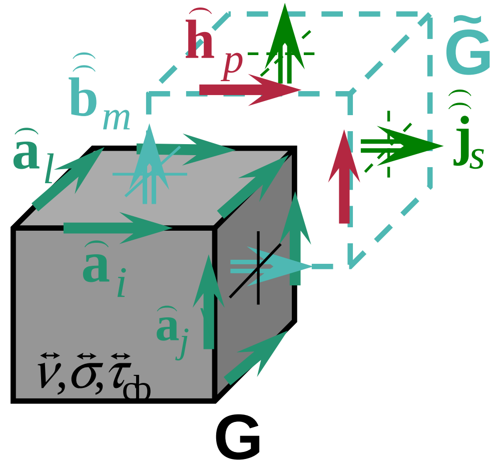


Finite Integration Technique (FIT)

$$\widehat{\widehat{\mathbf{b}}} = \mathbf{C}\widehat{\mathbf{a}}$$

$$\widehat{\mathbf{h}} = \mathbf{M}_{\vec{\vec{\nu}}}\widehat{\widehat{\mathbf{b}}}$$

$$\widehat{\widehat{\mathbf{j}}} = \tilde{\mathbf{C}}\widehat{\mathbf{h}}$$



$$\tilde{\mathbf{C}}\mathbf{M}_{\vec{\vec{\nu}}}\mathbf{C}\widehat{\mathbf{a}} + \mathbf{M}_{\vec{\vec{\sigma}}} \frac{d\widehat{\mathbf{a}}}{dt} + \tilde{\mathbf{C}} \nu_0 \mathbf{M}_{\vec{\vec{\tau}}_{\mathrm{cb}}} \mathbf{C} \frac{d\widehat{\mathbf{a}}}{dt} = \widehat{\widehat{\mathbf{j}}}_S$$

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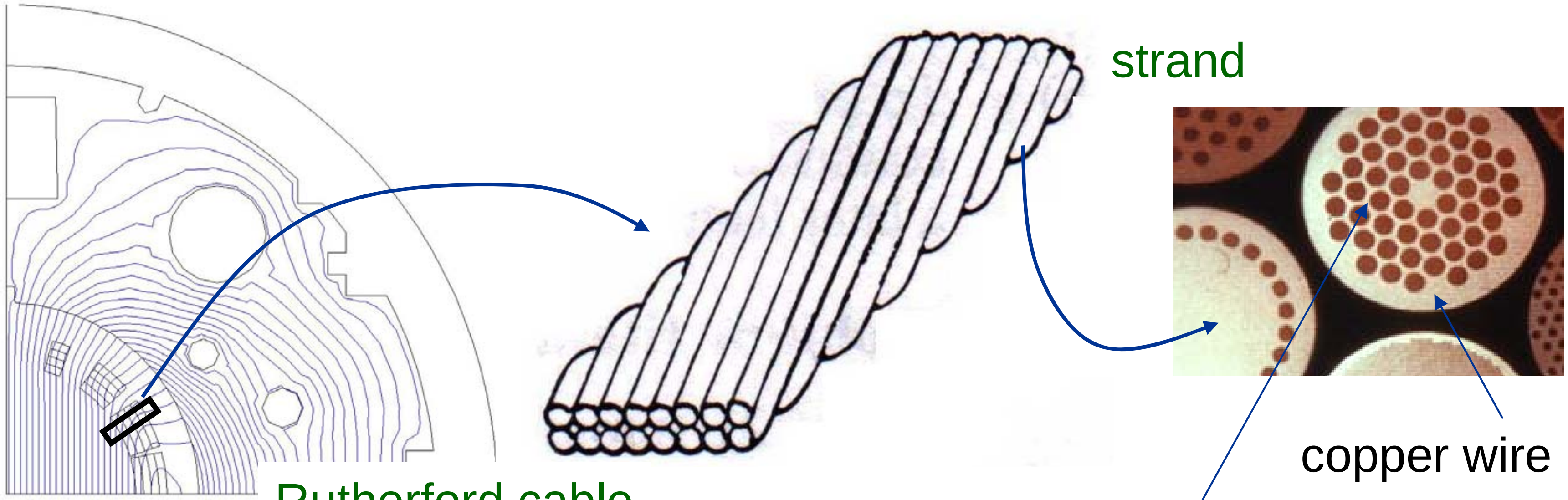
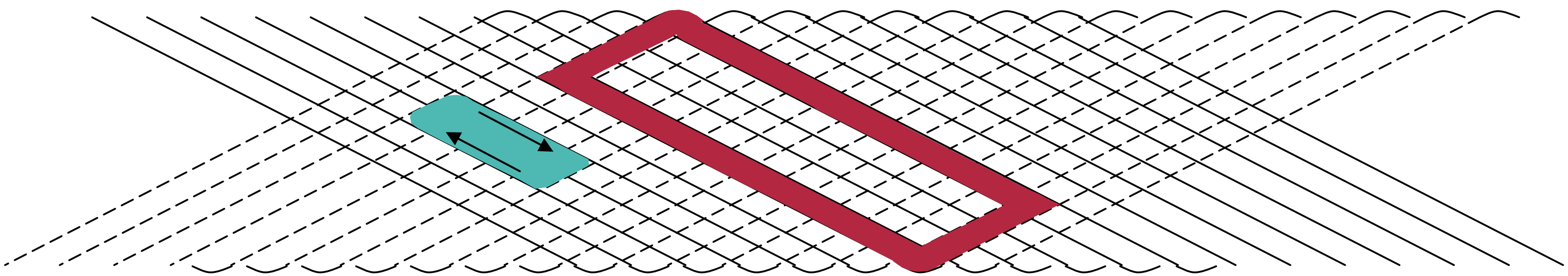
Rutherford Cable (1)





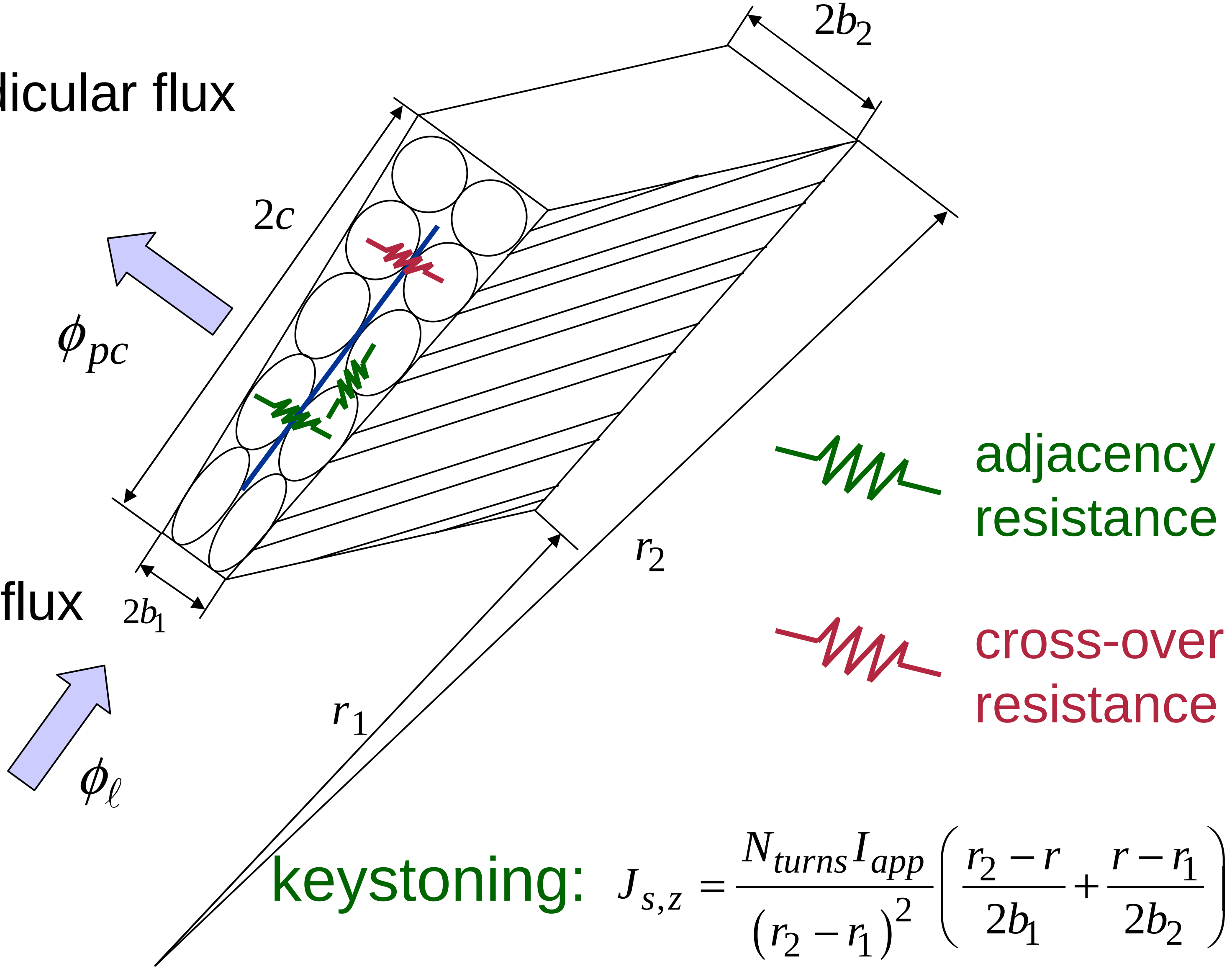
Rutherford Cable (2)

perpendicular flux

$B_p(r,\theta)$

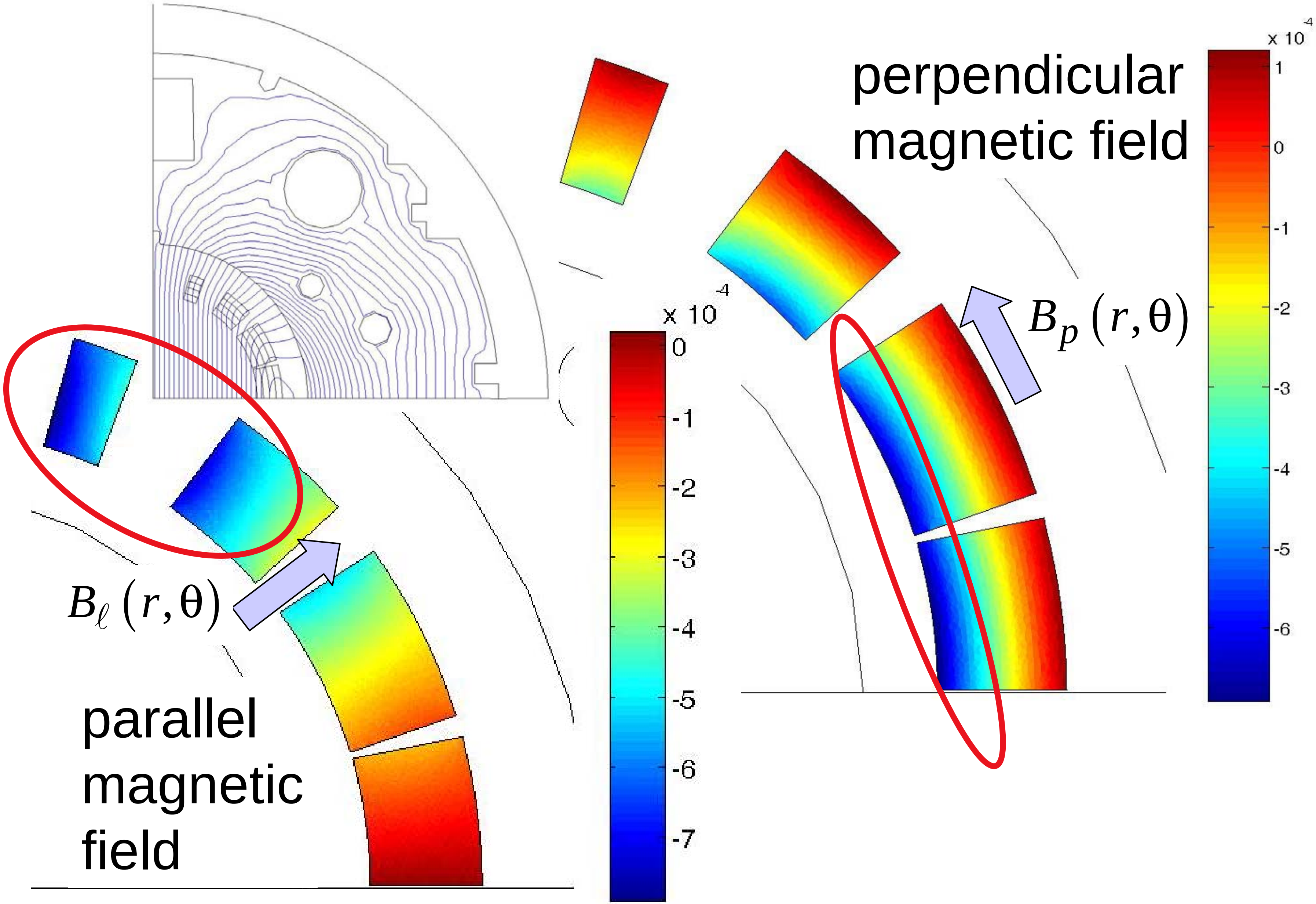
parallel flux

$B_\ell(r,\theta)$



keystoning: $J_{s,z} = \frac{N_{turns} I_{app}}{(r_2 - r_1)^2} \left(\frac{r_2 - r}{2b_1} + \frac{r - r_1}{2b_2} \right)$

Magnetic Flux Density



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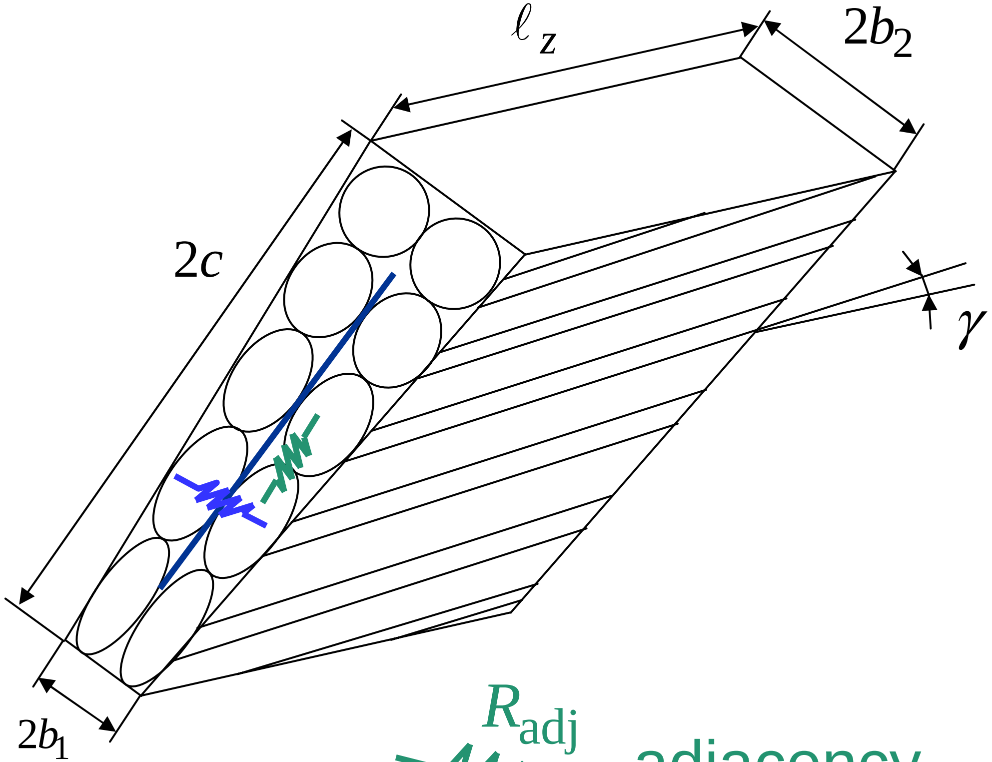
Adjacency Eddy Currents (1)



$$b = \frac{b_1 + b_2}{2}$$

adjacency resistance

adjacency resistance (+ core)

$$\vec{\vec{\sigma}}_{\text{cl}} = \begin{bmatrix} \sigma_r & & \\ & \sigma_\theta & \\ & & \sigma_z \end{bmatrix}$$

$$\begin{cases} \sigma_r = \dfrac{1}{R_{\text{adj}} \cos\gamma} \dfrac{b\ell_z}{2c} \\ \sigma_\theta = \dfrac{1}{R_{\text{core}}} \dfrac{2c\ell_z}{b} \\ \sigma_z = \dfrac{1}{R_{\text{adj}} \sin\gamma} \dfrac{2cb}{\ell_z} \end{cases}$$

Adjacency Eddy Currents (2)

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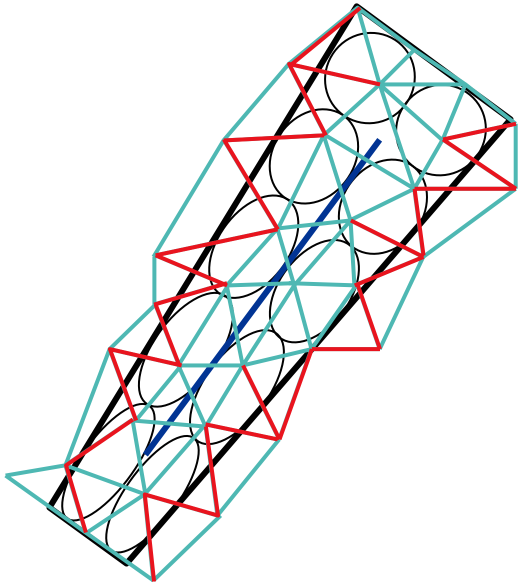
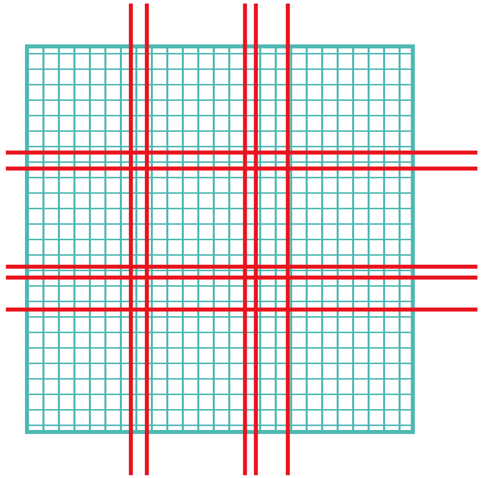


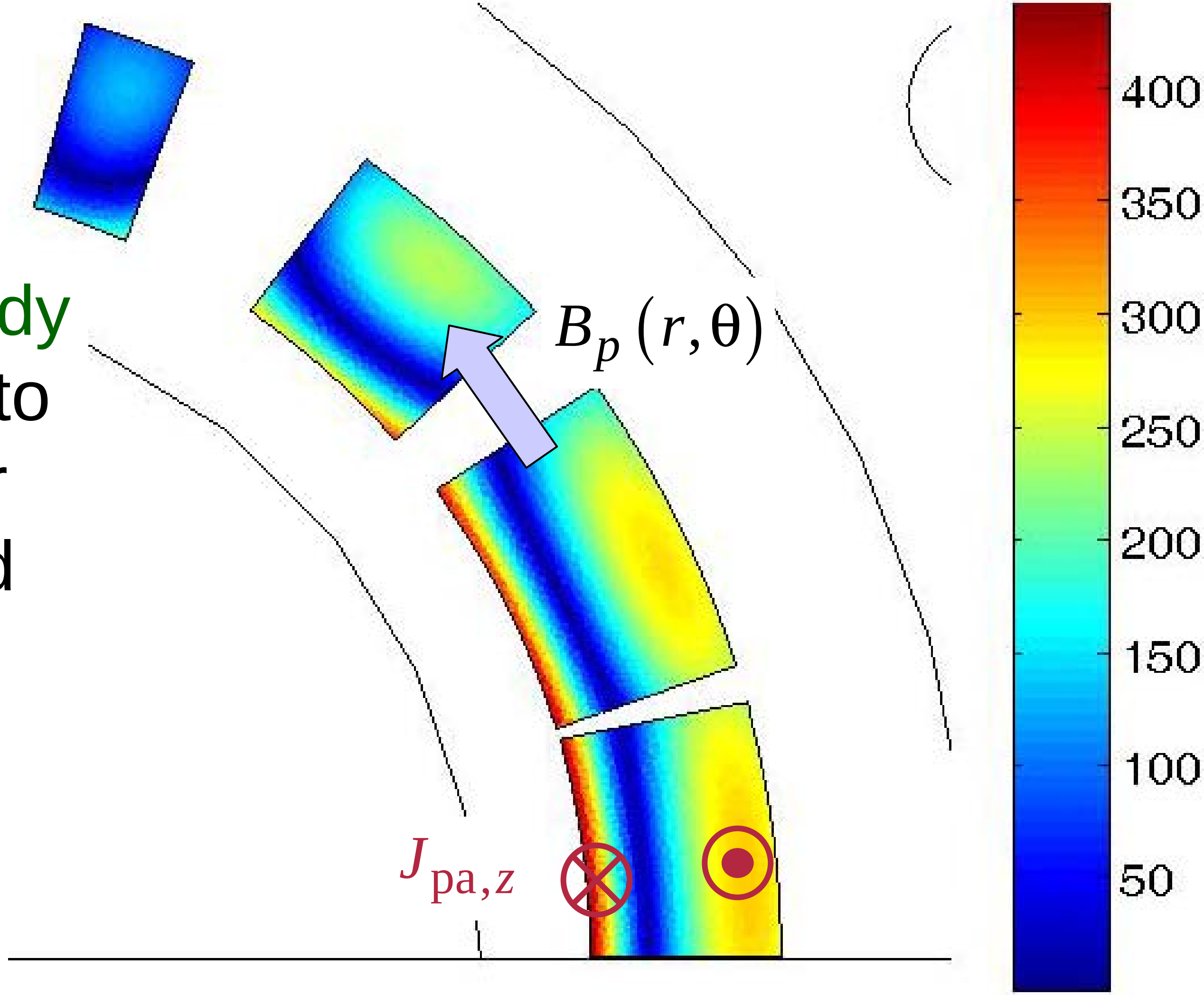
$$M^{(\mathrm{fe})}_{\vec{\vec{\sigma}}_{\mathrm{cl}},p,q} = \int_{\Omega} \vec{z}_p \cdot \vec{\vec{\sigma}}_{\mathrm{cl}} \vec{z}_q \, \mathrm{d}\Omega$$

$$M^{(\mathrm{fe})}_{\vec{\vec{\sigma}}_{\mathrm{cl}},p,q} =$$

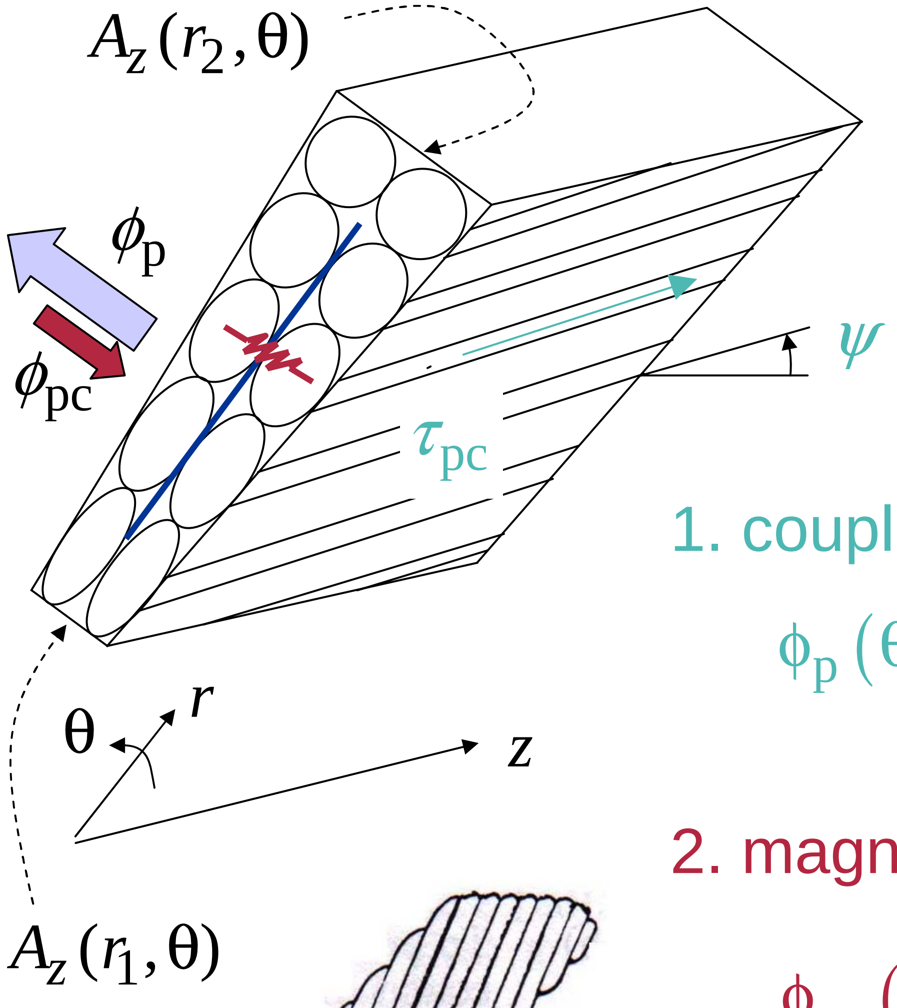


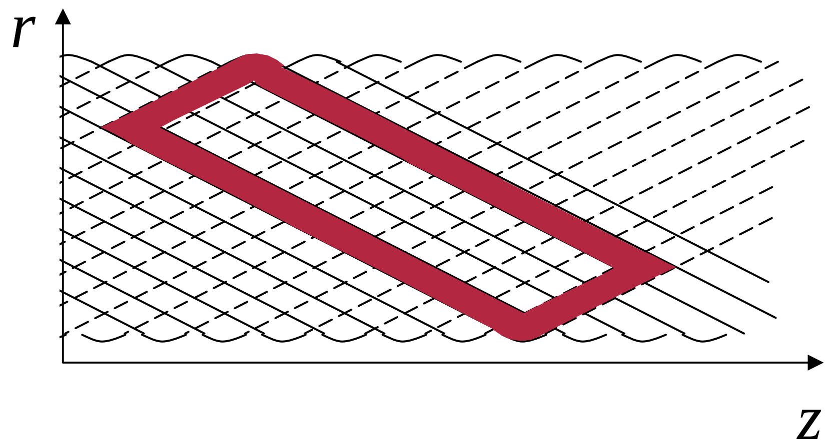
Adjacency Eddy Currents (4)

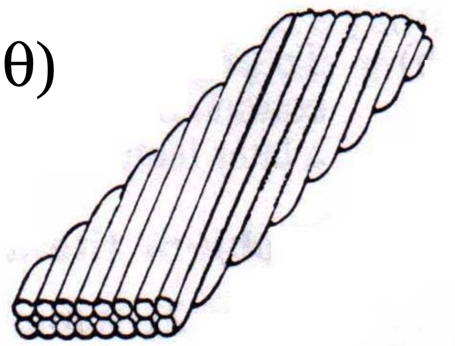
adjacency eddy currents due to perpendicular magnetic field



Cross-Over Eddy Currents (1)







1. coupled flux

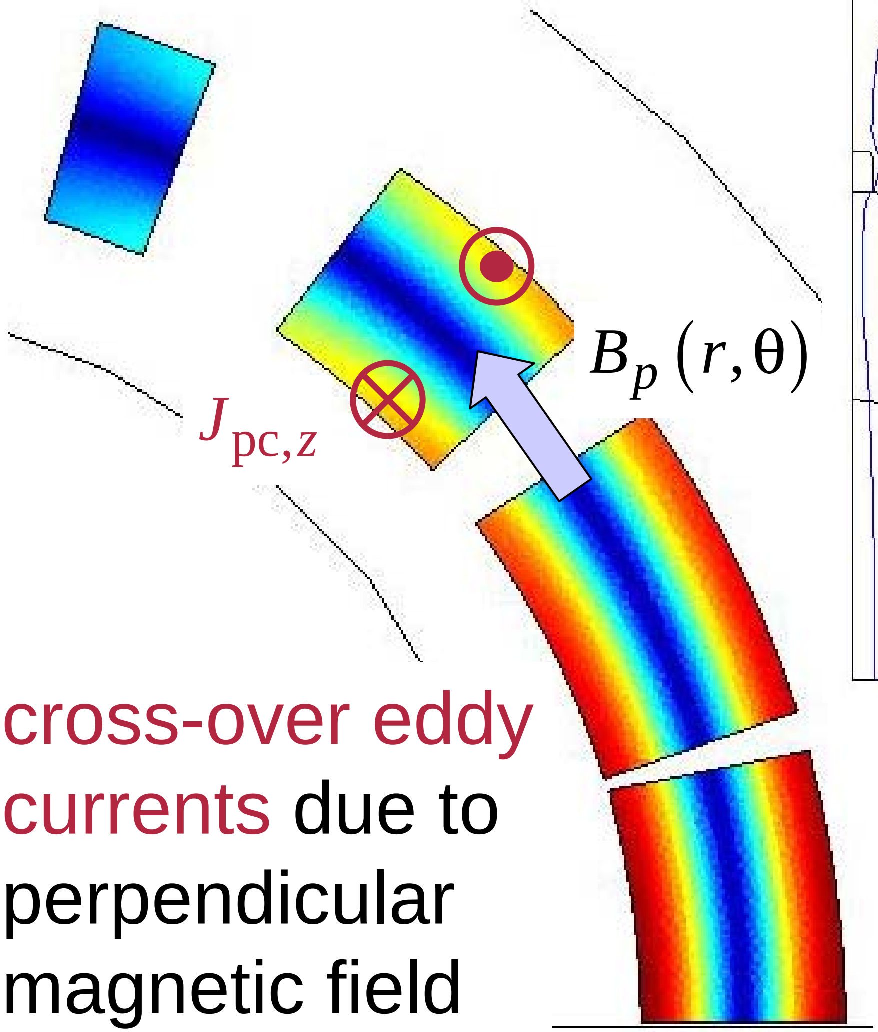
$$\phi_p(\theta) = \ell_z\left(A_z(r_2,\theta) - A_z(r_1,\theta)\right)$$

2. magnetisation

$$\phi_{pc}(\theta) = \tau_{pc}\frac{\partial\phi_p(\theta)}{\partial t}$$

time constant

Cross-Over Eddy Currents (2)



cross-over eddy currents due to perpendicular magnetic field

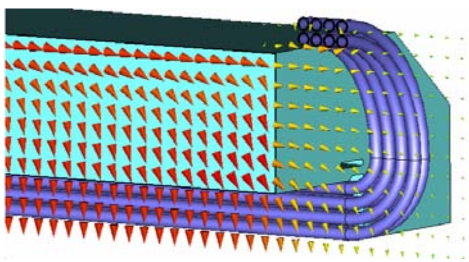
cross-over magnetisation

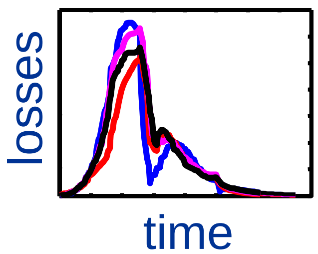
Outline

1. Motivation

2. Modelling and Simulation

3. Results





4. Summary

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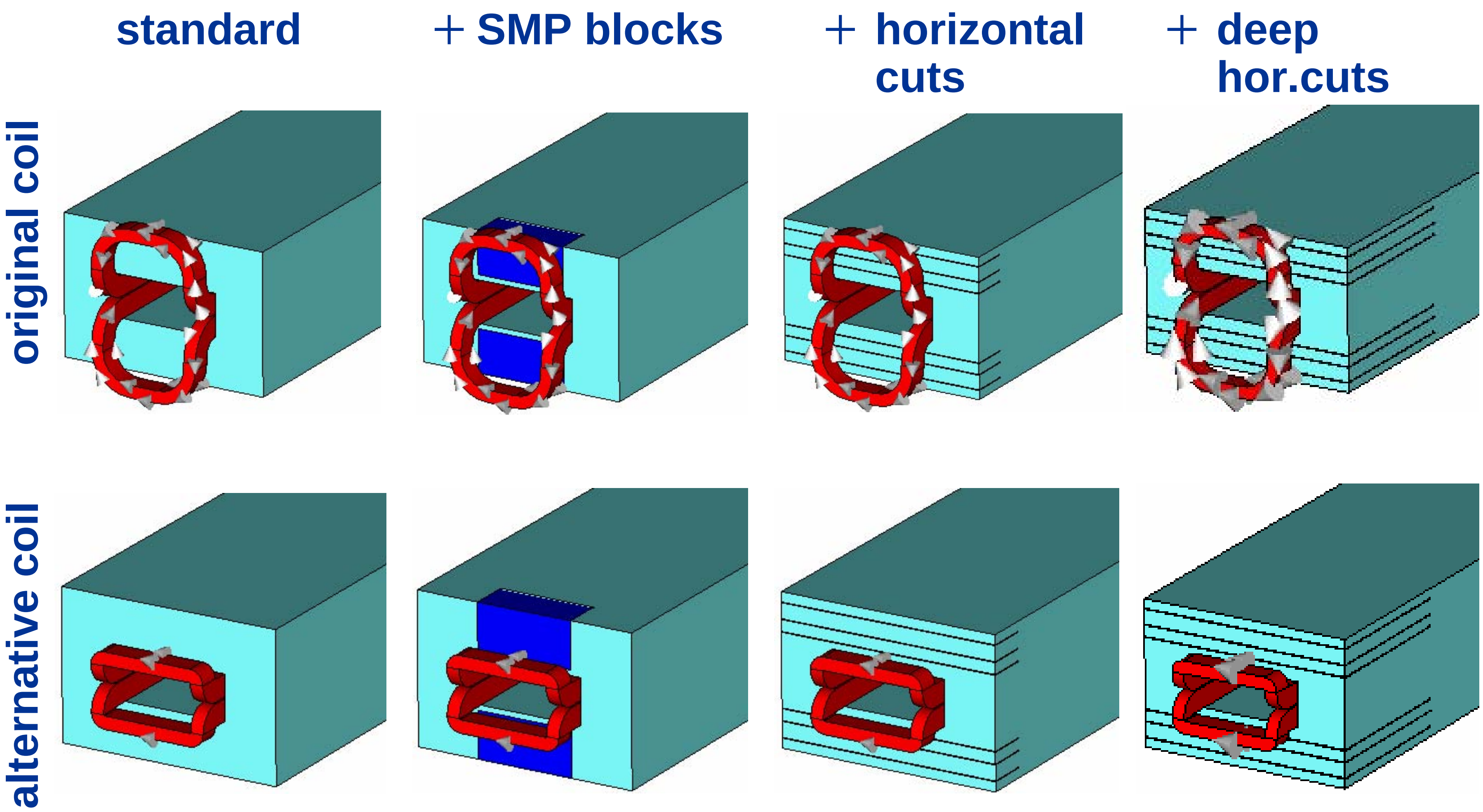
Computed Configurations



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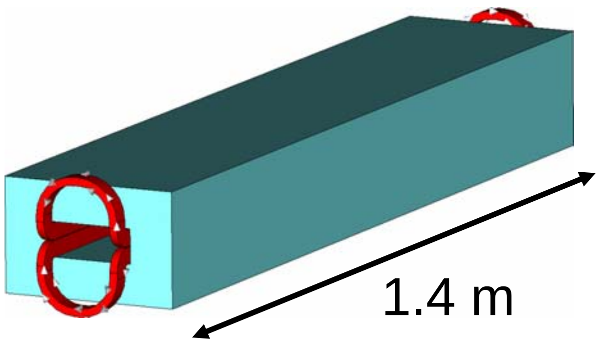
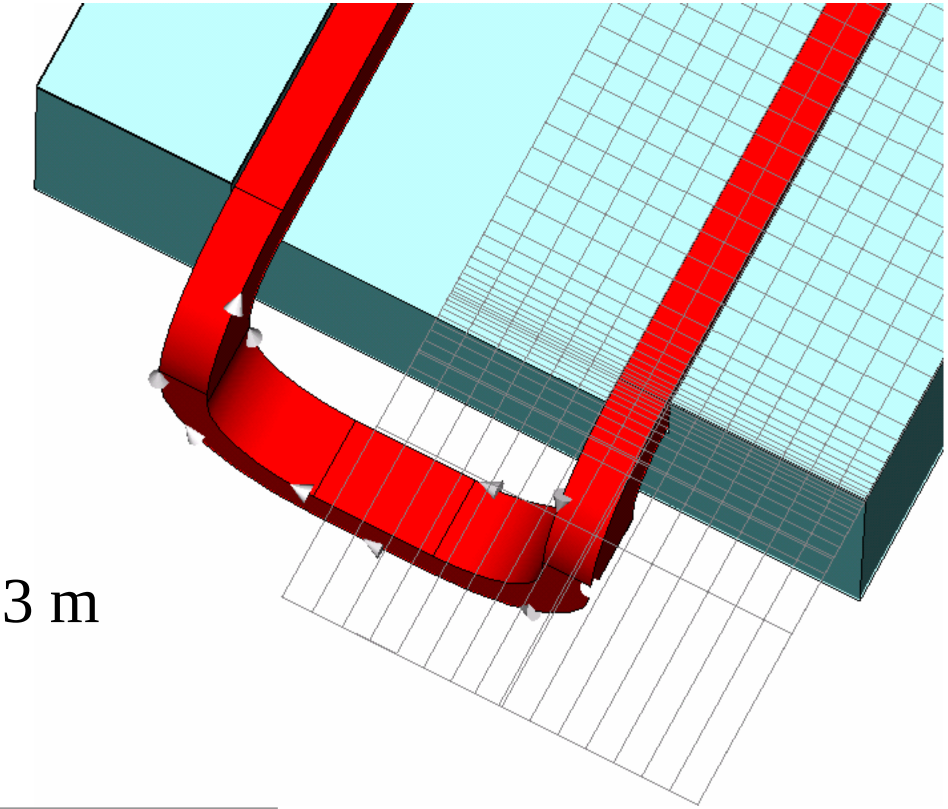
Meshing



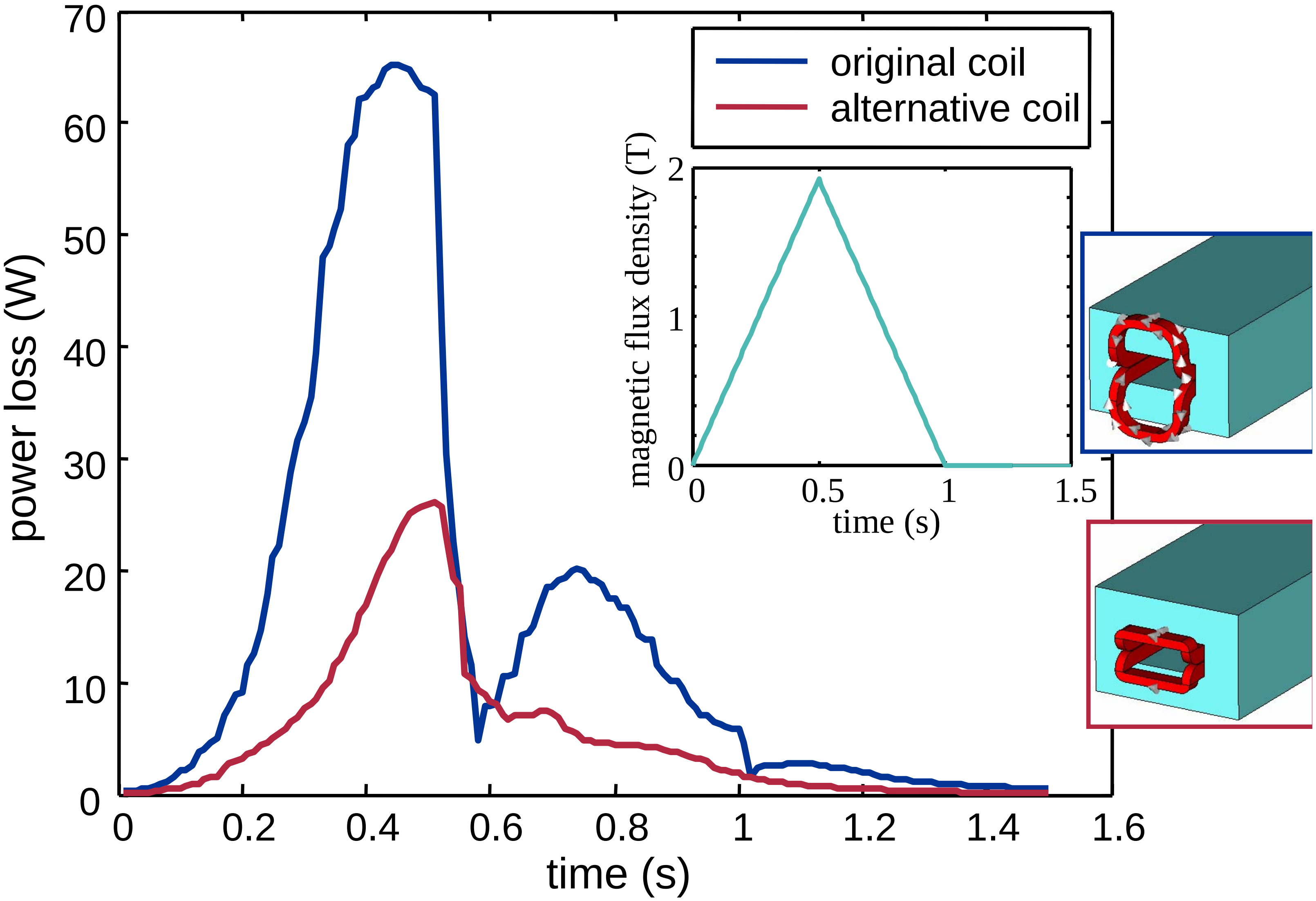


$$\delta_{\text{eddy}} = \sqrt{\frac{2}{\omega \mu_{xy} \sigma_{xy}}} = 0.003\ \text{m}$$

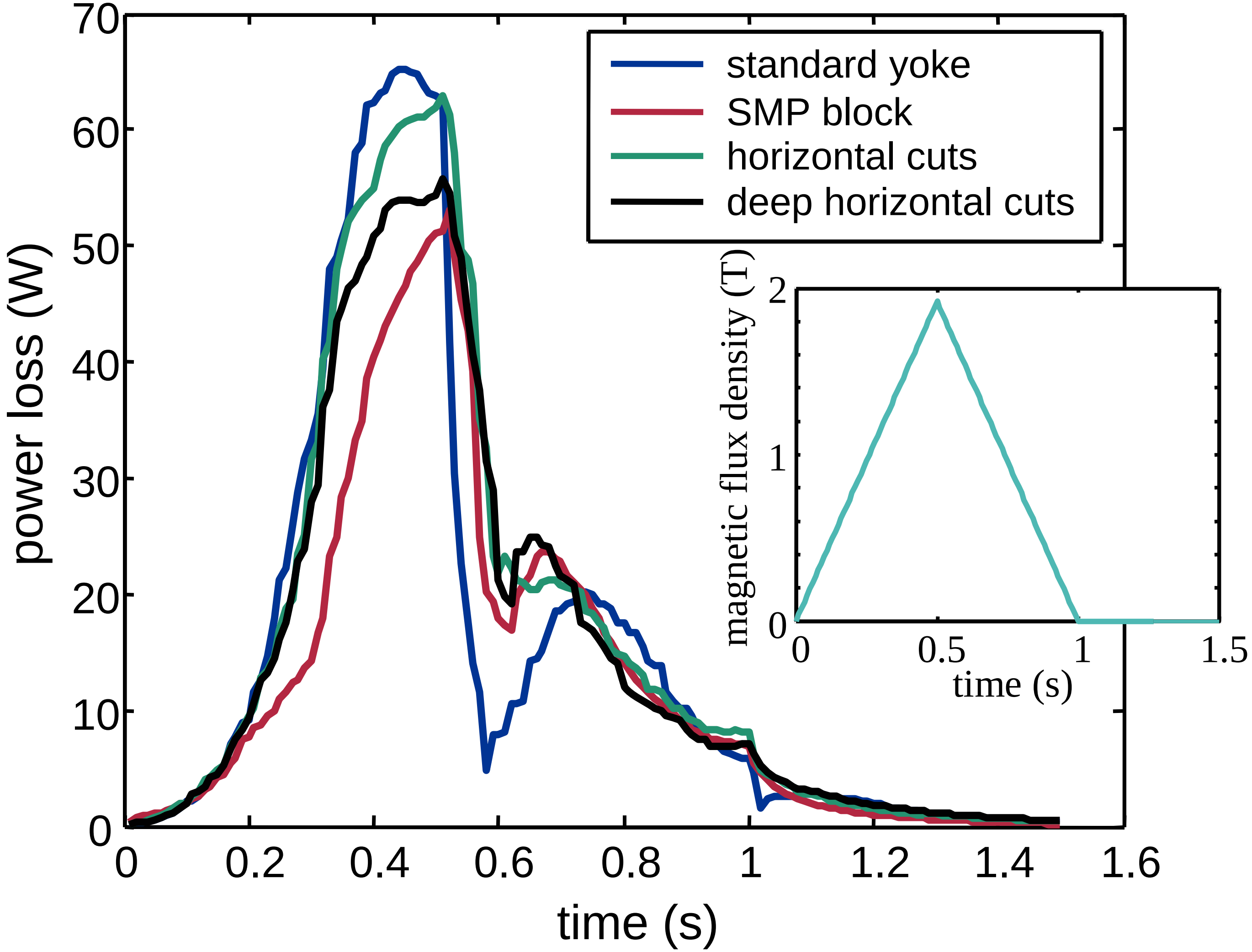
Skin depth has to be resolved!

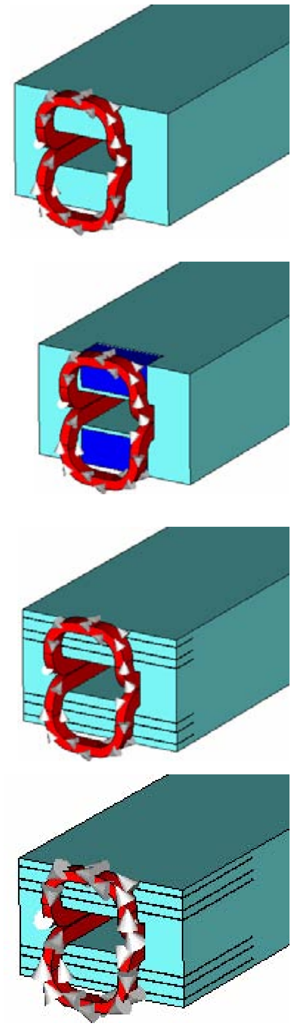
→ graded mesh in z-direction

Power Losses (1)

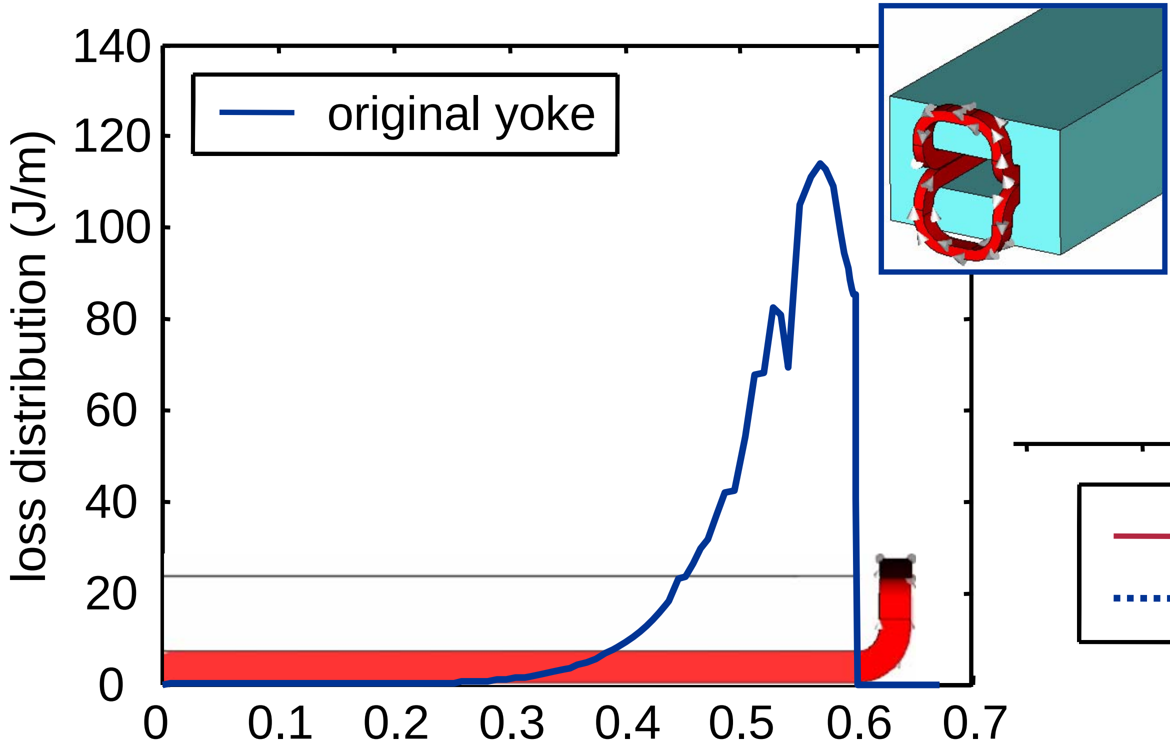


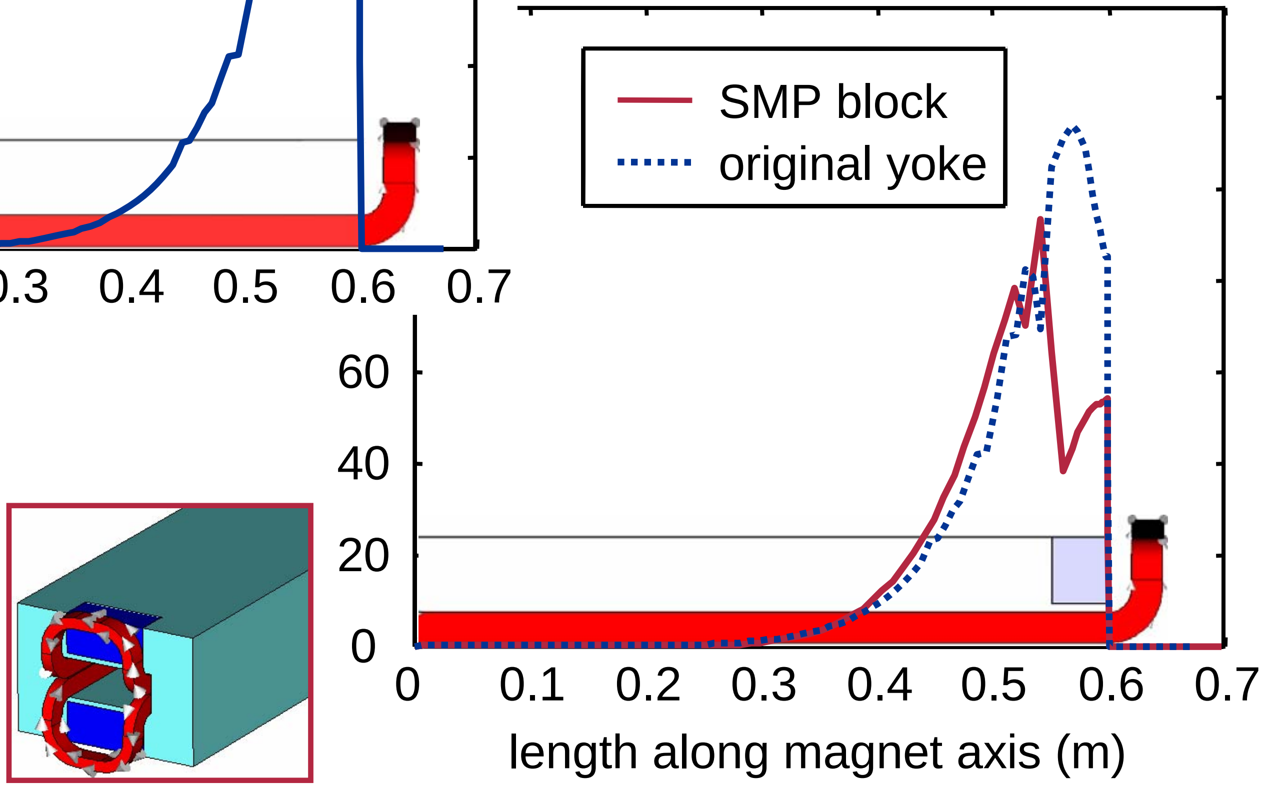
Power Losses (2)



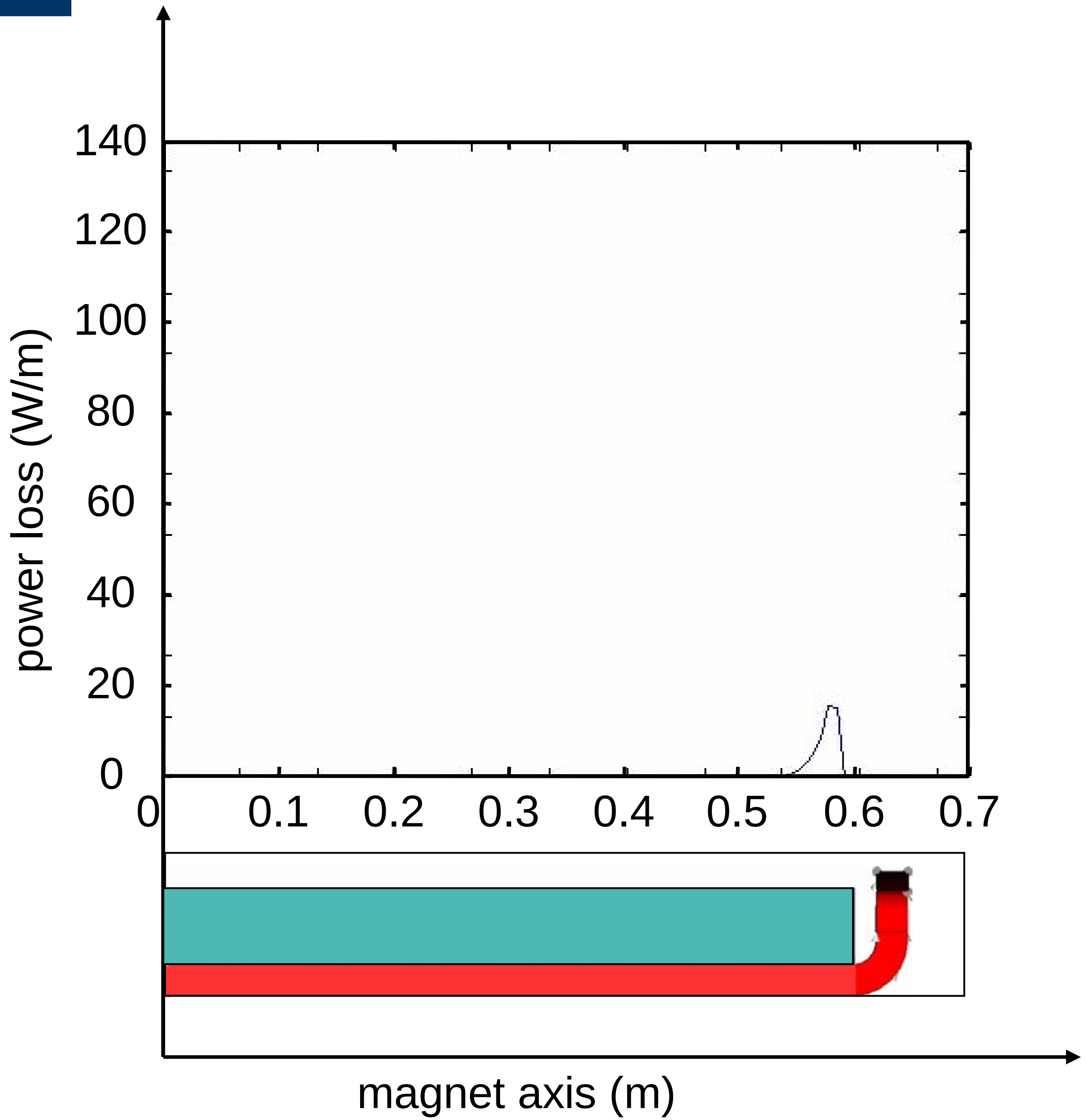


Loss Distribution





Power Losses (3)



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Summary and Outlook

3D transient, nonlinear simulation of a superconductive magnet

- ✓ homogenisation of the yoke lamination
- ✓ eddy currents in the yoke
- ✓ cable magnetisation
- ✓ different designs for the end plates

Future work:

- ➢ comparison with measurements
- ➢ increase resolution in both time and space
- ➢ repeat simulations with unstructured grids (FE)

Cable Models

1. cable magnetisation model

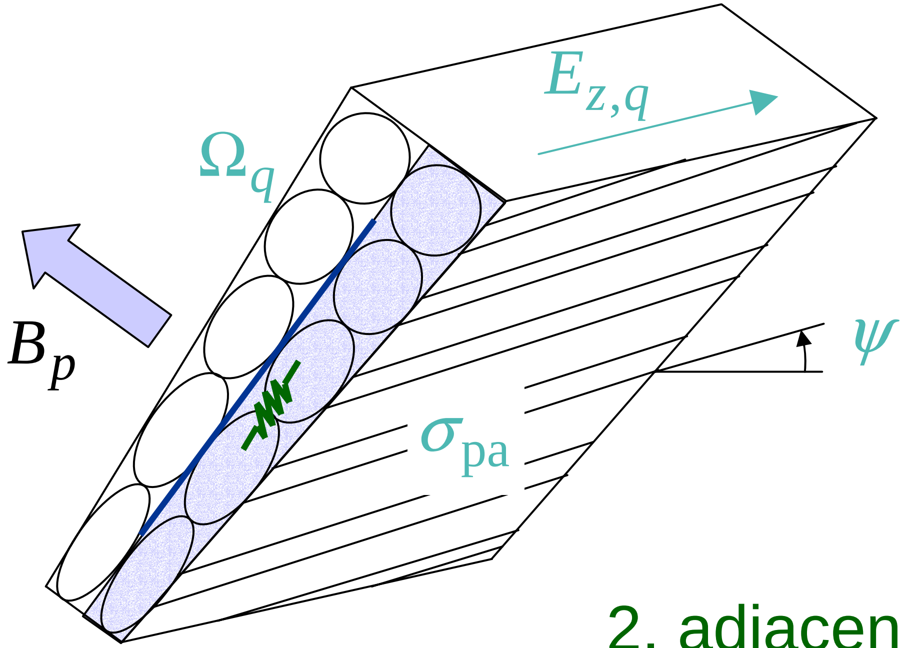
$$\tilde{\mathbf{C}}\mathbf{M}_{\vec{\nu}}\mathbf{C}\widehat{\mathbf{a}} + \mathbf{M}_{\vec{\sigma}}\frac{d\widehat{\mathbf{a}}}{dt} + \tilde{\mathbf{C}}\,\nu_0\mathbf{M}_{\vec{\tau}_{\mathrm{cb}}}\mathbf{C}\frac{d\widehat{\mathbf{a}}}{dt} = \widehat{\widehat{\mathbf{j}}}_s$$

2. cable eddy-current model

$$\begin{bmatrix} \tilde{\mathbf{C}}\mathbf{M}_{\vec{\nu}}\mathbf{C} + \mathbf{M}_{\vec{\sigma}}\frac{d}{dt} + \mathbf{M}_{\vec{\sigma}_{\mathrm{cl}}}\frac{d}{dt} & -\mathbf{M}_{\vec{\sigma}_{\mathrm{cl}}}\mathbf{Q}_{\mathrm{e}}^T \\ -\mathbf{Q}_{\mathrm{e}}\mathbf{M}_{\vec{\sigma}_{\mathrm{cl}}} & \underbrace{\mathbf{Q}_{\mathrm{e}}\mathbf{M}_{\vec{\sigma}_{\mathrm{cl}}}\mathbf{Q}_{\mathrm{e}}^T}_{\mathbf{G}_{\mathrm{e}}} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{a}} \\ \widehat{\mathbf{e}}_{\mathrm{e}} \end{bmatrix} = \begin{bmatrix} \widehat{\widehat{\mathbf{j}}}_s \\ 0 \end{bmatrix}$$

$$\tilde{\mathbf{C}}\mathbf{M}_{\vec{\nu}}\mathbf{C}\widehat{\mathbf{a}} + \mathbf{M}_{\vec{\sigma}}\frac{d\widehat{\mathbf{a}}}{dt} + \left(\mathbf{M}_{\vec{\sigma}_{\mathrm{cl}}} - \mathbf{M}_{\vec{\sigma}_{\mathrm{cl}}}\mathbf{Q}_{\mathrm{e}}^T\mathbf{G}_{\mathrm{e}}^{-1}\mathbf{Q}_{\mathrm{e}}\mathbf{M}_{\vec{\sigma}_{\mathrm{cl}}}\right)\frac{d\widehat{\mathbf{a}}}{dt} = \widehat{\widehat{\mathbf{j}}}_s$$

Adjacency Eddy Currents (1)



due to perpendicular magnetic field

1. additional discretisation for unknown electric field $E_{z,q}$:

$$E_z(x,y) = \sum_q E_{z,q} M_q(x,y)$$

2. adjacency eddy current density :

$$J_{pa,z}(r,\theta) = \sigma_{pa} E_{z,q} - \sigma_{pa} \frac{\partial A_z}{\partial t}$$

3. netto current through Ω_q = 0

$$I_{z,q} = \int_{\Omega_q} J_{pa,z}(r,\theta)\, d\Omega = 0$$

additional constraint !

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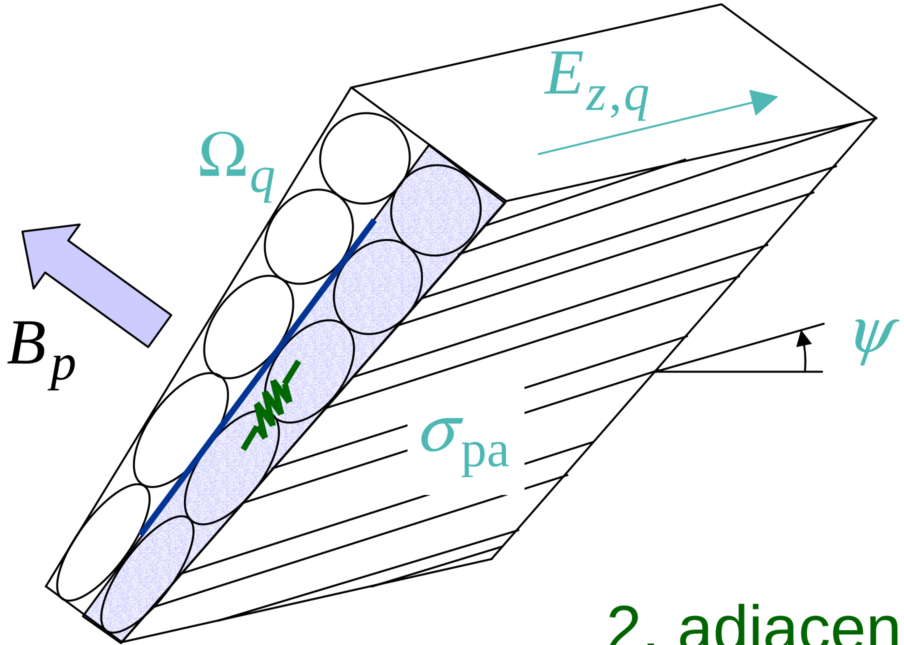
Adjacency Eddy Currents (1)



due to perpendicular magnetic field

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$$J_{\mathrm{pa},z}(r,\theta) = \sigma_{\mathrm{pa}} E_{z,q} - \sigma_{\mathrm{pa}} \frac{\partial A_z}{\partial t}$$

3. netto current through Ω_q = 0

$$I_{z,q} = \int_{\Omega_q} J_{\mathrm{pa},z}(r,\theta)\, d\Omega = 0$$

} *additional constraint !*

Adjacency Eddy Currents (2)

additional load term for magnetic FE model

$$g_{\mathrm{pa}} = M_{\mathrm{pa}} \frac{\partial u}{\partial t} - Z_{\mathrm{pa}} e_{\mathrm{pa}}$$

additional constraint

$$-Z_{\mathrm{pa}}^{\mathrm{T}} \frac{\partial u}{\partial t} + G_{\mathrm{pa}} e_{\mathrm{pa}} = 0$$

degrees of freedom for $E_{z,q}$

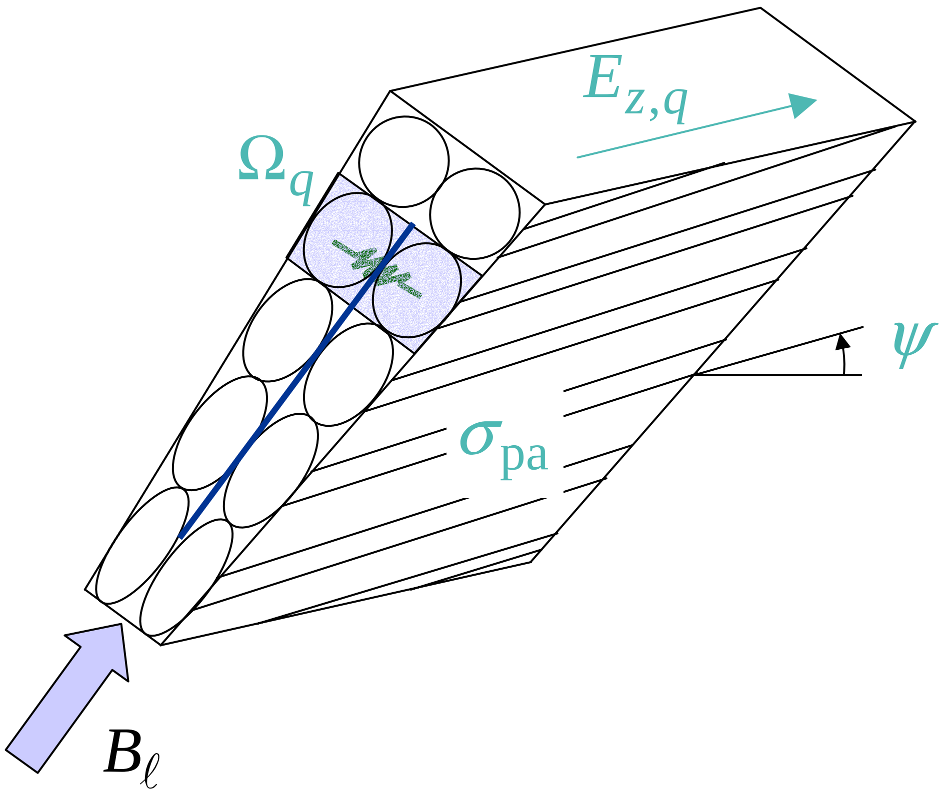
$$\begin{bmatrix} M_{\mathrm{pa}} & 0 \\ Z_{\mathrm{pa}}^{\mathrm{T}} & 0 \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} u \\ e_{\mathrm{pa}} \end{bmatrix} + \begin{bmatrix} K & Z_{\mathrm{pa}} \\ 0 & G_{\mathrm{pa}} \end{bmatrix} \begin{bmatrix} u \\ e_{\mathrm{pa}} \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

$$M_{\mathrm{pa},ij} = \int_\Omega \sigma_{\mathrm{pa}} N_i(x,y) N_j(x,y)\,\mathrm{d}\Omega$$

$$Z_{\mathrm{pa},iq} = \int_\Omega \sigma_{\mathrm{pa}} N_i(x,y) M_q(x,y)\,\mathrm{d}\Omega$$

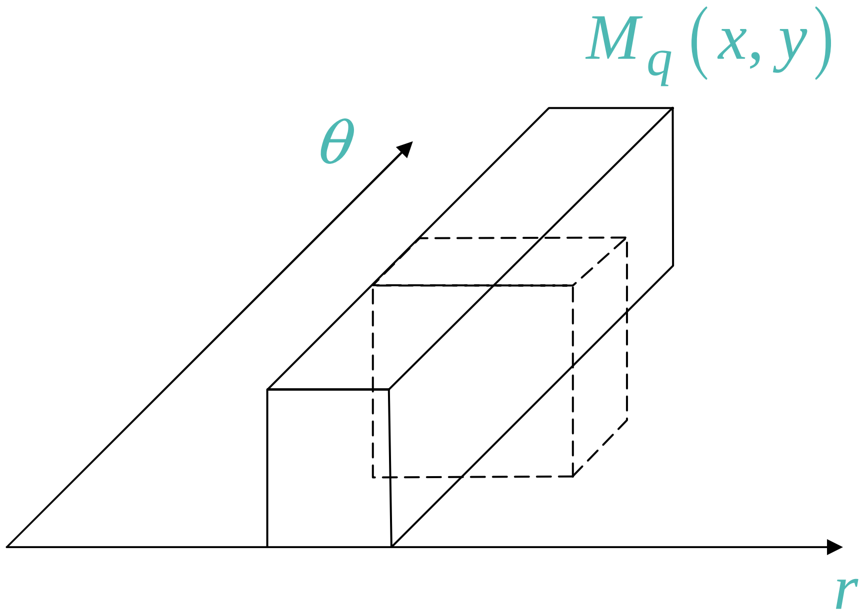
$$G_{\mathrm{pa},pq} = \int_\Omega \sigma_{\mathrm{pa}} M_p(x,y) M_q(x,y)\,\mathrm{d}\Omega$$

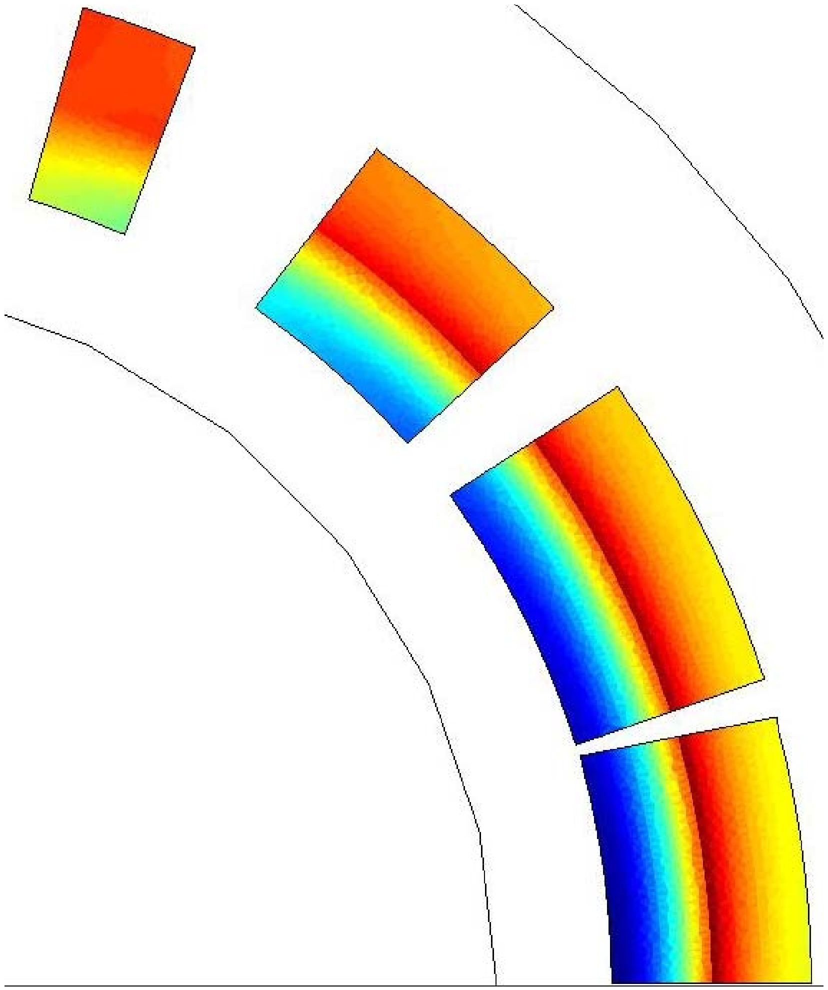
Adjacency Eddy Currents (3)

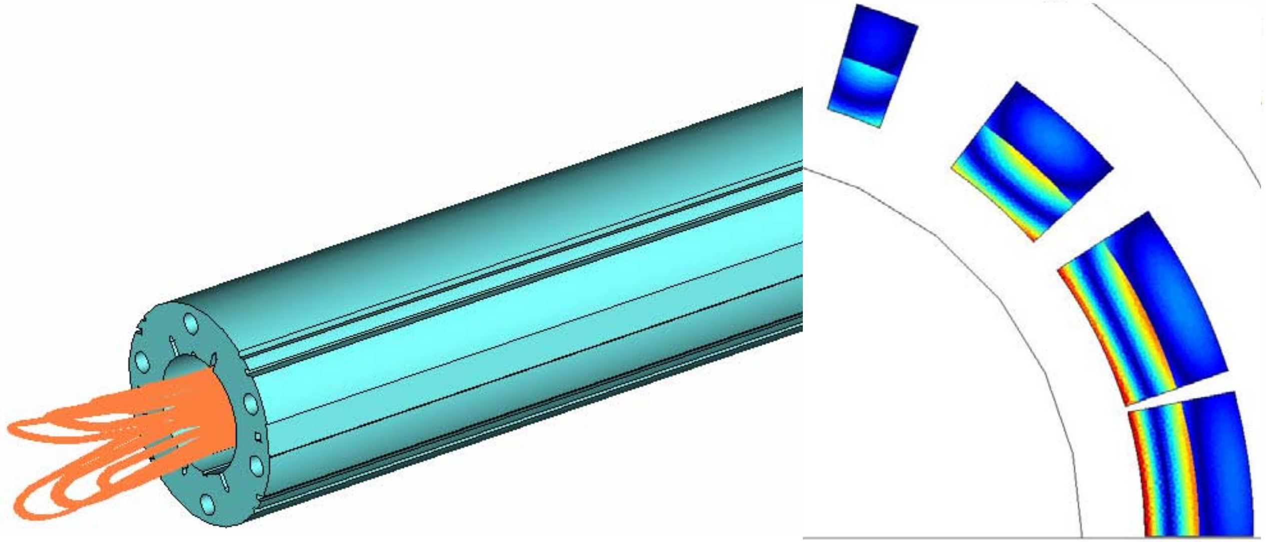


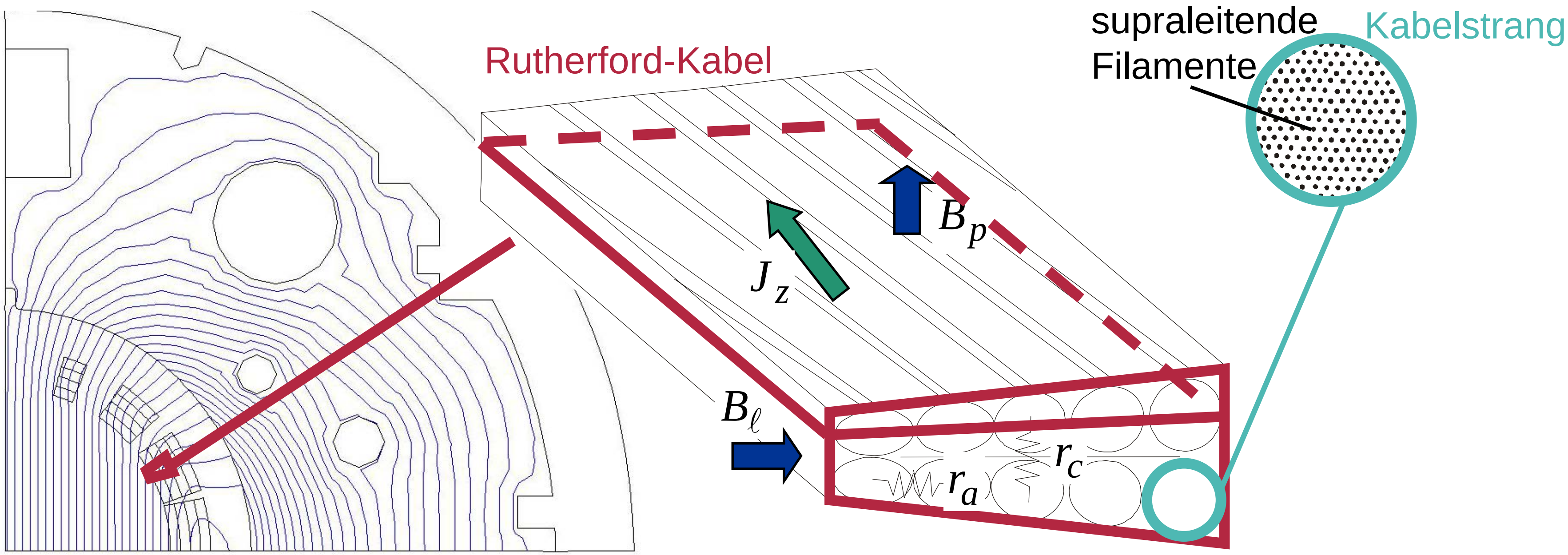
due to parallel magnetic field

shape functions $M_q(x,y)$ related (but not necessarily equal) to the zones of current redistribution








Rutherford-Kabel
supraleitende
Filamente
Kabelstrang
B_p
J_z
B_ℓ
r_a
r_c